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THE UNIVERSITY OF MISSISSIPPI**

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**Compiled by  
Elizabeth A. Furr  
National Center for Physical Acoustics  
The University of Mississippi  
University, MS 38677**

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**ELASTICITY OF LEUCITE THROUGH HIGH-TEMPERATURE PHASE  
TRANSITIONS**

**DONALD ISAAK<sup>1</sup>, ANDY SHEN<sup>2</sup>, ORSON ANDERSON<sup>1</sup>, JOHN CARNES<sup>1</sup>**

**<sup>1</sup>UNIVERSITY OF CALIFORNIA AT LOS ANGELES**

**<sup>2</sup>UNIVERSITY OF CAMBRIDGE, UK**



# **Preview**

- **Elasticity of Leucite**
  - **Phase transition**
- **Rutile**
  - **Info on  $K'$  from high-T RUS**
- **RUS and pressure -  $\text{SiO}_2$**

**Yes**

**No**

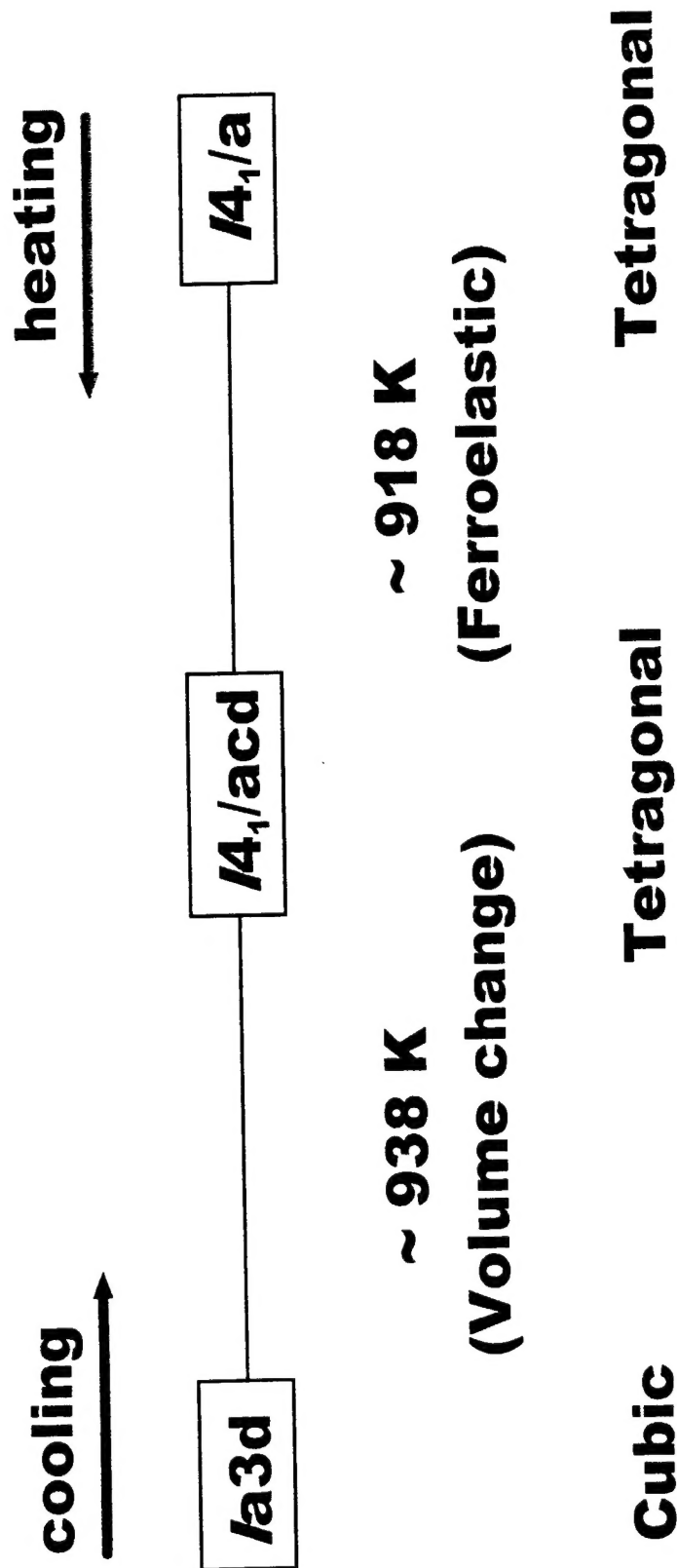
**Leucite**

- ⇒ **White, gray mineral**  
 **$\text{KAlSi}_2\text{O}_6$**
- ⇒ **Important**  
**constituent in alkali**  
**rocks (esp. lava)**

**Lucite**

- ⇒ **Trademark name for**  
**a transparent**  
**synthetic resin:**  
**polymerized methyl**  
**methacrylate**

# Phase Transitions in Leucite



# **Sample Description**

**Locality: Vesuvius, Italy**

**Density: 2.461 g/cm<sup>3</sup>**

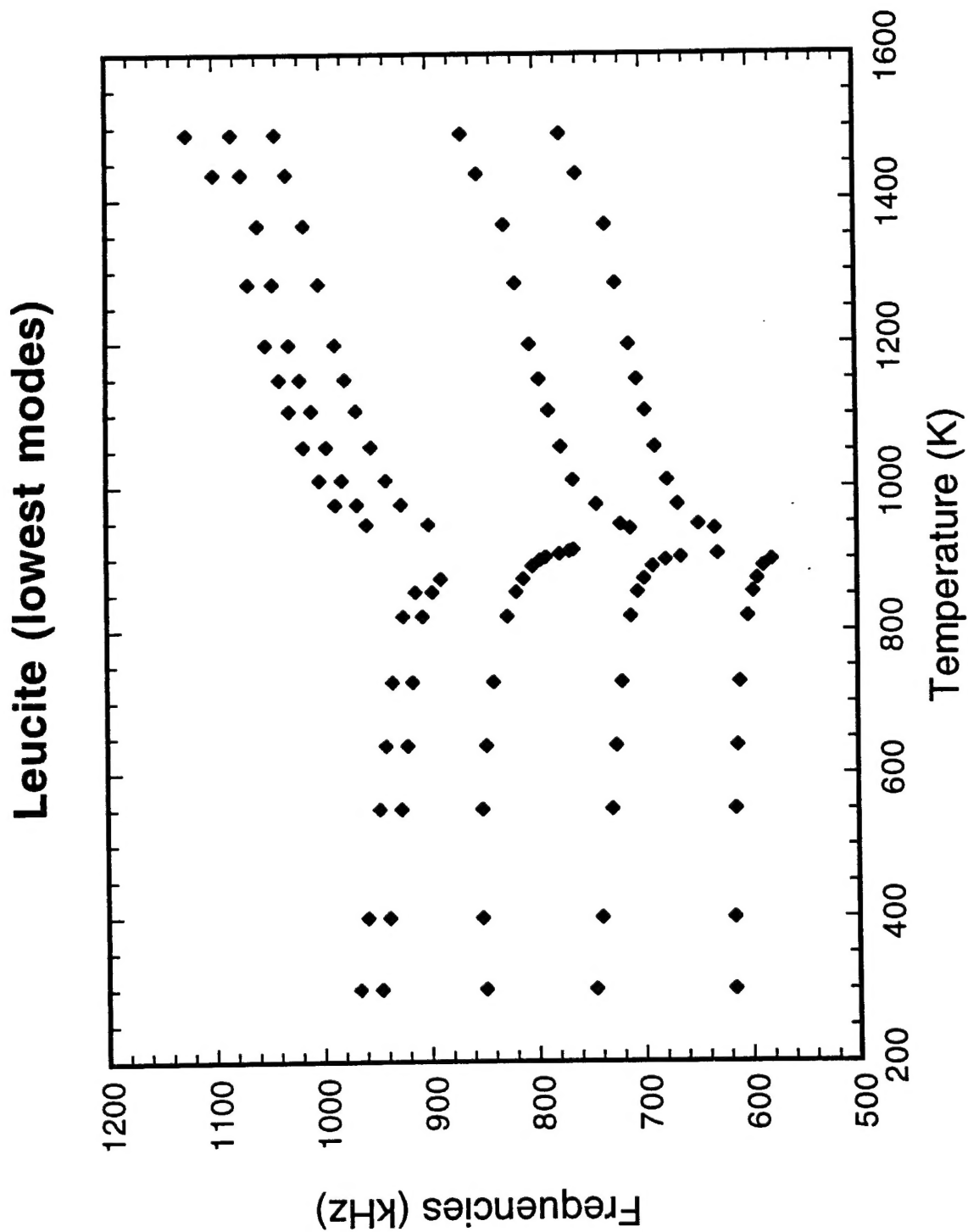
**Dimensions: 1.739 mm x 1.944 mm x 1.969 mm**

**Starting elastic moduli for the high temperature phase (*la3d*)**

**$C_{11} = 170$  GPa**

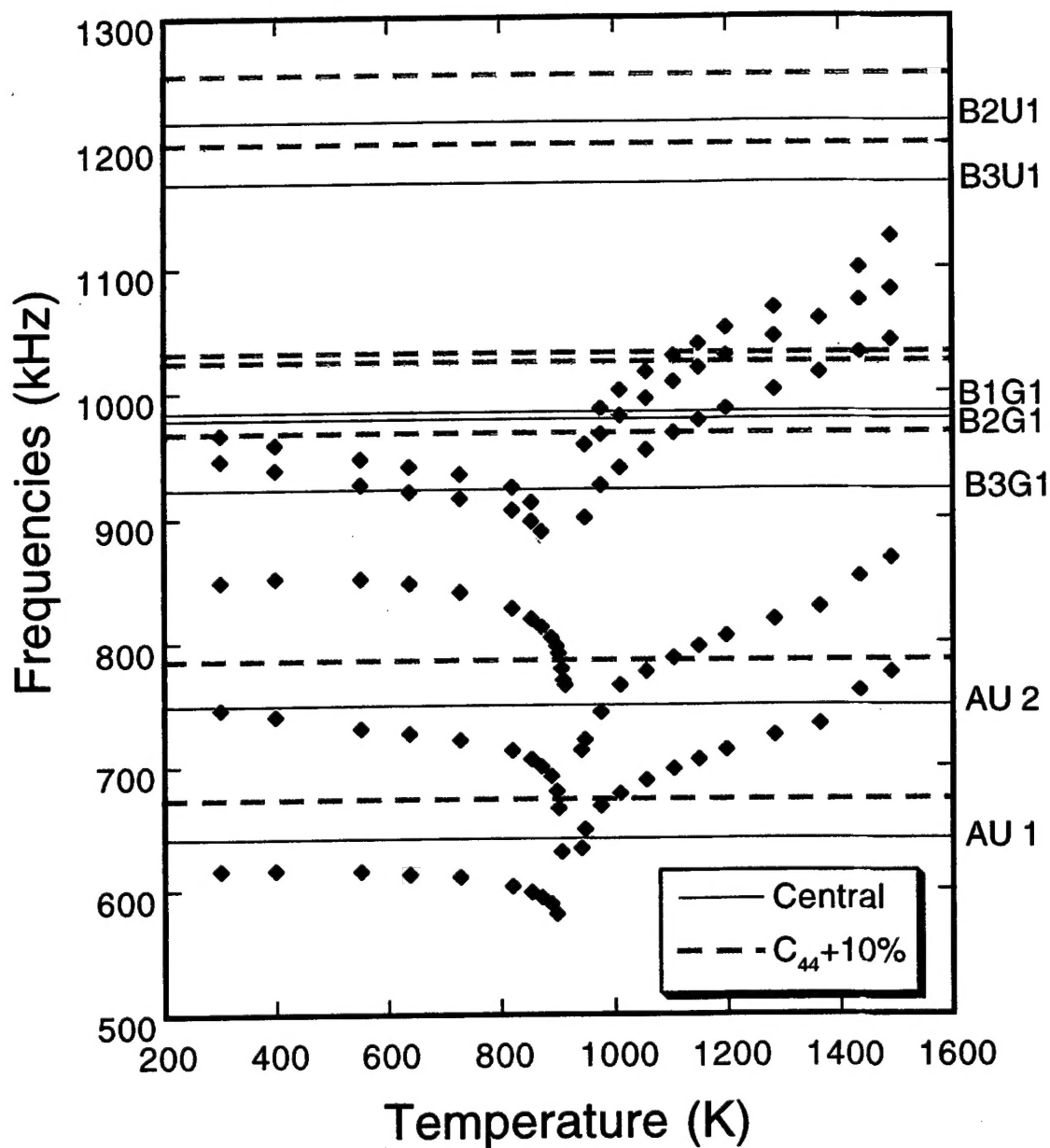
**$C_{12} = 67$  GPa**

**$C_{44} = 20$  GPa**

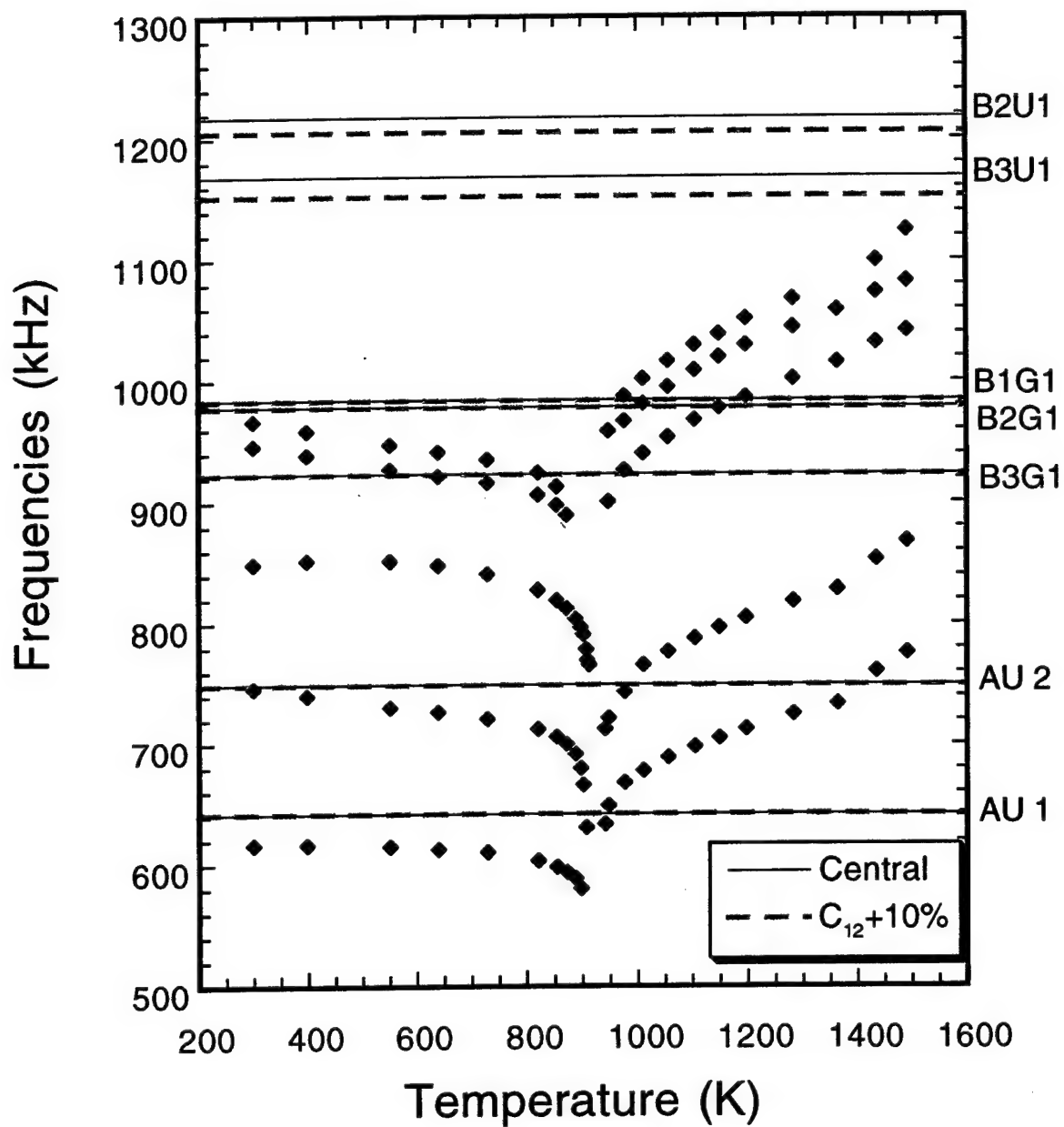


TR-5

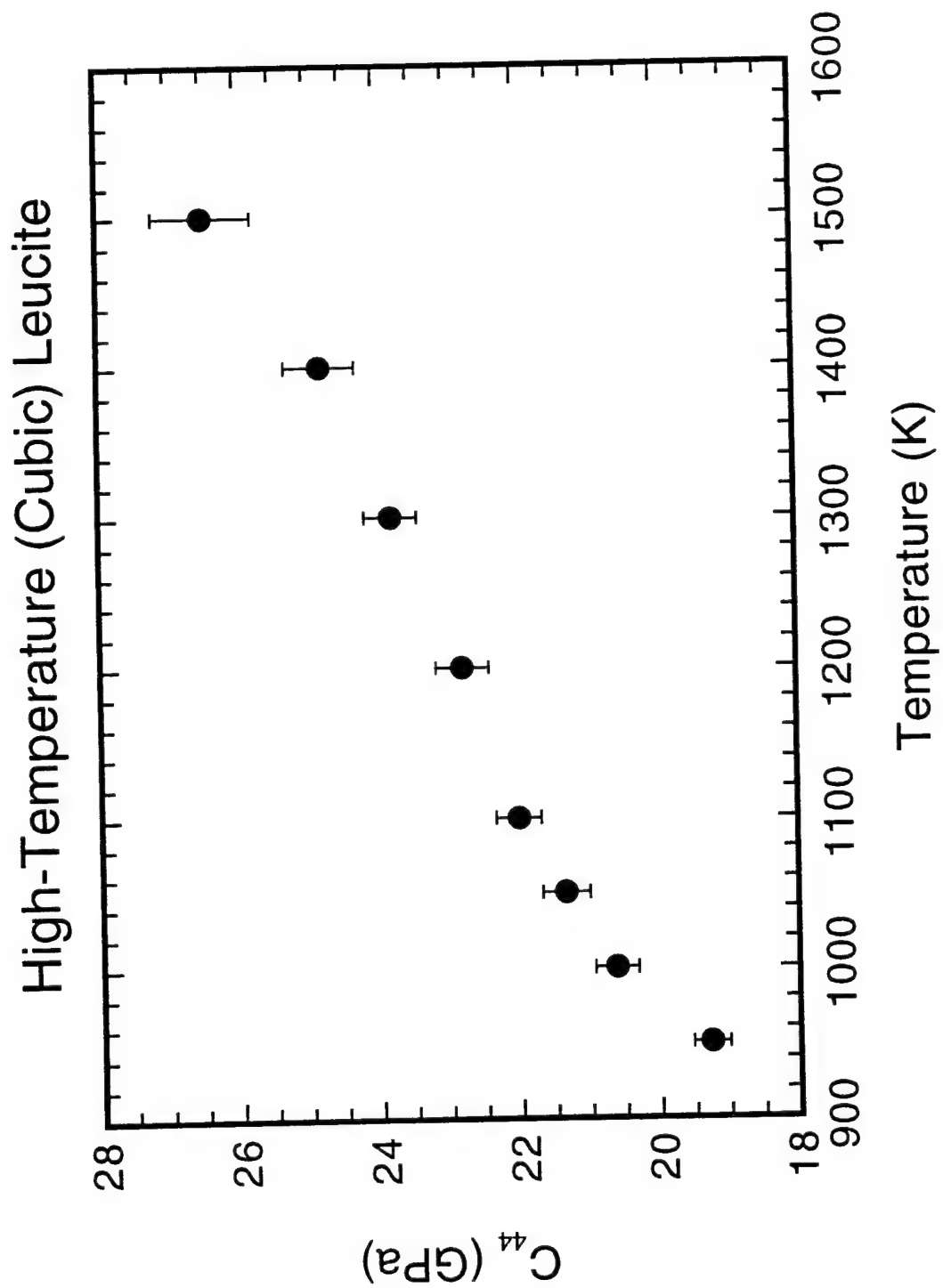
**- Leucite -**  
**Variation of Frequencies with  $C_{44}$**



# **- Leucite -** **Variation of Frequencies with $C_{12}$**

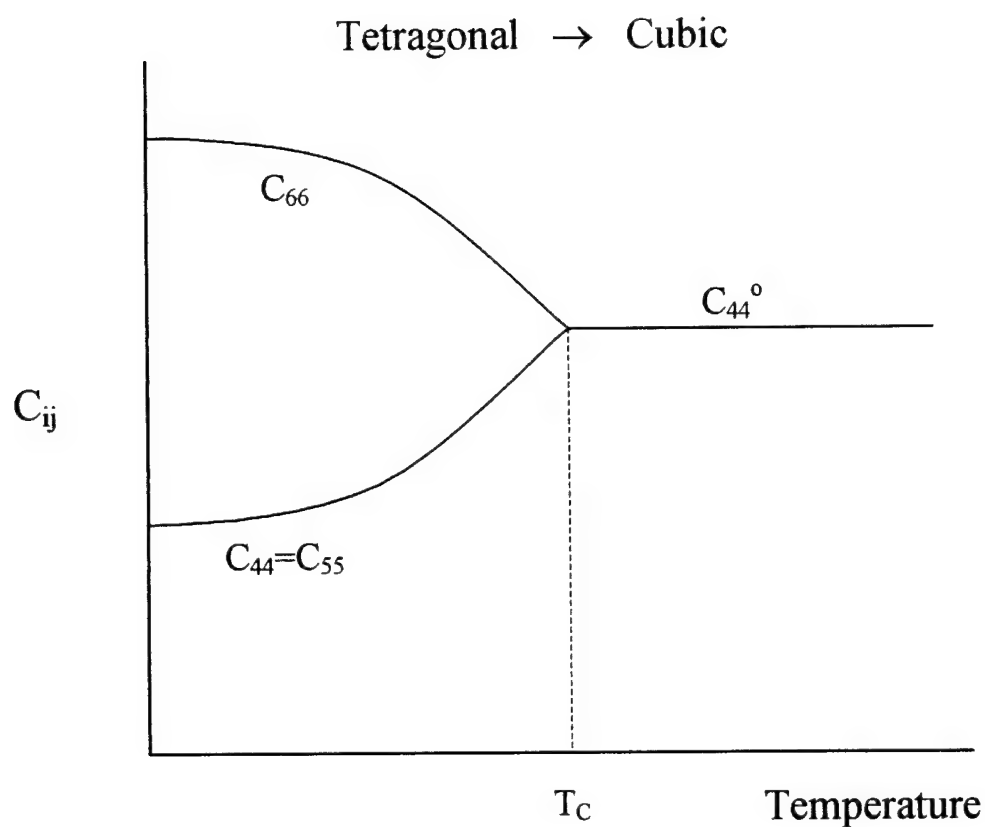


TR-7



TR-8





After  $T_C$ :

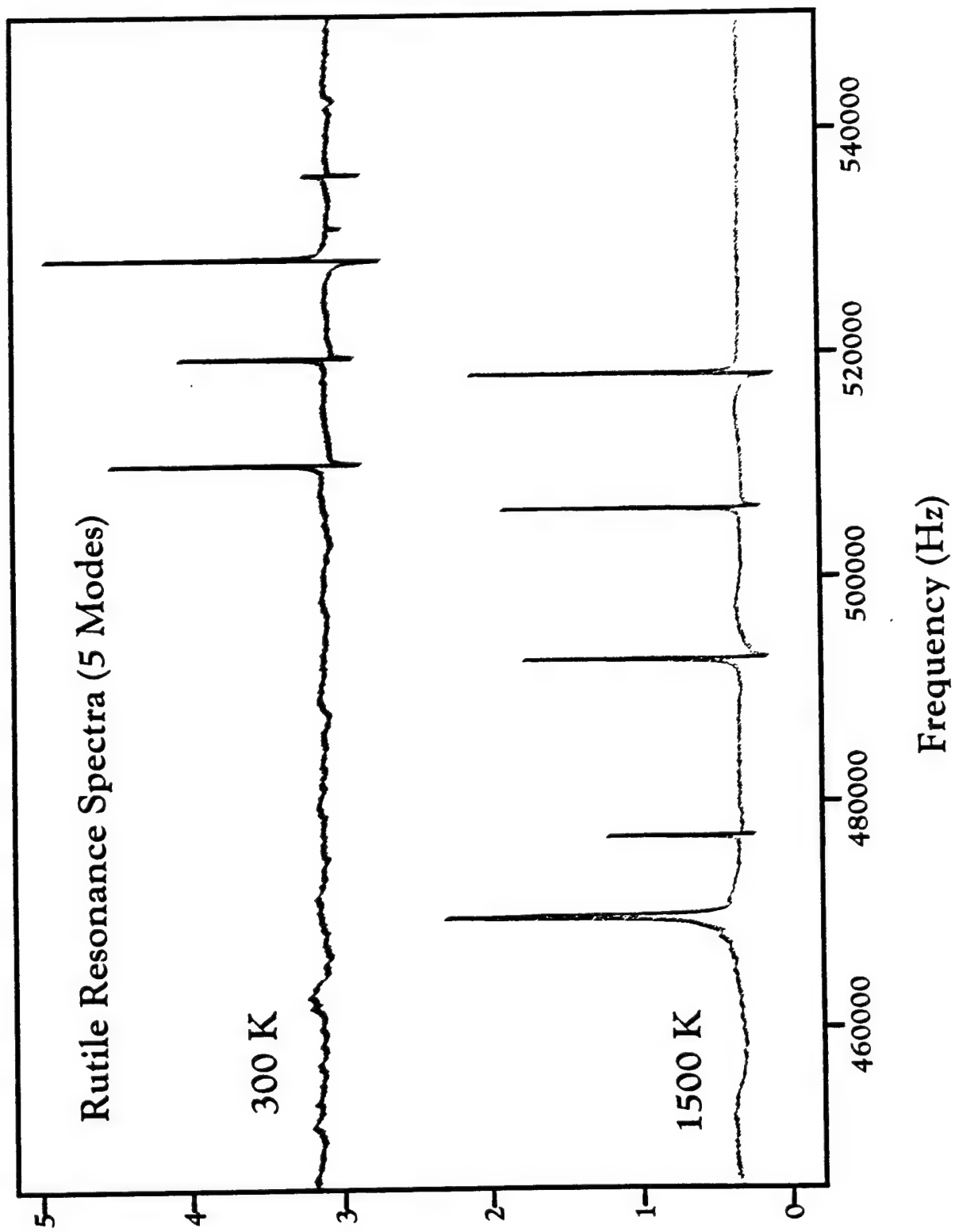
$$C_{44} = C_{55} = C_{66} = C_{44}^0$$

$C_{44}^0$  is the normal  $C_{44}$  without influence from transition

(See Carpenter et al., *Eur. J. Mineral.*, 10, 621-691, 1998)

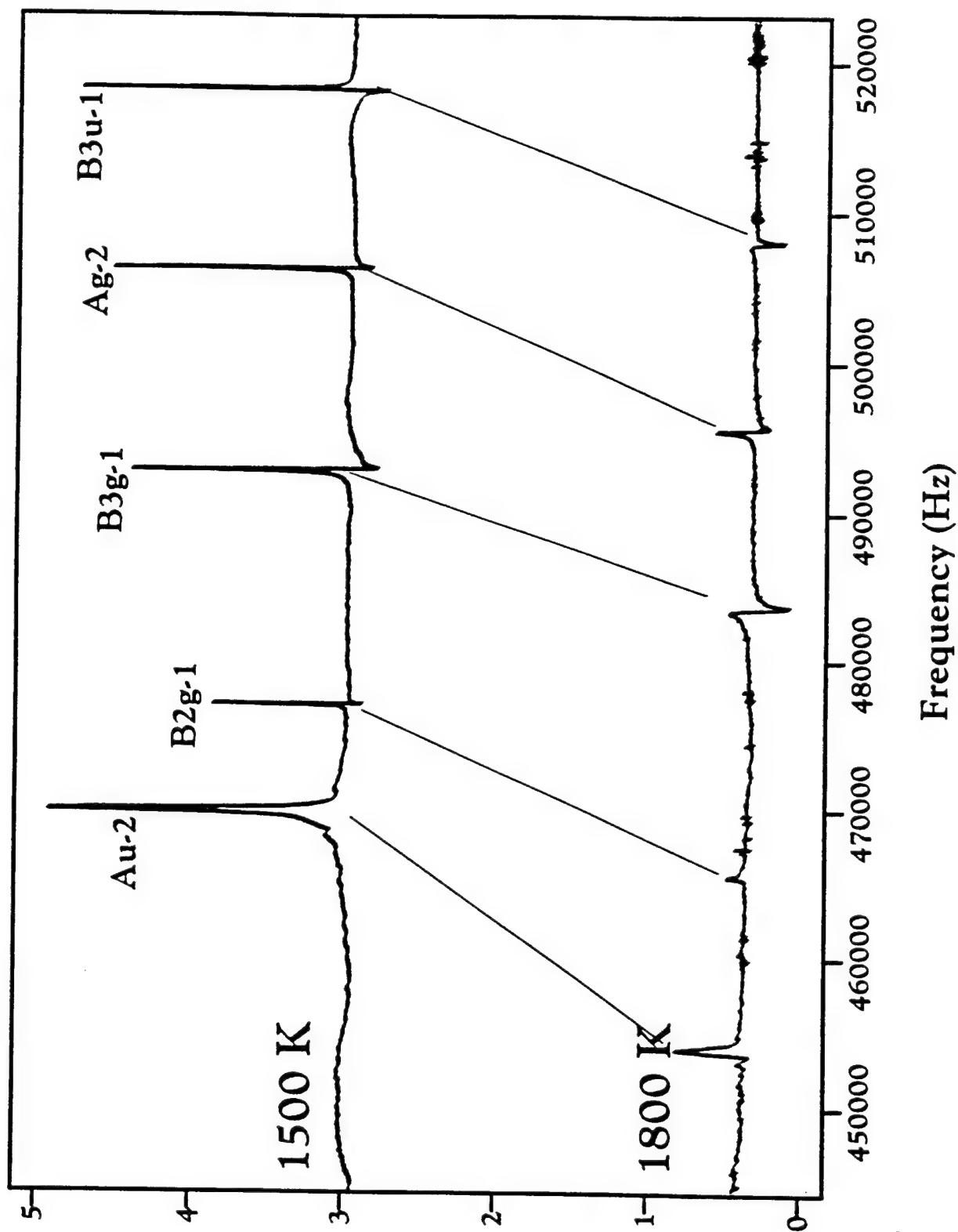
Our Observations with Leucite:

- Wide range of  $T$  over which  $T_C$  influences  $C_{44}$  in leucite  
(Compare to Migliori et al. 1993)
- $C_{44}$  does not equal  $C_{44}^0$  unless  $dC_{44}/dT > 0$  (which seems implausible)



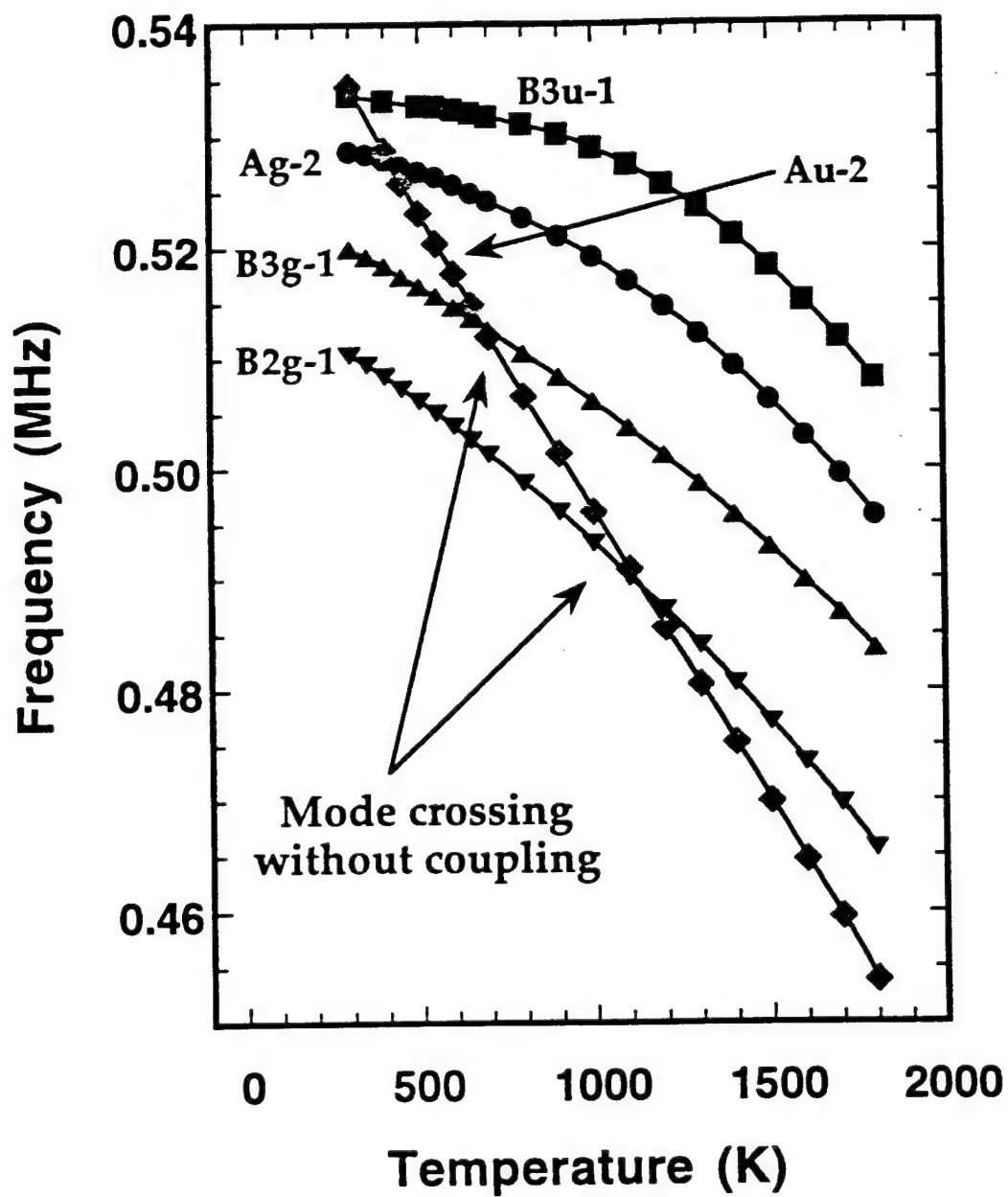
TR-10

# Rutile Resonance Spectra (5 Modes)



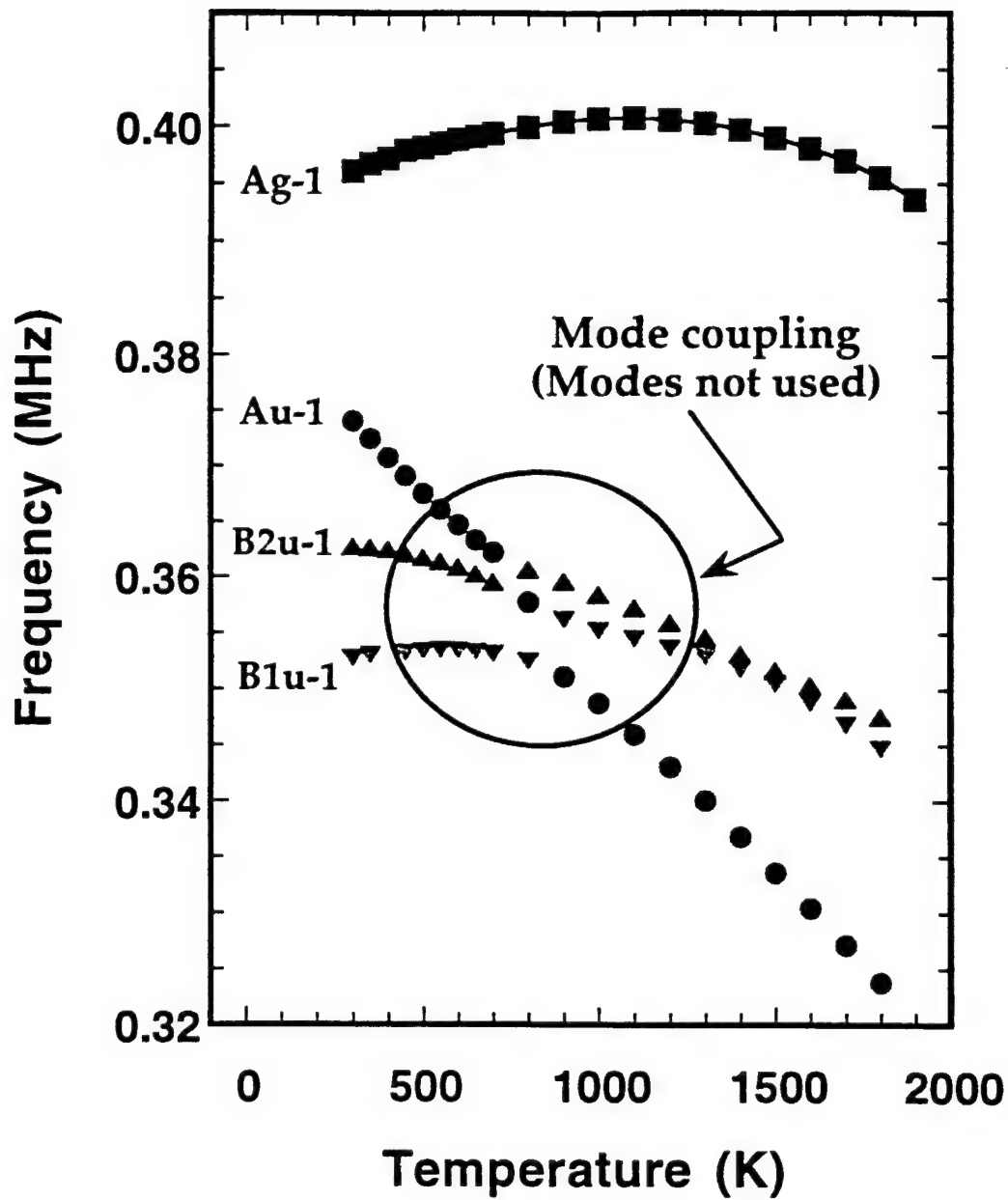
TR-11

## Temperature Variation of Modes

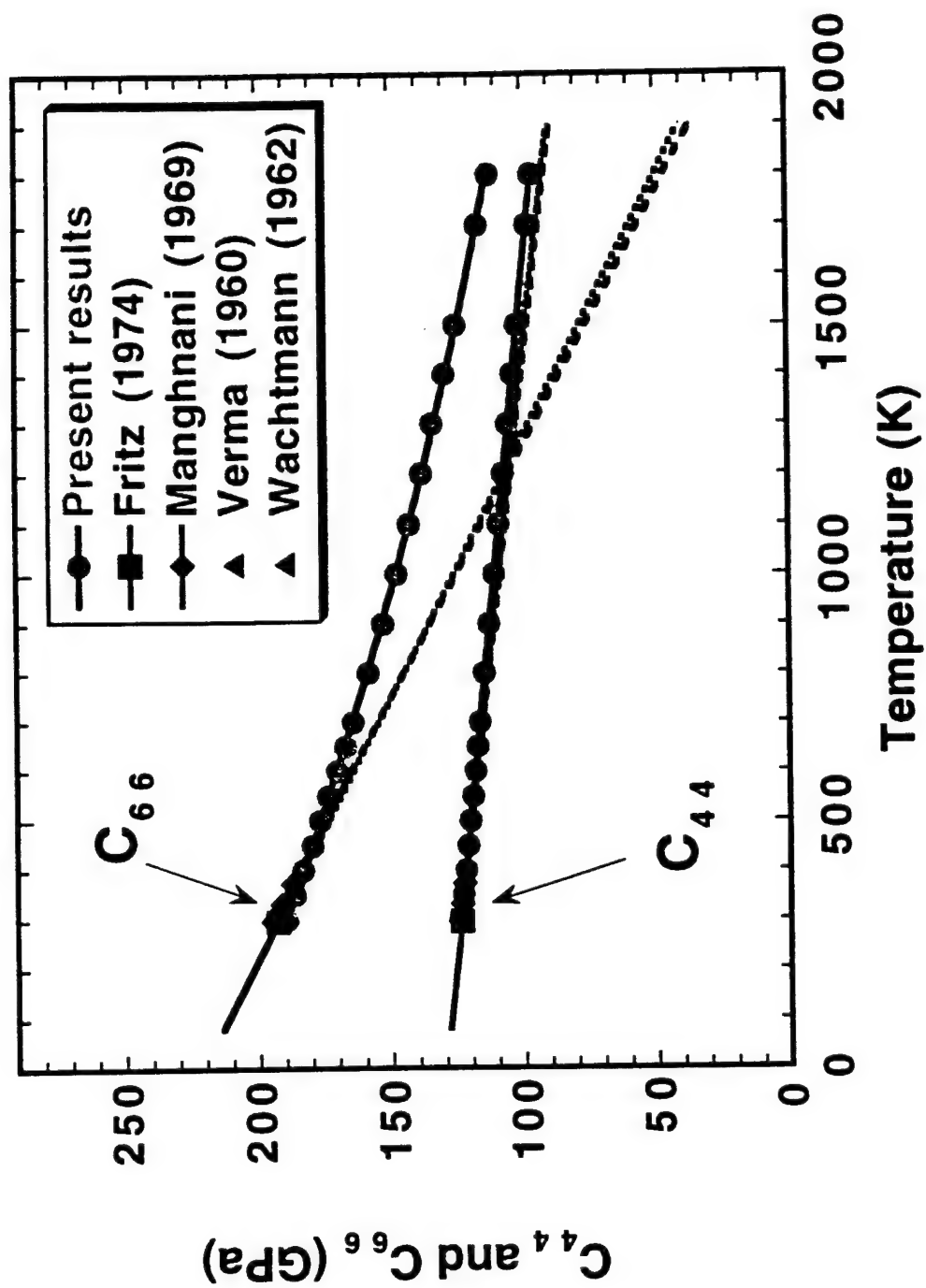


TR-12

## Temperature Variation of Modes

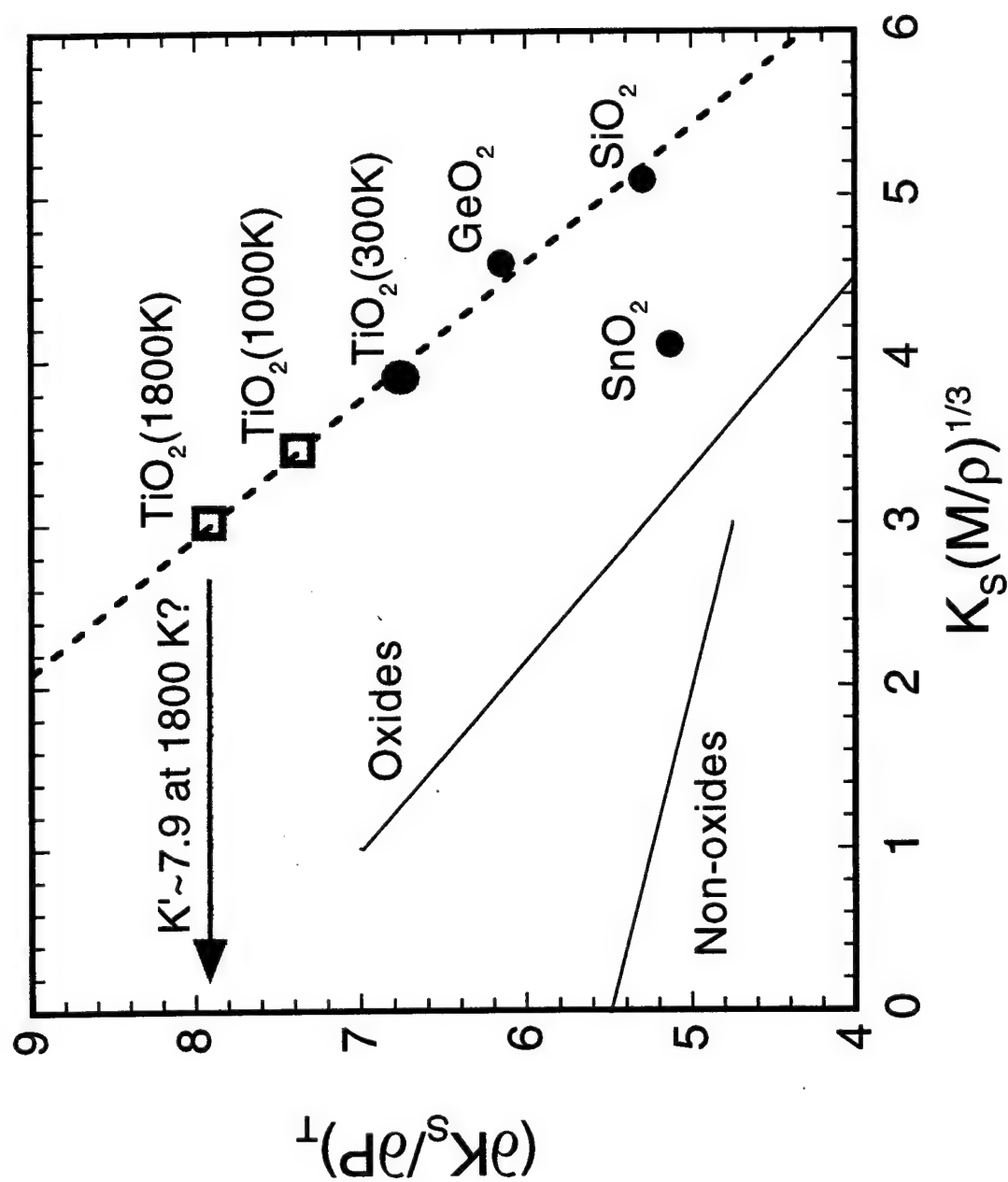


TR-13

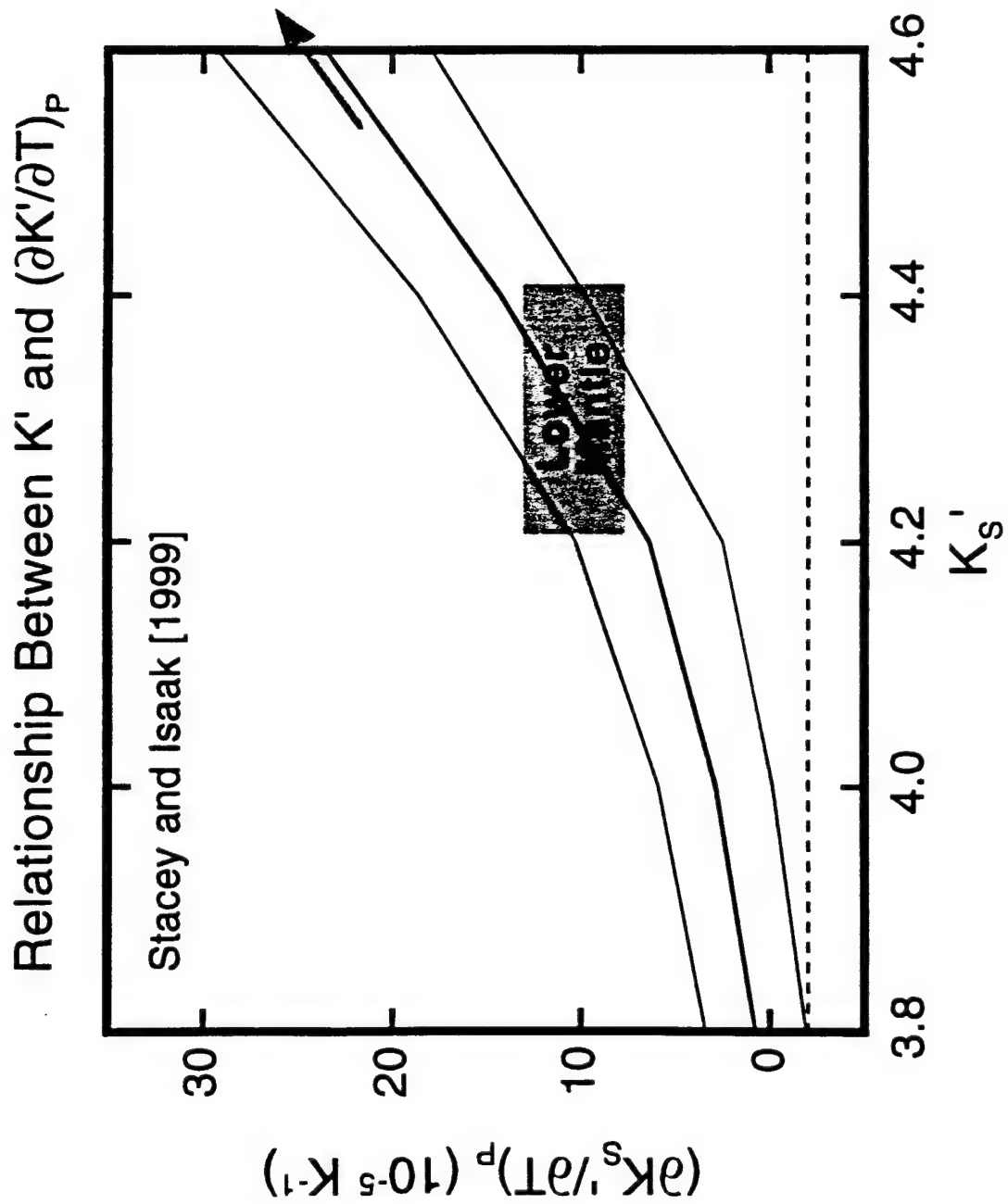


TR-14

# Systematics in Rutile Structure

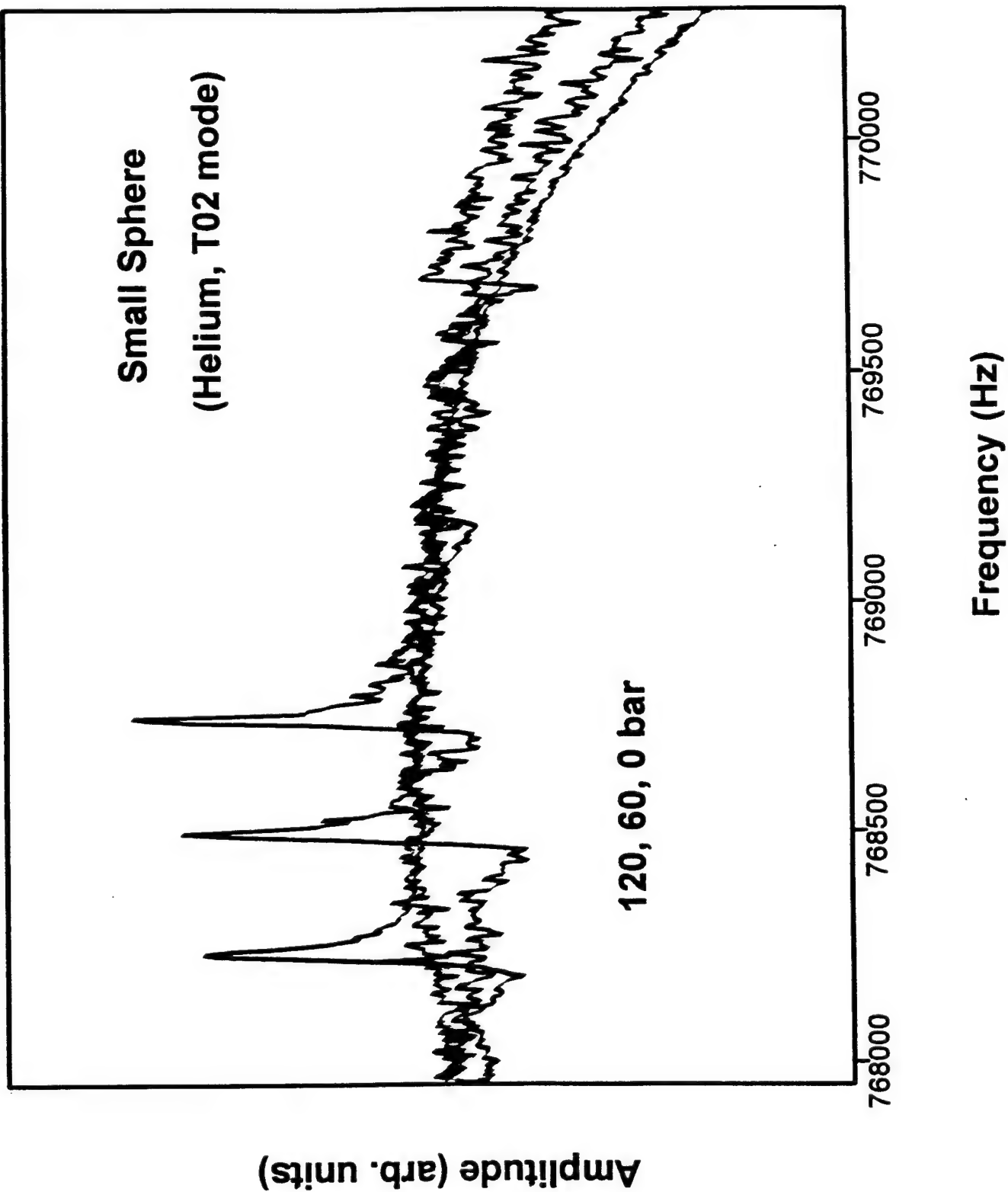


TR-15



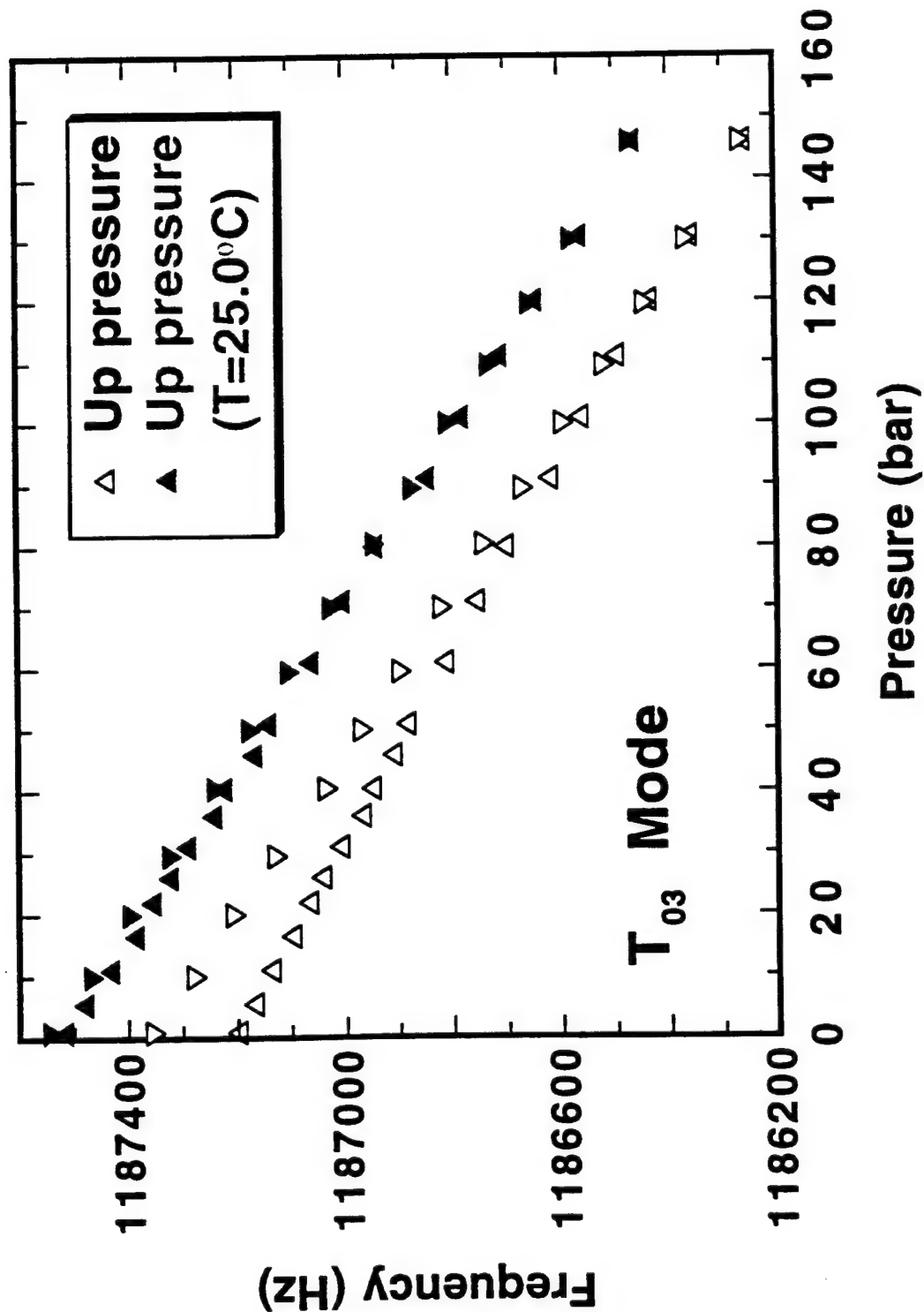
TR-16



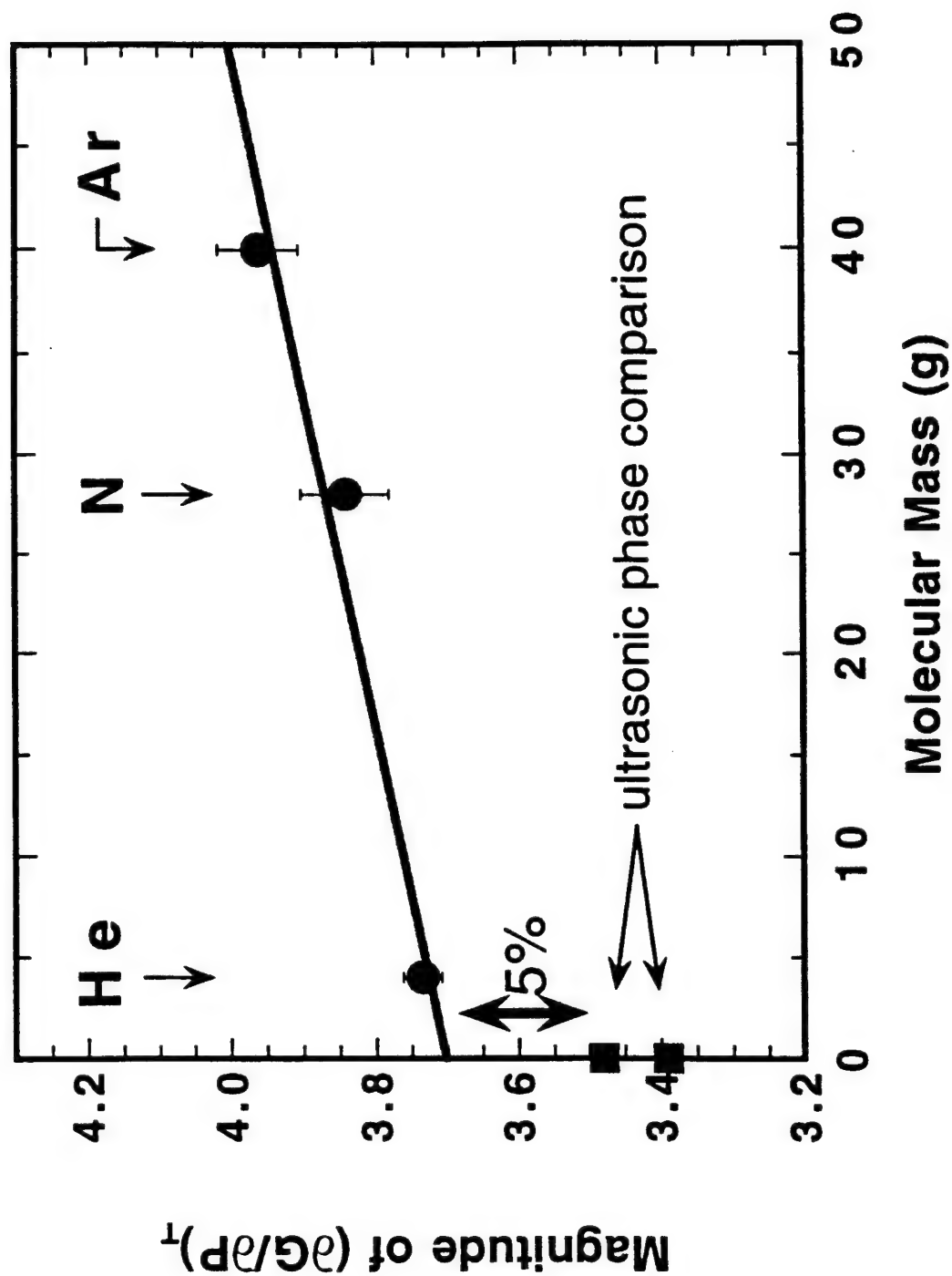


TR-17

# Pressure Dependence of Frequencies - Nitrogen Gas -



# Dependence of $(\partial G/\partial P)_T$ on Molecular Mass of Gas - Helium, Nitrogen, Argon -



TR-19

# Basic Equations

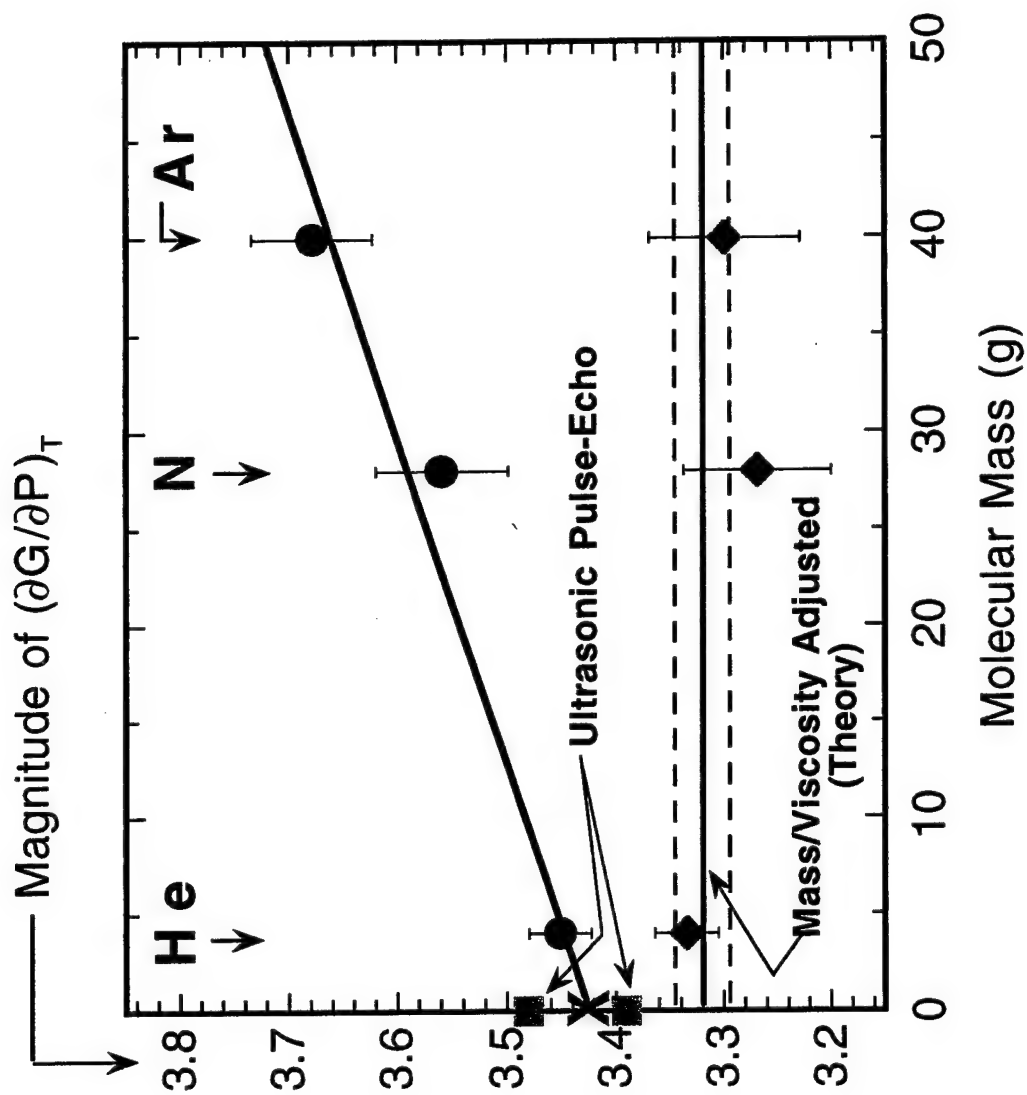
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- Frequency,  $f$ , depends on  $G$  and  $r$

$$\delta \mathbf{f} = \left( \frac{\partial \mathbf{f}}{\partial \mathbf{G}} \right)_{r,m} \delta \mathbf{G} + \left( \frac{\partial \mathbf{f}}{\partial \mathbf{r}} \right)_{G,m} \delta \mathbf{r}$$

- Include pressure,  $P$ , and do algebra

$$\frac{dG}{dP} = \frac{\frac{df}{dP} + \frac{f}{6K_T}}{\left( \frac{\partial \mathbf{f}}{\partial \mathbf{G}} \right)_{r,m}}$$



TR-21

$$\delta f = \left(\frac{\partial f}{\partial G}\right)_{r,m} \delta G + \left(\frac{\partial f}{\partial r}\right)_{G,m} \delta r + \left(\frac{\partial f}{\partial m}\right)_{G,r} \delta m$$

$$\frac{dG}{dP} = \frac{\frac{df}{dP} + \frac{f}{6K_T} + \left(\frac{f}{2m}\right)\left(\frac{dm}{dP}\right)}{\left(\frac{\partial f}{\partial G}\right)_{r,m}}$$

where  $\frac{dm}{dP} = \frac{d(\rho_g V_g)}{dP}$

$$= \frac{2\sqrt{\pi} r^2}{P} \sqrt{\frac{\rho_g \eta_g}{f}}$$

# Resonant Ultrasound Spectroscopy of Metal-Hydrogen Systems

Bob Leisure  
*Department of Physics*  
*Colorado State University*

## Acknowledgments

Keir Foster - *Colorado State University*  
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Alexander Skripov - *Ekaterinburg, Russia*  
Albert Migliori - LANL

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TR-1

## OUTLINE

### A. Background

1. Metal-hydrogen systems
2. Hydrogen motion and ultrasonic attenuation

### B. Measurements on a Laves phase material, $\text{TaV}_2$ , containing hydrogen(deuterium)

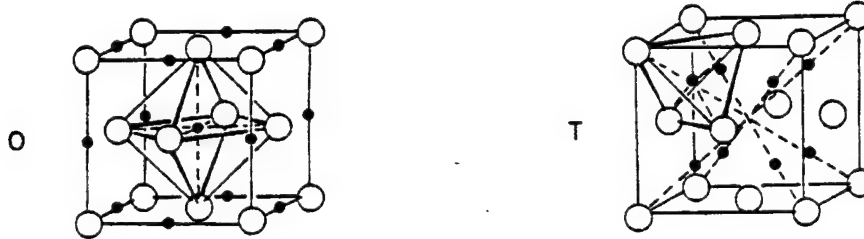
### C. Measurements on a Ti-Zr-Ni quasicrystal and a Ti-Zr-Ni crystalline approximant containing hydrogen

### D. Summary



# METAL-HYDROGEN SYSTEMS

- Many metals absorb large amounts of hydrogen.  
e.g. transition metals, rare earths, actinides
- H typically sits on interstitial sites of host metal lattice.

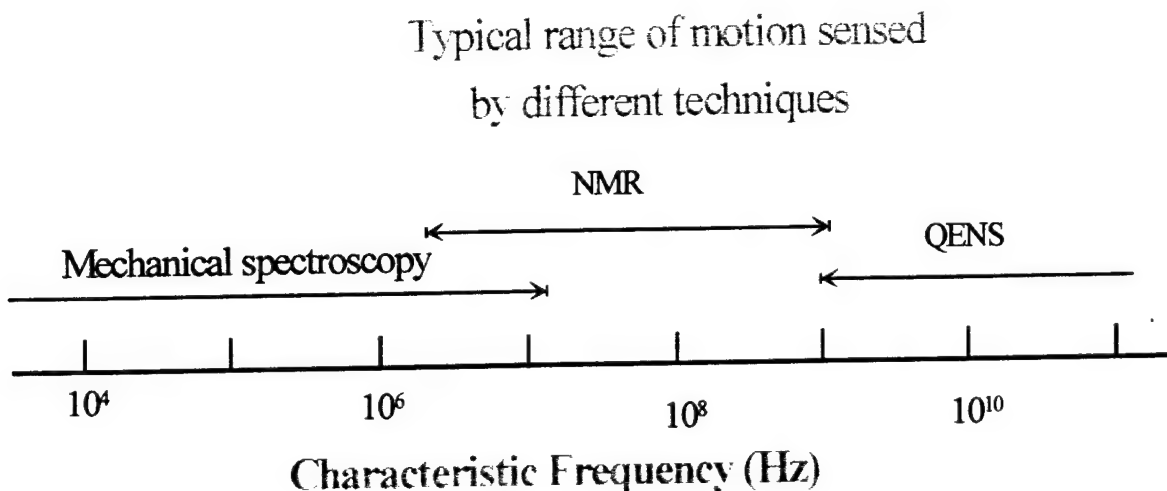


Octahedral and tetrahedral sites in the fcc lattice

- Site occupancy may be random (solid solution), or ordered (hydrides).
- H is often highly mobile and diffuses by hopping between interstitial sites.
- Many physical effects and practical applications are associated with hydrogen in metals, e.g.
  - Hydrogen storage
  - Metal hydride batteries
  - Switchable optical and magnetic properties
  - Hydrogen embrittlement

## HYDROGEN MOTION

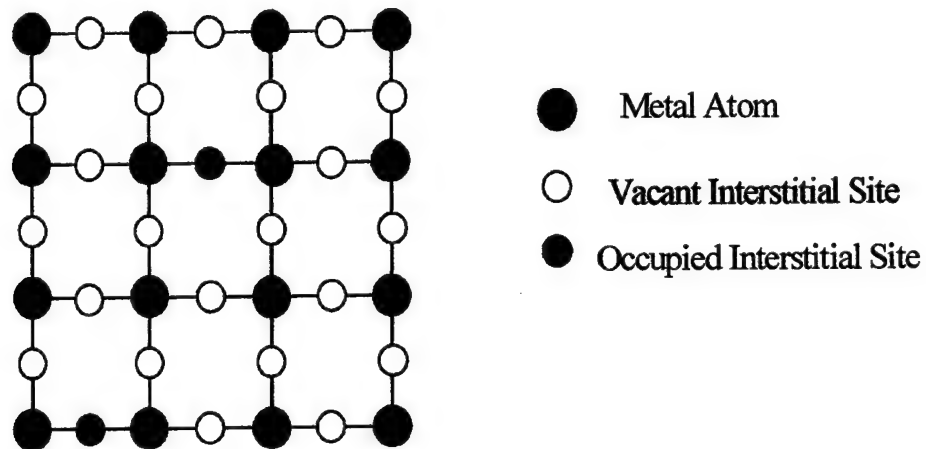
- H motion is studied by a variety of techniques.



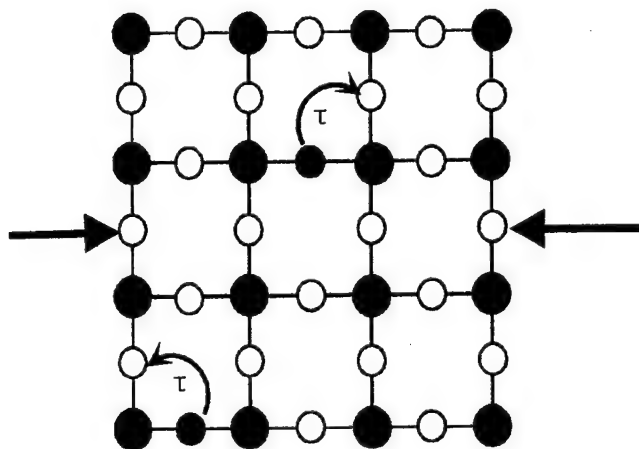
- Ultrasonic attenuation arises due to a stress-induced rearrangement of H on the interstitial sites (relaxation).
- Defect symmetry must be lower than crystal symmetry for relaxation attenuation to occur (*Anelastic relaxation in crystalline solids*, Nowick and Berry).

## Illustration of relaxation effects for sites on the face of a simple cubic lattice

I. With no stress, all interstitial sites energetically equal



II. With stress, sites no longer energetically equal, rearrangement of interstitial atoms occurs.



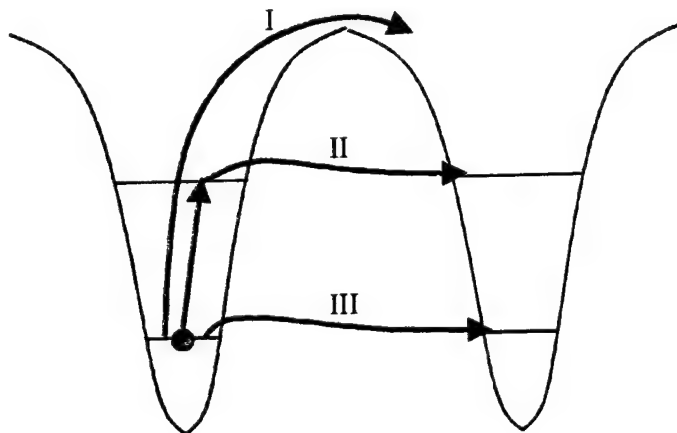
TR-5

## RELAXATION ATTENUATION

- The loss is given by

$$\frac{1}{Q} = \frac{\Delta C}{C} \frac{\omega \tau_R}{1 + \omega^2 \tau_R^2} \quad . \text{ Usually } \frac{\Delta C}{C} \propto \frac{1}{T}$$

- Hydrogen can "hop" between sites by several different processes.



- Classical Arrhenius behavior may be observed at high temperatures, process I.

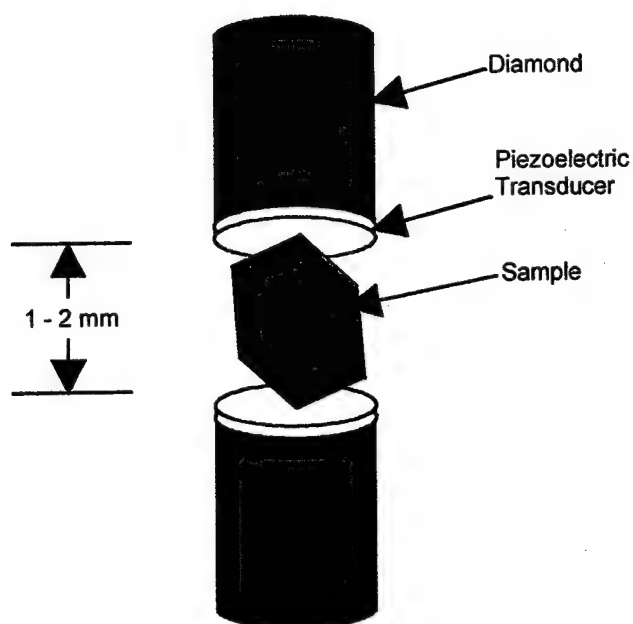
$$\tau_R = \tau_{R0} \exp(E_a / kT)$$

A distribution of  $E_a$  is frequently required.

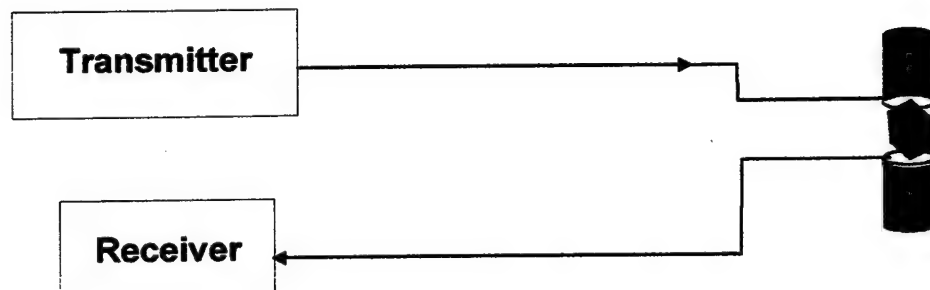
- Deviation from classical behavior is common, especially at lower temperatures.

# Resonant Ultrasound Spectroscopy

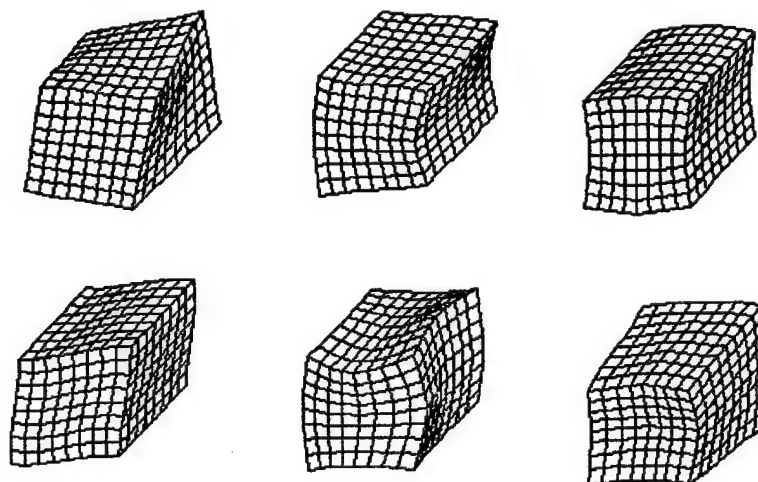
## I. Sample-Transducer Arrangement

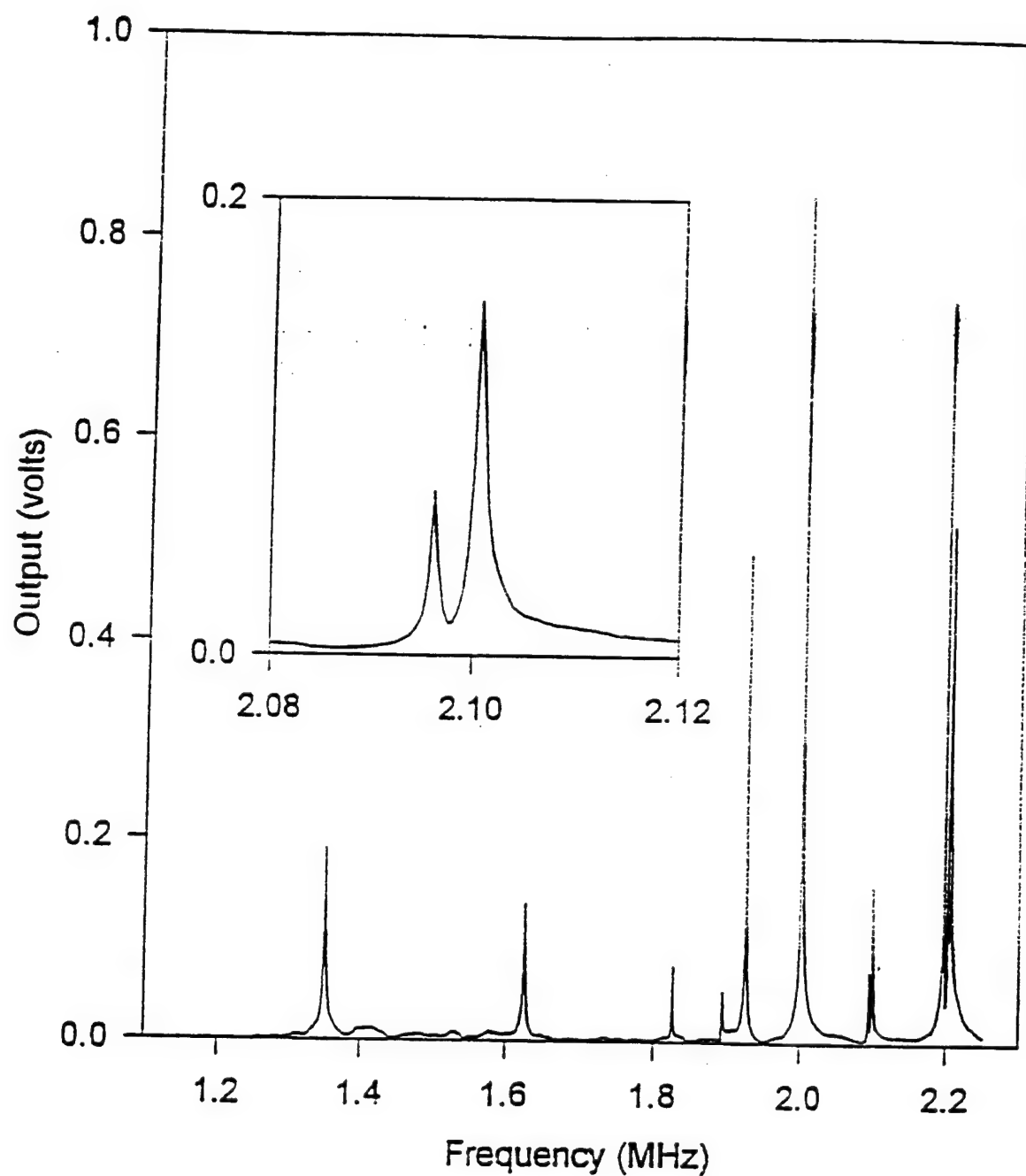


## II. Spectrometer



## III. Typical Modes

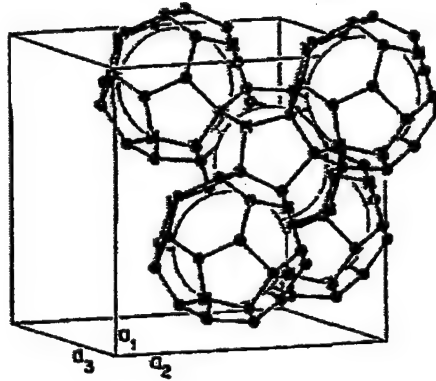




TR-8

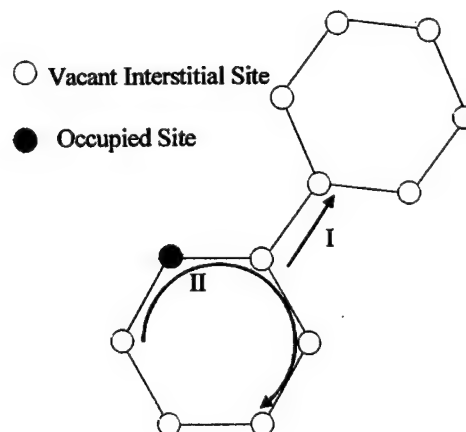
## The C-15 Laves-phase compound $\text{TaV}_2$

- Many C-15 materials, including  $\text{TaV}_2$ , absorb large amounts of hydrogen.
- The hydrogen is highly mobile.
- Large number of tetrahedral sites.



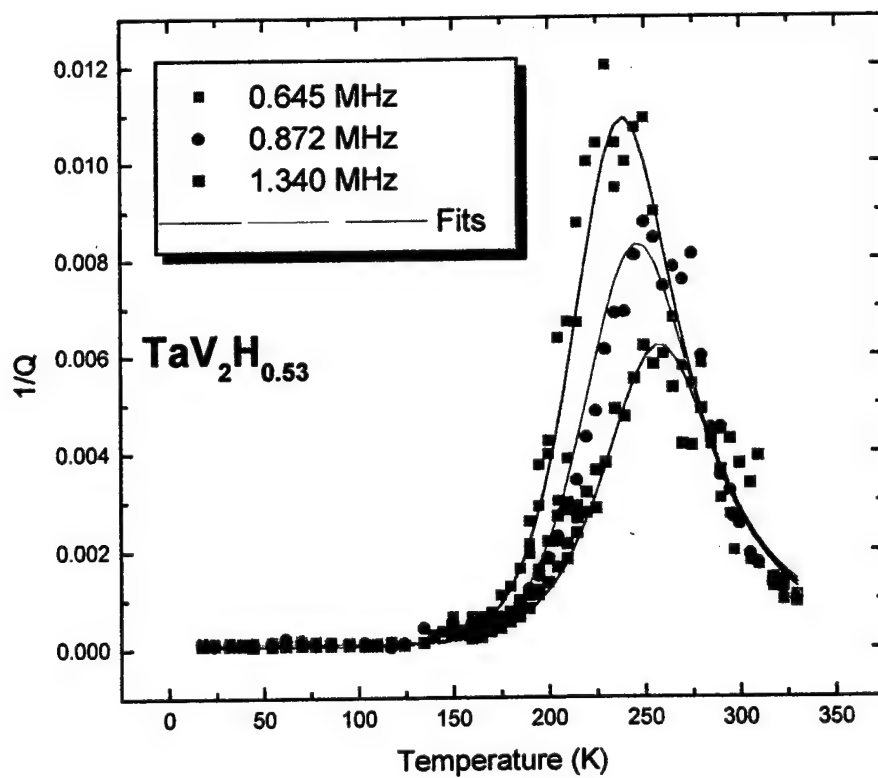
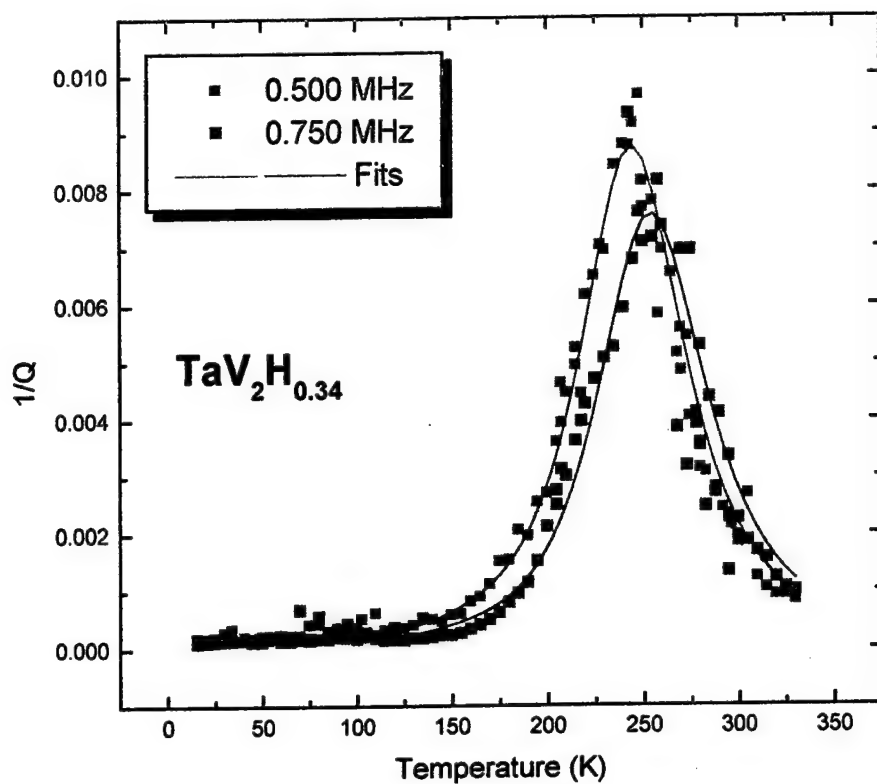
Interstitial sites in  $\text{TaV}_2$

- Previous neutron scattering and NMR work suggests two types of motion in these materials: I. "Slow", Long range; II. "Fast", Local



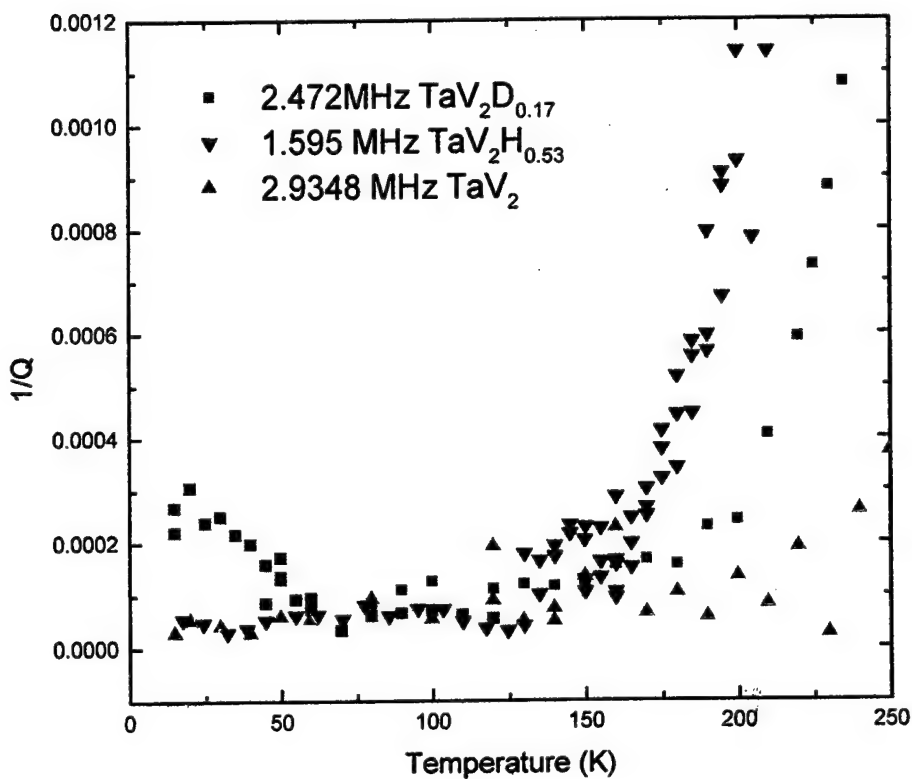
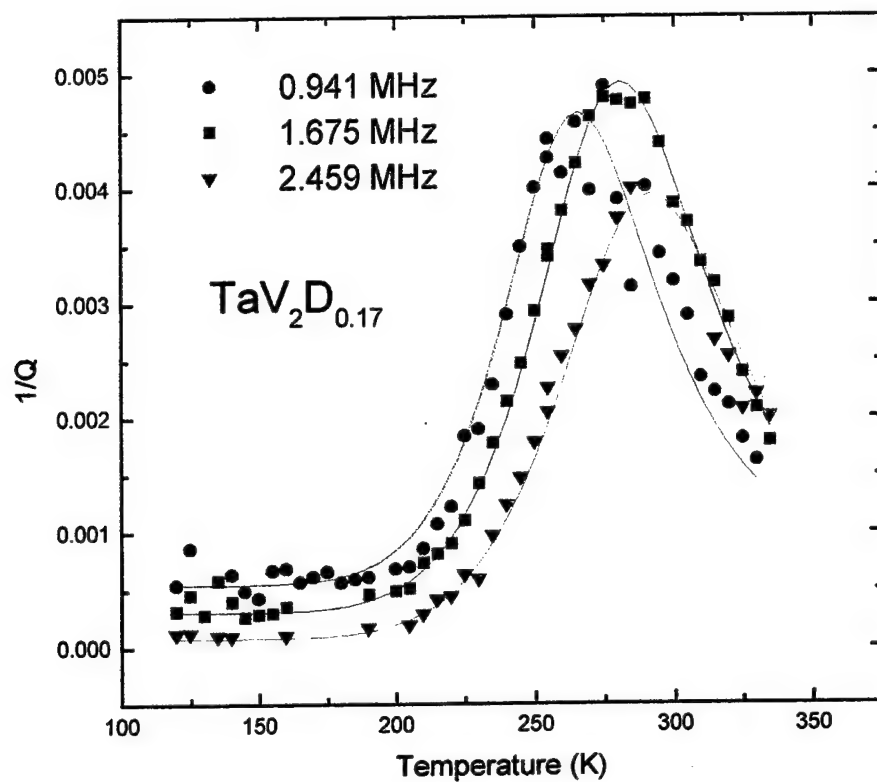
2-D representation of  $\text{H(D)}$  in  $\text{TaV}_2$

- Objective of present study - Investigate this motion in a different frequency range using RUS



TR-10





TR-11

## Ti-based quasicrystals and crystalline approximants

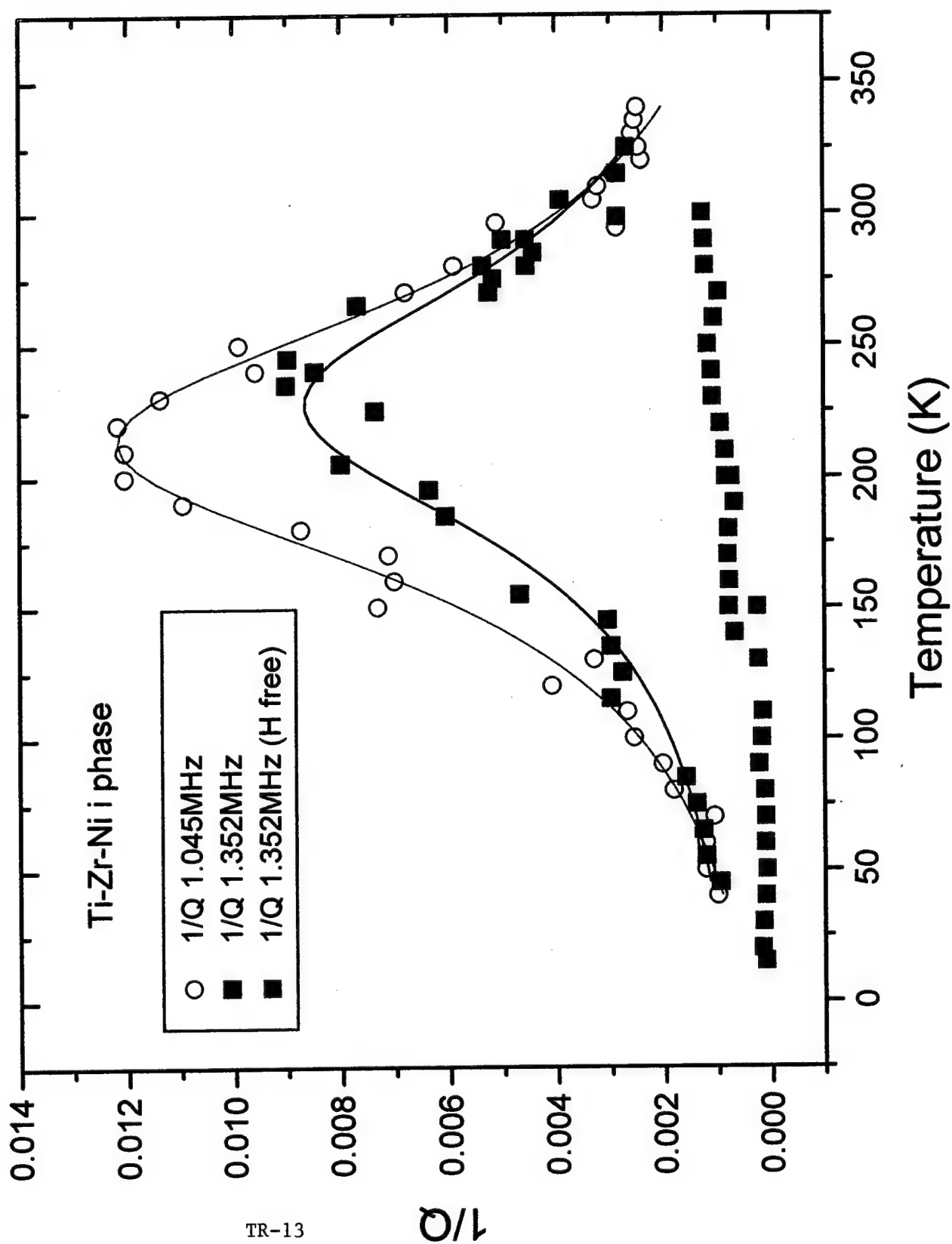
- Can absorb H up to  $H/M = 1.6$
- Little previous study of H motion in such materials.
- Offers opportunity to study diffusion in a non-periodic lattice.
- Materials studied

i-phase quasicrystal  $\text{Ti}_{41.5}\text{Zr}_{41.5}\text{Ni}_{17}$   
Loaded with hydrogen to  $H/M = 0.79$

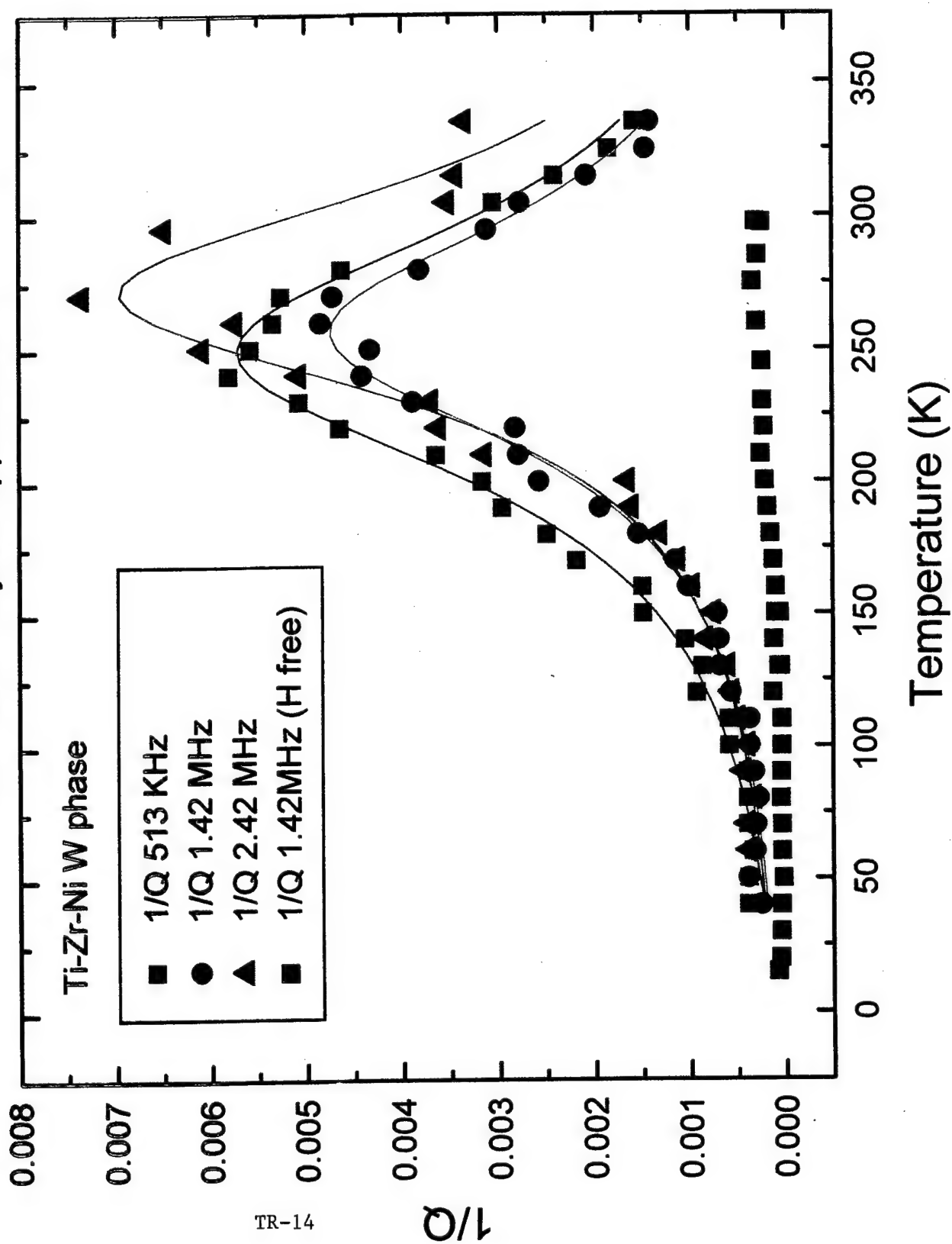
W-phase 1/1 approximant  $\text{Ti}_{44}\text{Zr}_{40}\text{Ni}_{16}$   
Loaded with hydrogen to  $H/M = 0.20$

- H enters in solid solution.

# Ultrasonic loss data versus temperature for Ti-Zr-Ni phase quasicrystal.



# Ultrasonic loss data versus temperature for Ti-Zr-Ni 1/1 crystal approximant.



## SUMMARY

- Evidence for two types of motion in  $\text{TaV}_2\text{D}_x$  with different characteristic frequencies.
- First ultrasonic measurements of H motion in quasicrystals. Evidence for a distribution of activation energies.
- Resonant ultrasound spectroscopy is a sensitive probe of internal motion in solids in a frequency range which is complementary to NMR and neutron scattering.

## TaV<sub>2</sub>H(D)<sub>x</sub> Fitting Parameters

- **TaV<sub>2</sub>H<sub>x</sub>**

$$\tau_R^{-1} = \tau_1^{-1} + \tau_2^{-1}$$

$$\tau_i = \tau_{io} \exp(E_i / k_B T)$$

	$E_{a1}$ (eV)	$\tau_{o1}(10^{-12}\text{sec})$	$E_{a2}$ (eV)	$\tau_{o1}(10^{-8}\text{sec})$
<b>TaV<sub>2</sub>H<sub>0.34</sub></b>	<b>0.25</b>	<b>3.1</b>	<b>0.082</b>	<b>3.6</b>
<b>TaV<sub>2</sub>H<sub>0.53</sub></b>	<b>0.23</b>	<b>4.8</b>	<b>0.091</b>	<b>1.9</b>

- **TaV<sub>2</sub>D<sub>0.17</sub>**

$$\tau_R = \tau_{Ro} \exp(E_a / k_B T)$$

$$\tau_{Ro} = 3.4 \times 10^{-12} \text{ sec} \quad E_a = 0.25 \text{ eV}$$

# Industrial Applications

Sort good vs defective parts during  
manufacturing

Types of defects

Existing methods for inspection of parts

RUS

Problems

The future

# Defect Types

Cracks  
Chips  
Dimensional variations  
Material Composition  
Microcracking  
Hardness



# Existing Methods

Eddy Current

Metals only

Dye Penetrant

Messy, environmental issues, visual

Mag Particle

Visual

X-Ray

Training, certification

5/30/1999

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# **Inspection via RUS**

**Works on many materials**

**Ceramics**

**Metals**

**Glass**

**Environmentally friendly**

**Not limited to surface defects**

**Automated**

5/30/1999

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4

# NDT & RUS

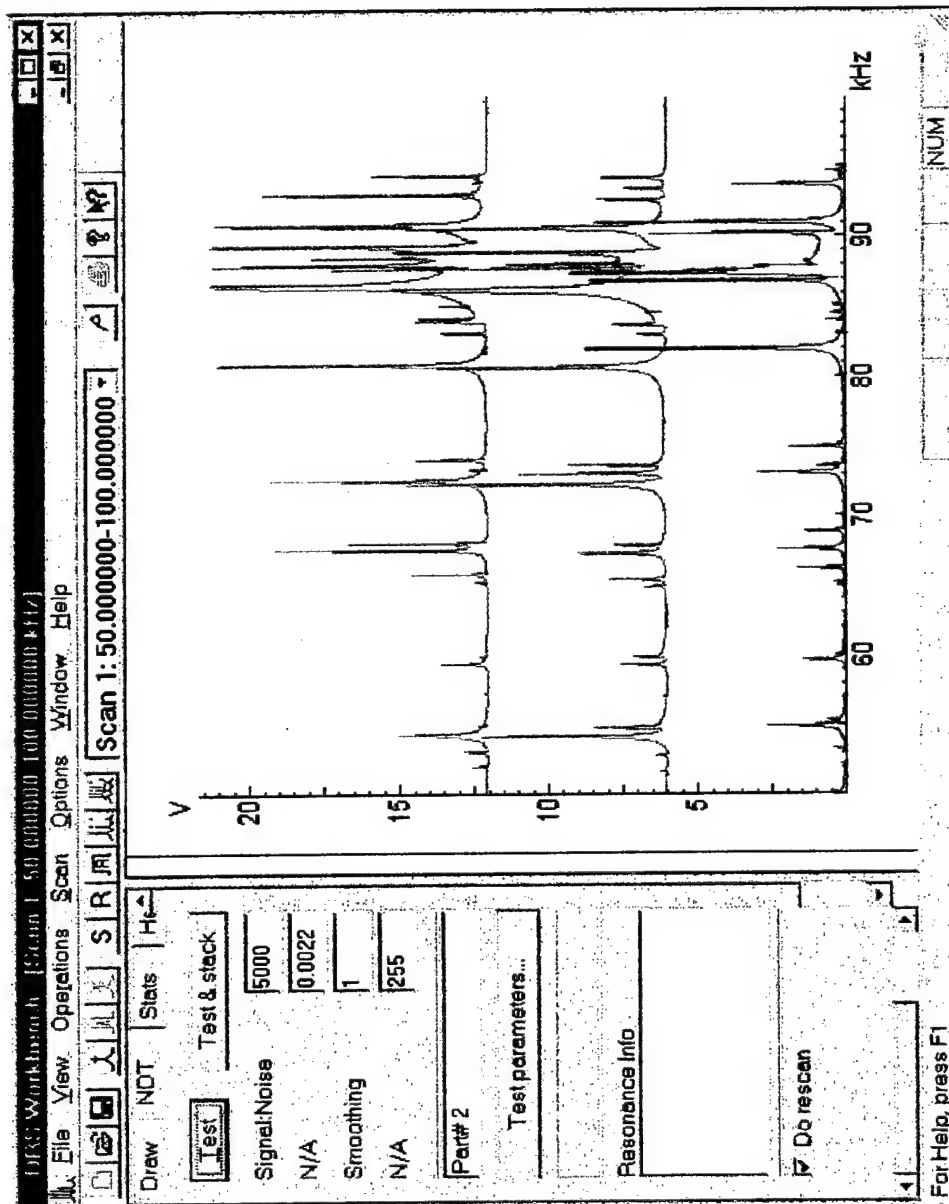
Must use known good and defective parts to define spectra

Determine differences in spectra between good and bad sets of parts

Sort unknown parts based on measured resonances

"Good" parts may contain defects

# 2 Good, 1 cracked cups



5/30/1999

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# Types of Tests (Patterns)

Simple position

Correct number of peaks in a frequency region

Doublet

2 peaks separated by the correct amount

An ideal part might only show 1 peak

Separation of doublet may be related to size of defect

# More test types

Linked peaks

2 peaks linearly related

Can also be used to correct for density or other variations

Q

Amplitude

Phase

# Successful Applications

## Ceramic Gas Seal

~14 mm diameter ceramic 'o-ring'

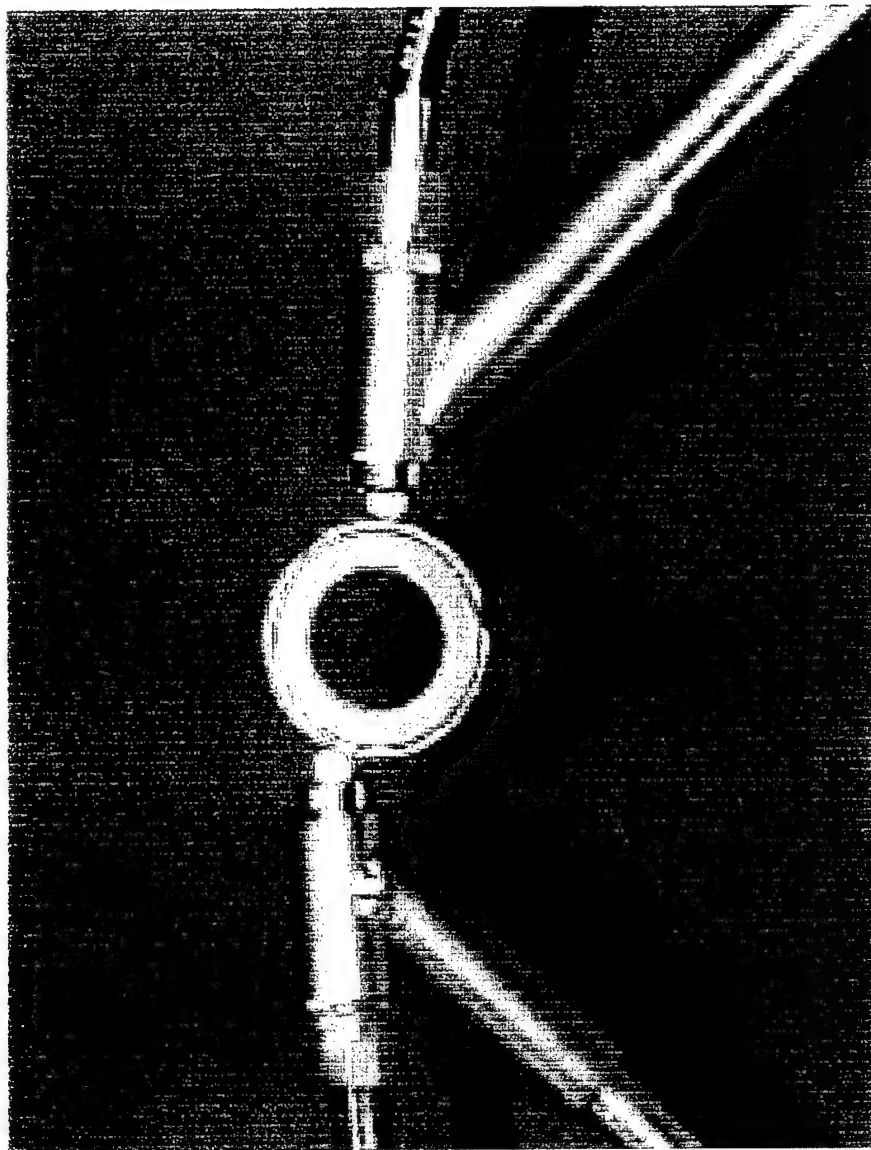
Tolerance on diameters ~0.006"

Saw correlation between chip size and doublet separation

Tolerance on diameters  $dV \sim 9 \text{ mm}^3$  same as ~2 mm diameter chip

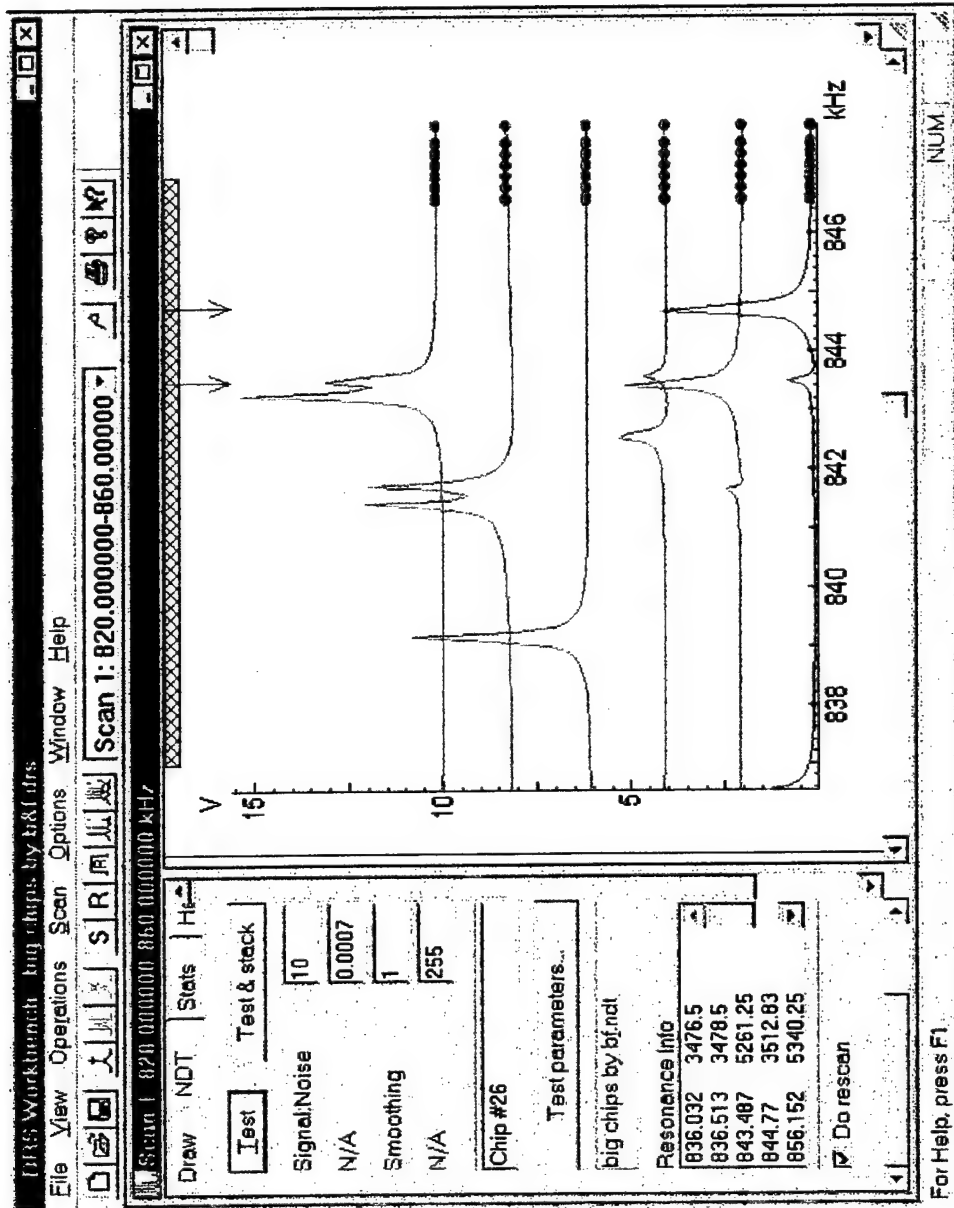
Could detect chips down to ~1 mm<sup>3</sup>

# Gas Seal





# Gas Seal



5/30/1999

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11

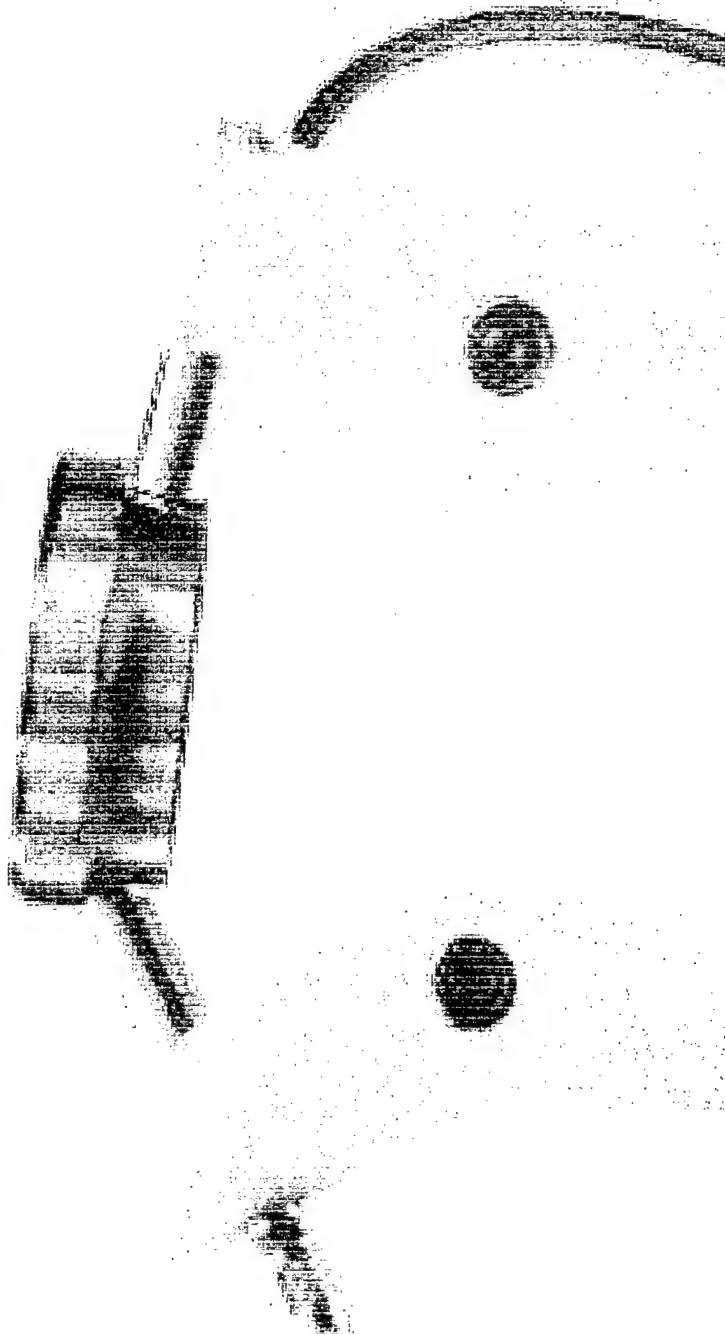
# Successful Applications

Machined steel bearing race

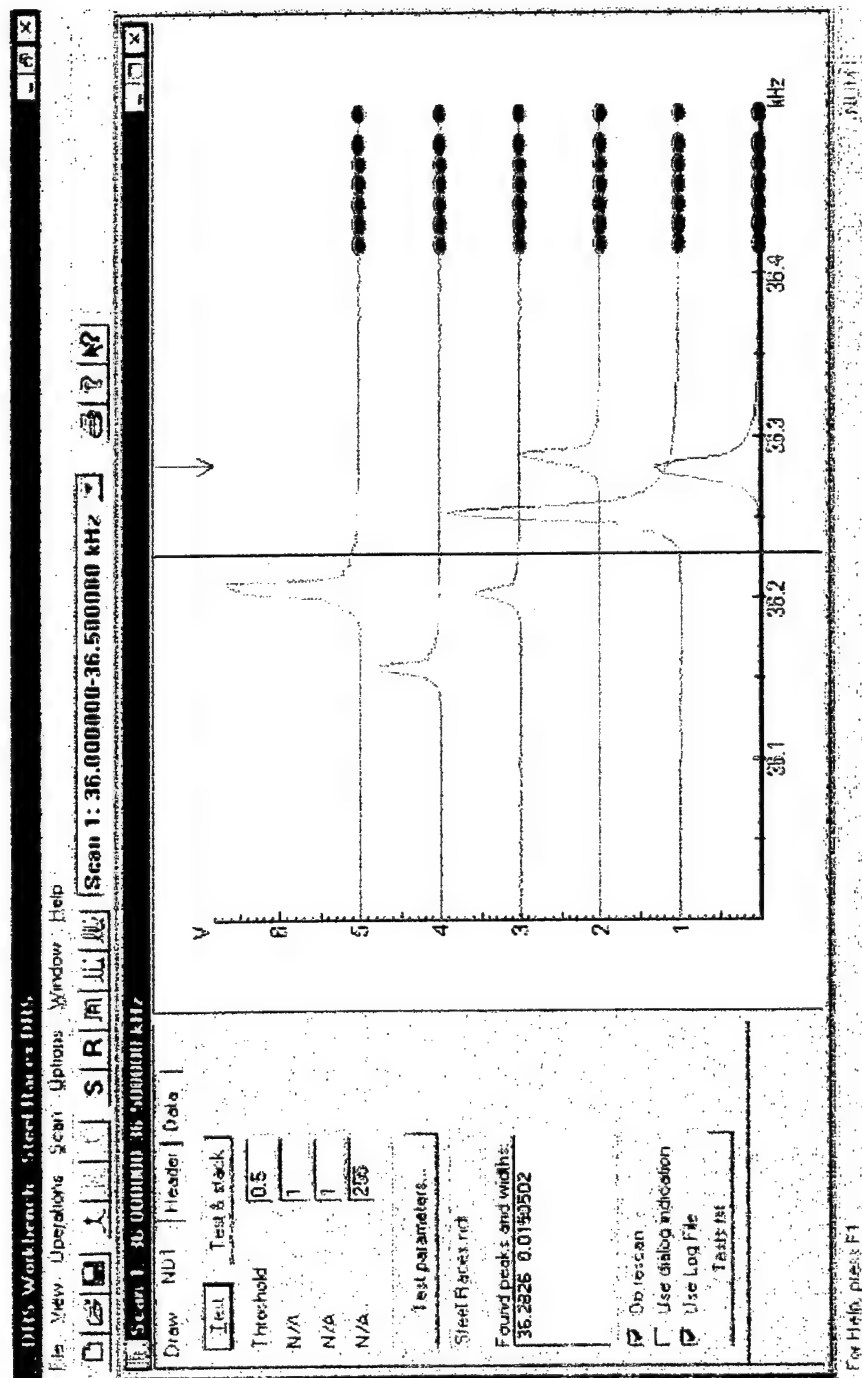
Grinding burns were the primary defect  
sought

Combined 2 simple position tests to improve  
sort

# Bearing Race



# Bearing Race



# Problems

## Pattern recognition

- Peaks do not line up - variations in material or density
- Peaks missing - High density of modes in many cases
- Parts are large - Lowest resonances often at 100's Hz
- Combinations of tests may be needed

# Problems

Powdered Metal Parts  
Composition varies?

All parts  
Patterns may be due to dimensional  
variations  
Good parts often contain defects

# **Next Generation System(s)**

Lower Frequency

Broad Spectrum

Do FFT on complicated signal

Advanced pattern recognition

# The Future

Tolerances on parts will be tighter  
Quality of raw materials will improve  
Manufacturing process control will improve

All will make RUS easier to implement and more reliable

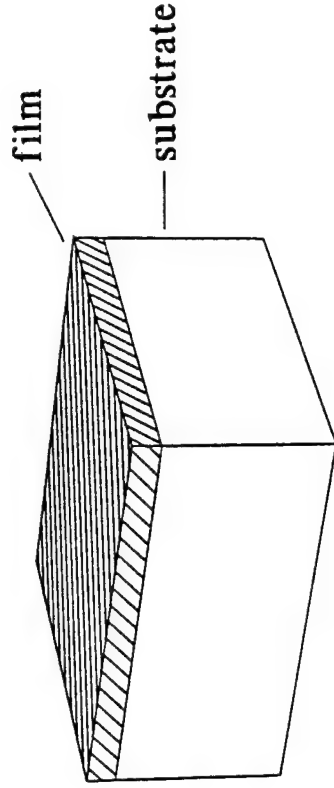


# Thin Film Characterization Using Resonant Ultrasound Spectroscopy

J. D. Maynard, Jason White, and Jin Hyun So  
The Pennsylvania State University

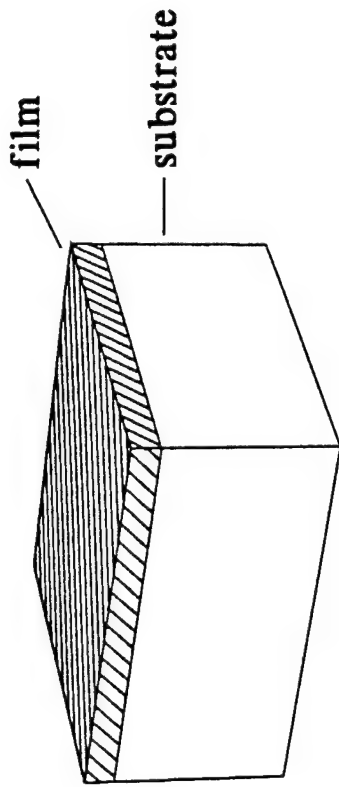
## Objective

- To study physical properties, through measurements of elastic stiffness and damping, for thin films deposited on substrates.



## Applications

- Thin films in magnetic recording media
- Protective or lubricating coatings, such as diamond, quasicrystalline, or carbon nanotube films
- Superconducting films
- Micro- (nano-) electronics

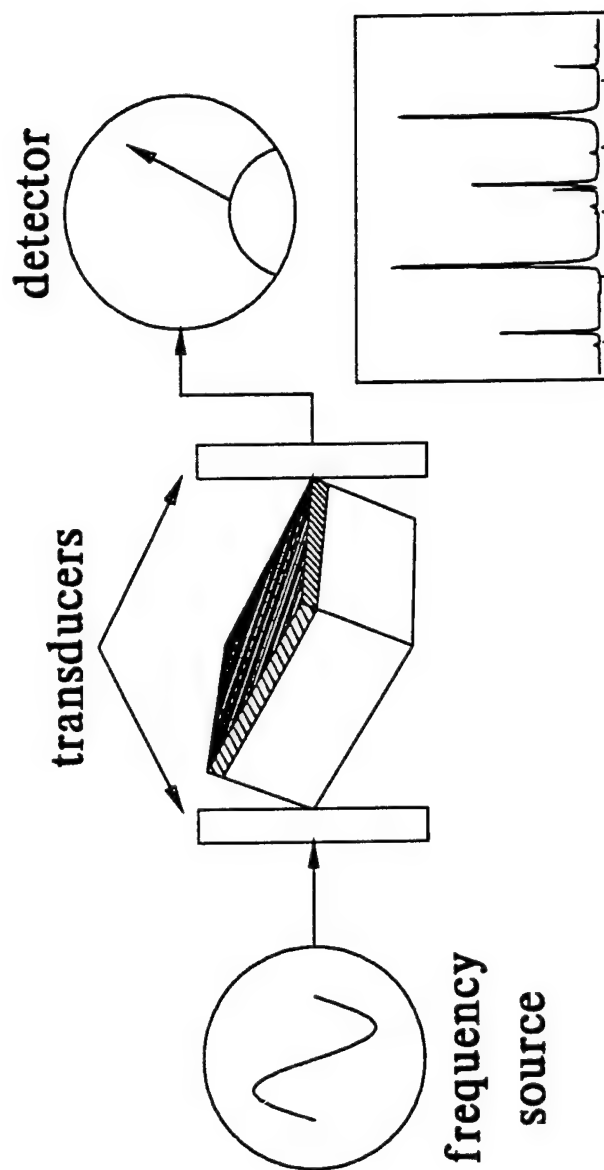


## Scientific Issues

- The physics of thin deposited films is effected by:
  - the reduced dimensionality of the film (2D rather than 3D)
  - the influence of the substrate, such as strain or disorder induced in the film by lattice mismatch, magnetic interactions, etc.
- Novel properties of unique systems such as mats of carbon nanotubes
- Determine properties for practical applications, such as elastic constants, friction, and damping

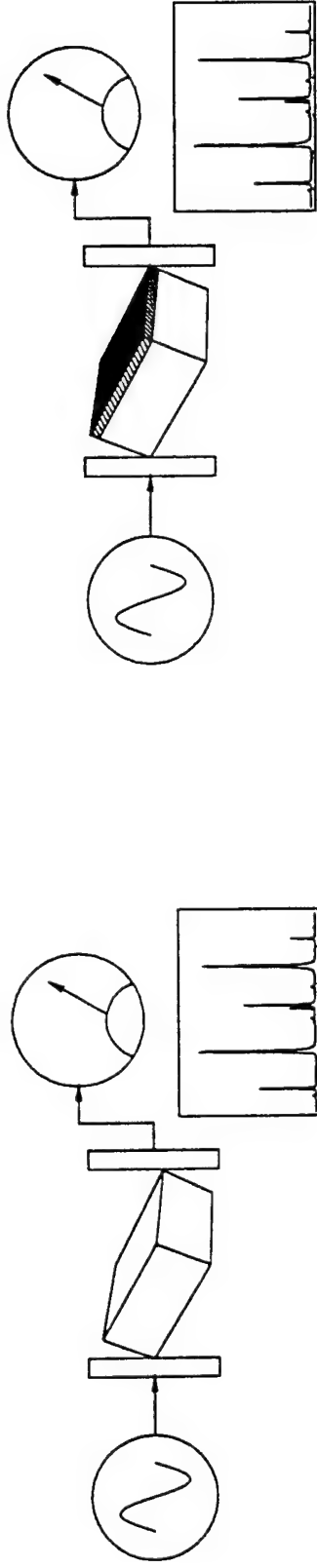
## Approach

Use Resonant Ultrasound Spectroscopy to measure elastic constants and damping for the film, assuming known properties of the substrate.



## Approach, cont.

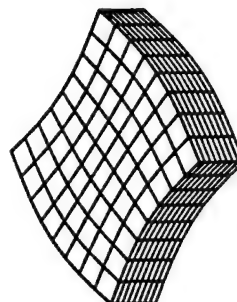
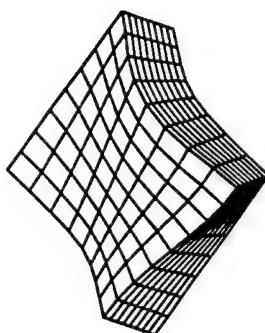
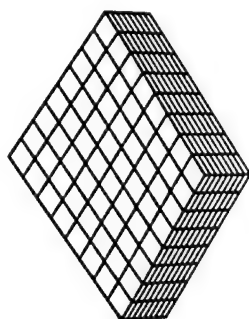
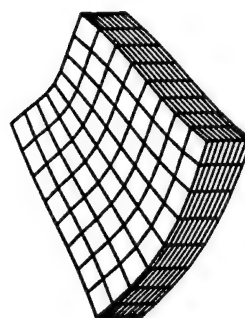
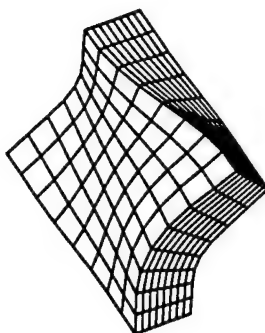
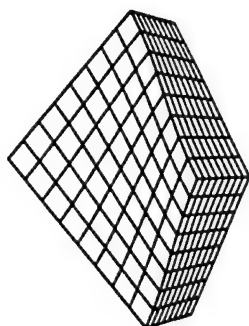
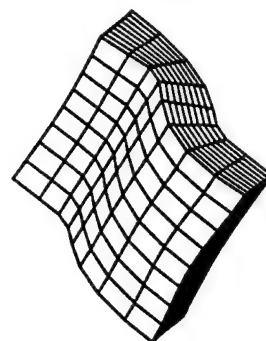
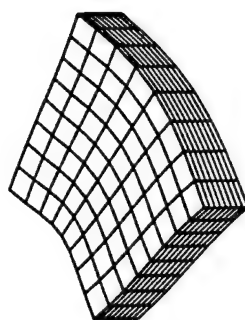
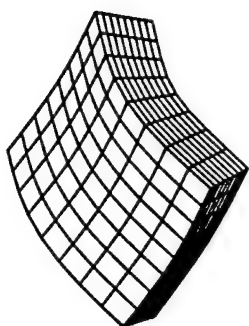
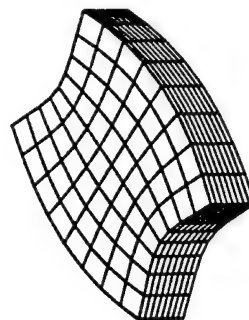
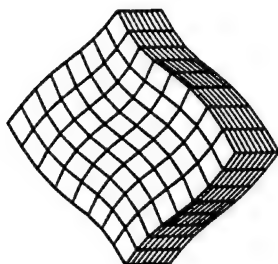
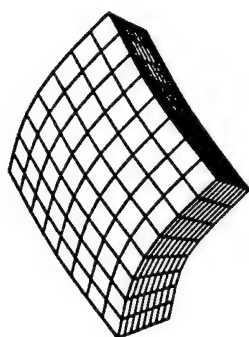
Measurements are made for the substrate with and without the film.



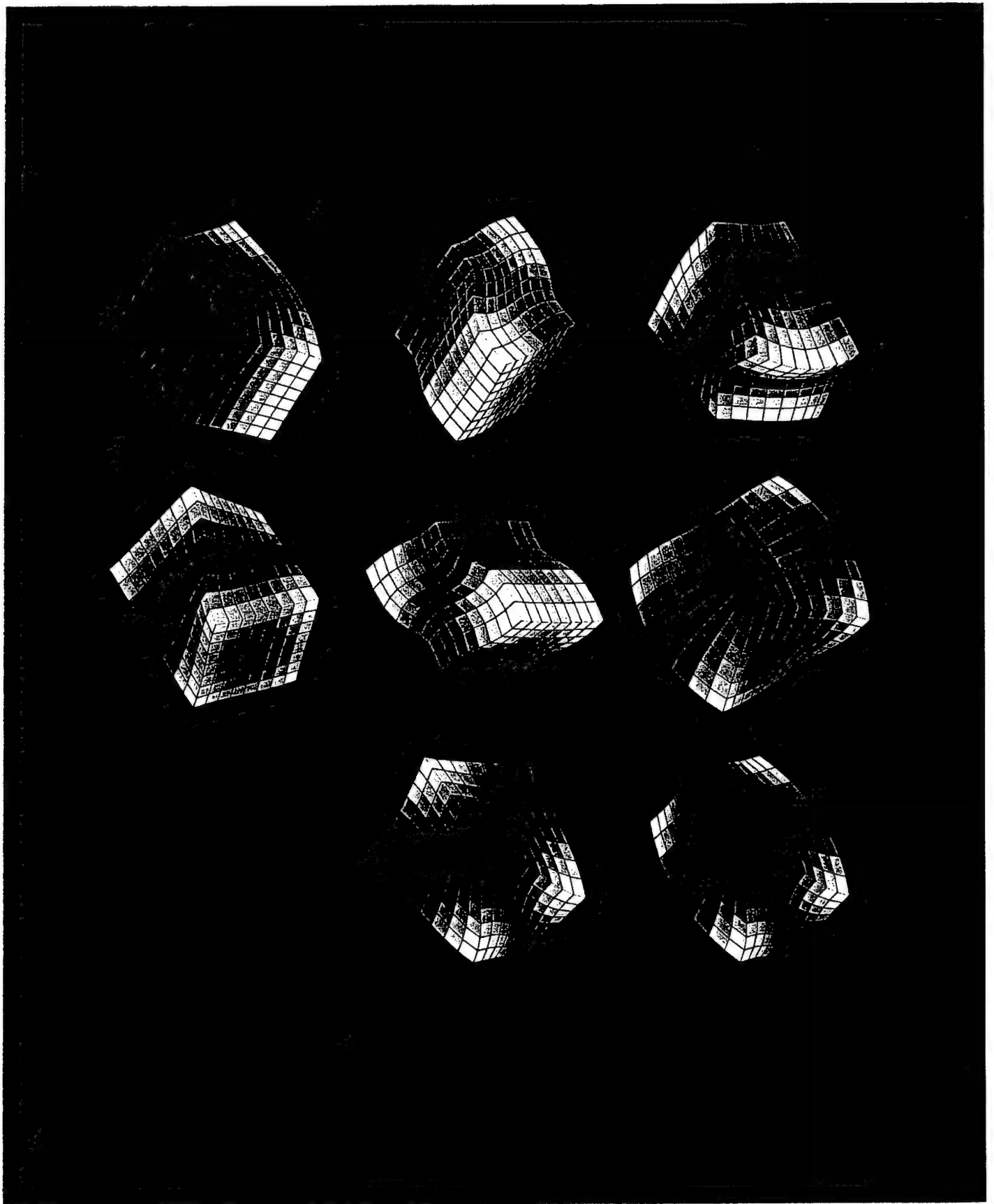
TR-4  
61

## Challenges

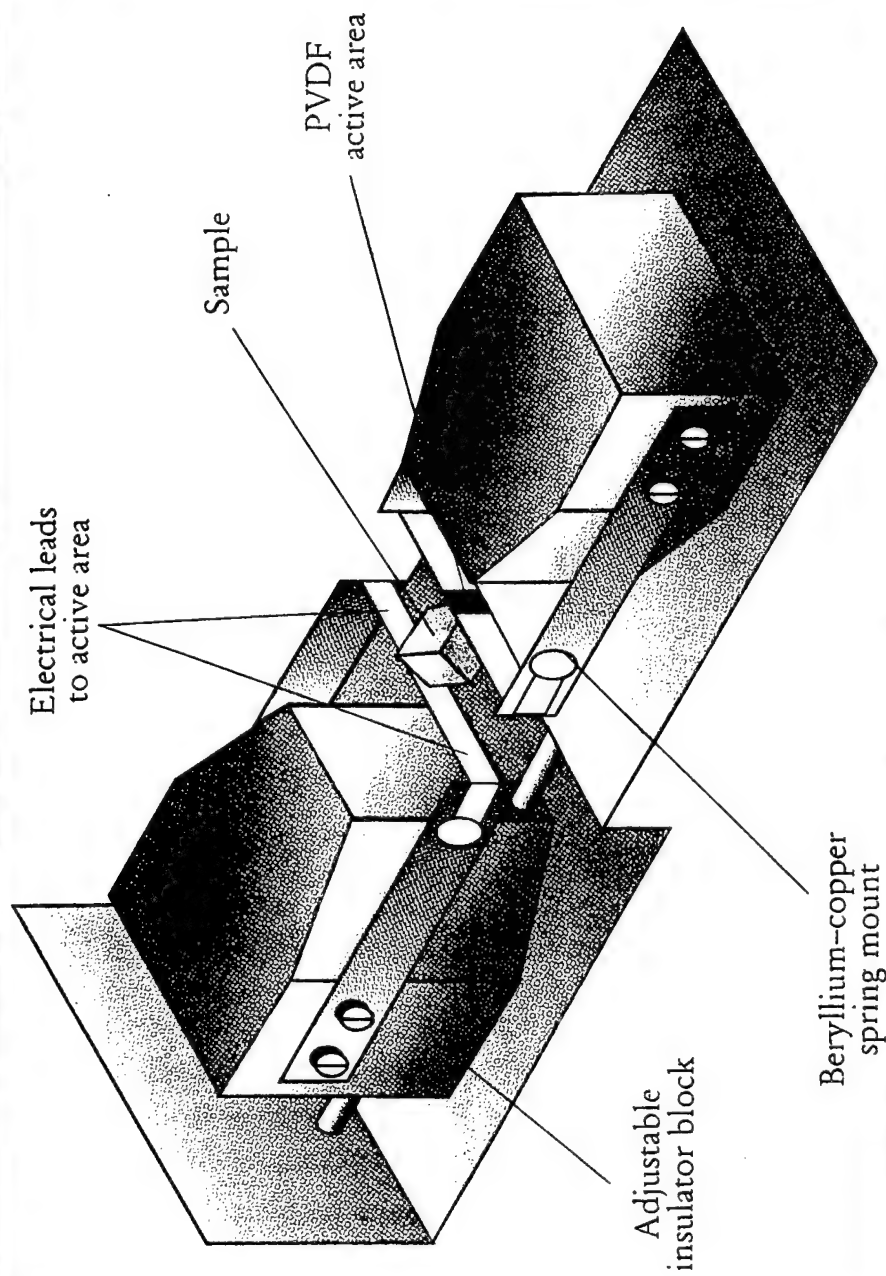
- The film must be a suitable fraction of the thickness of the sample
  - Typically: 100 nm films on 100  $\mu\text{m}$  substrates
- For anisotropic films, the lateral dimensions should be small as well
  - Necessary to excite thickness modes of film



TR-5



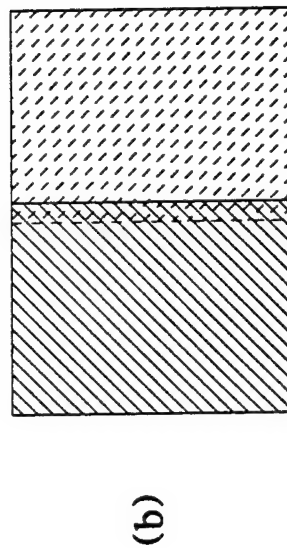
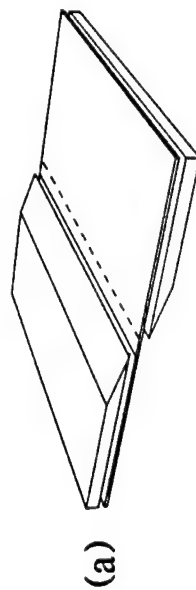
TR-6



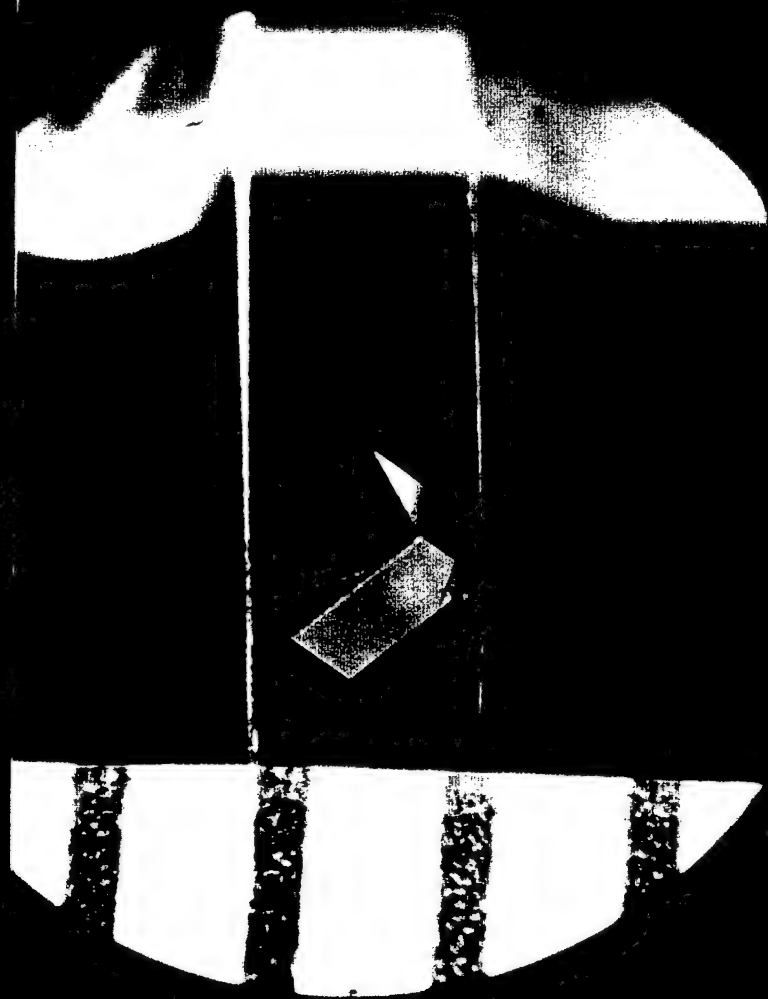
# Small transducers for small samples

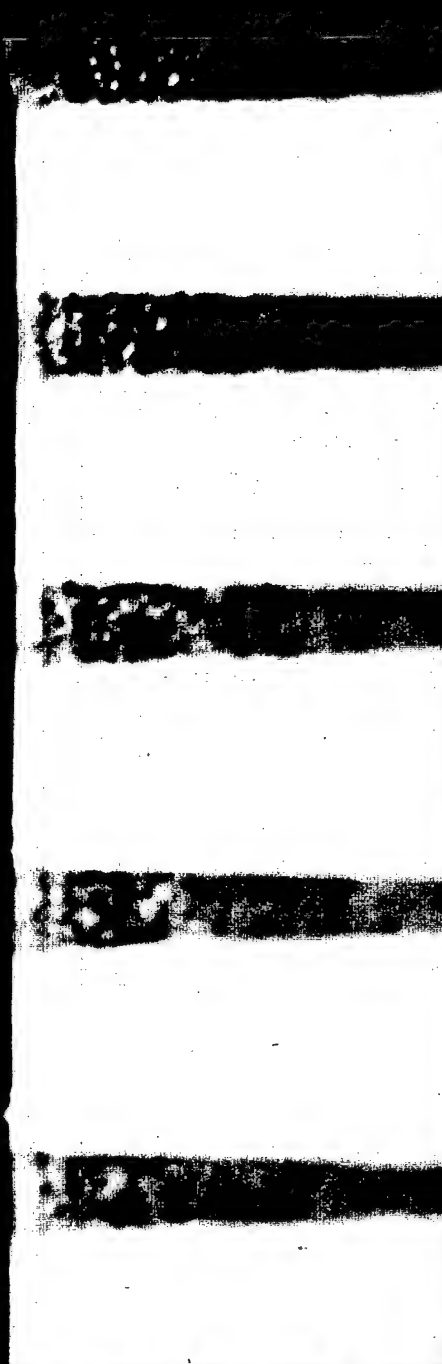
Piezoelectric Film: Polyvinylidene Fluoride (PVDF, tradename Kynar)  
Available in 9  $\mu\text{m}$  thick sheets

	units	BaTiO	Quartz	PVDF
Electric field/Pressure	Vm/N	5	50	200
Quality factor	Q	$10^3$	$10^5$	10
Acoustic impedance, $\rho c$	$10^6 \text{ kg/m}^2\text{-s}$	30	15	3
Frequency response		MHz	MHz	GHz









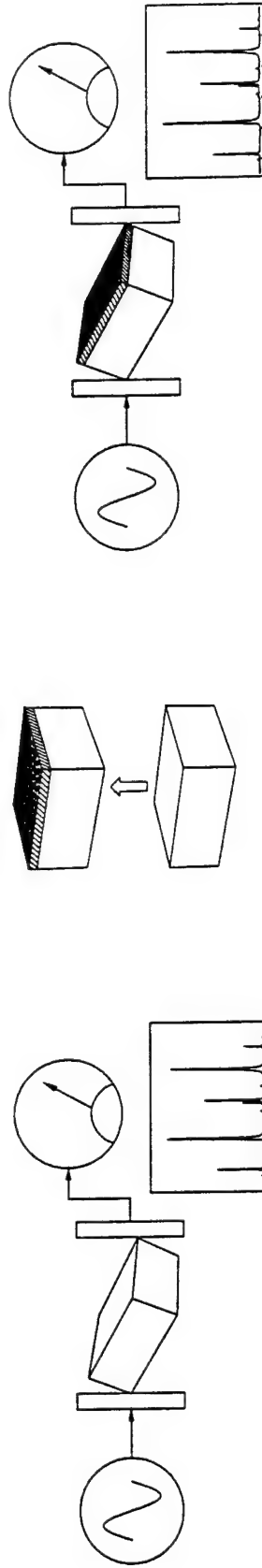
## Feasibility

The elastic constants for the film,  $c'_{ij}$ , should be measured with suitable precision

$$\frac{\Delta c'_{ij}}{c'_{ij}} = \frac{\Delta f}{f} \left[ \frac{c'_{ij}}{f} \frac{\partial f}{\partial c'_{ij}} \right]^{-1}$$

**Problem:** Measuring the substrate with and without the film may require mounting and remounting the sample.

TR-11



- The precision in measuring frequency,  $\Delta f/f$ , may be limited by changes resulting from remounting.

# Reproducibility in Mounting and Remounting Sample

Silicon sample #1: a = 768  $\mu\text{m}$ , b = 640  $\mu\text{m}$ , c = 97  $\mu\text{m}$

Mode	1st	2nd	3rd	4th	Avg	ppm
1	1.334500	1.334500	1.333750	1.333750	1.333750	375
2	1.919250	1.920000	1.919625	1.919250	1.919373	221
3	2.137250	2.137250	2.136500	2.136875	2.136833	193
4	2.502500	2.503000	2.501500	2.501500	2.136833	362
5	3.407250	3.407625	3.406500	3.406500	3.406500	220
6	3.859750	3.859375	3.860500	3.860875	3.860125	153
7	4.069375	4.069000	4.068625	4.068625	4.068775	94
8	4.142125	4.142500	4.141750	4.141375	4.141643	134
10	4.780000	4.780500	4.779250	4.779000	4.781000	192
11	5.223375	5.223750	5.219625	5.227500	5.224375	494
12	5.463000	5.464875	5.465250	5.464900	5.464375	180

# Reproducibility in Mounting and Remounting Sample

Strontium titanate sample #2: a = 804  $\mu\text{m}$ , b = 722  $\mu\text{m}$ , c = 445  $\mu\text{m}$

Mode	1st	2nd	3rd	Avg	ppm
1	2.373750	2.373625	2.373625	2.373667	25
2	3.051250	3.051625	3.051250	3.051375	58
3	3.661500	3.661375	3.661375	3.661417	16
4	3.787750	3.787750	3.787750	3.787750	00
7	4.273500	4.273500	4.273625	4.273542	14
8	4.368625	—	4.368750	4.368688	14
9	4.520812	4.520687	4.520937	4.520812	23
10	4.714000	4.714375	4.714750	4.714375	65
11	4.777500	4.777500	4.778000	4.777667	49
12	5.138250	5.138625	5.138625	5.138500	34
14	—	5.429875	5.430000	5.429938	12

# Precision in Determining Film Elastic Constants

Strontium titanate sample #2:  $a = 804 \text{ } \mu\text{m}$ ,  $b = 722 \text{ } \mu\text{m}$ ,  $c = 445 \text{ } \mu\text{m}$

Film thickness  $t = 450 \text{ nm}$

Mode	Freq	$\partial f / \partial t$	$\Delta 11\%$	$\Delta 12\%$	$\Delta 44\%$
1	2.372141	-0.792E-03	-	-	3.61
2	3.047371	-0.111E-02	4.28	7.03	-
3	3.667886	-0.382E-03	5.00	9.71	-
4	3.778126	-0.875E-03	-	-	-
5	4.007881	-0.379E-03	-	-	-
6	4.210928	-0.450E-03	-	-	-
7	4.278163	-0.366E-03	4.61	9.03	-
8	4.370969	-0.898E-03	2.19	3.98	4.65
9	4.520948	-0.960E-03	2.70	4.57	-
10	4.691540	-0.370E-03	-	-	-
11	4.793241	-0.477E-03	-	-	-
12	5.152972	-0.491E-03	7.52	-	-
13	5.195106	-0.252E-03	-	-	-
14	5.445382	-0.421E-03	3.52	8.41	-

## Review of Basic RUS analysis

Assume stress-free boundary conditions and  $\cos(\omega t)$  time dependence ( $\omega = 2\pi f$ ):

$$\epsilon_{ij} = \frac{1}{2} \left( \frac{\partial \psi_i}{\partial x_j} + \frac{\partial \psi_j}{\partial x_i} \right)$$

Strain

$$\sigma_{ij} = c_{ijkl} \epsilon_{kl} = c_{ijkl} \frac{\partial \psi_k}{\partial x_l}$$

Hooke's Law

$$\rho \omega^2 \psi_i + c_{ijkl} \frac{\partial^2 \psi_k}{\partial x_j \partial x_l} = 0$$

Newton's Law

$$\sigma_{ij} n_j = c_{ijkl} \frac{\partial \psi_k}{\partial x_l} n_j = 0$$

Stress-free boundary condition

Boundary value problem has eigenvalues  $\omega_n \rightarrow$  resonant frequencies  $f_n = \omega_n / 2\pi$   
Equivalent to minimizing the Lagrangian:

$$L = \iiint \left( \rho \omega^2 \psi_i \psi_i - c_{ijkl} \frac{\partial \psi_i}{\partial x_j} \frac{\partial \psi_k}{\partial x_l} \right) dV$$

## Rayleigh-Ritz Approximation

Expand with basis functions  $\psi_i = \sum_{p=1}^N a_{pi} \Phi_p$

Lagrangian  $L = (\omega^2 E_{piqj} - \Gamma_{piqj}) a_{pi} a_{qj}$

where  $E_{piqj} = \delta_{ij} \int \int \int \rho \Phi_p \Phi_q dV$

and  $\Gamma_{piqj} = \int \int \int c_{ikjl} \frac{\partial \Phi_p}{\partial x_k} \frac{\partial \Phi_q}{\partial x_l} dV$

Let  $\mu$  represent the combination  $pi$ , and  $\nu$  represent  $qj$ . Minimizing  $L$  with respect to the coefficients  $a_\mu$  yields the  $3N \times 3N$  matrix eigenvalue problem

Matrix eigenvalue problem  $\Gamma_{\mu\nu} a_\nu = \omega^2 E_{\mu\nu} a_\nu$



## Visscher Basis Set:

Use  $\Phi_p = x^l y^m z^n$  : Integrals for  $\Gamma_{\mu\nu}$  and  $E_{\mu\nu}$  may be evaluated analytically for prisms, spheroids, ellipsoids, shells, bells, eggs, potatoes, sandwiches, and others. Normalize to improve matrix conditioning:

$$\Phi_p = \begin{pmatrix} x \\ - \\ a \end{pmatrix}^l \begin{pmatrix} y \\ - \\ b \end{pmatrix}^m \begin{pmatrix} z \\ - \\ c \end{pmatrix}^n$$

where  $a, b, c$  are half the edge lengths of the sample. The integrals, with fixed limits  $(-1, 1)$ , become simple expressions.

## Consequences of Symmetry

- If the elastic tensor has orthorhombic symmetry or better, then the matrices become **block diagonal**.
- Result of parity of  $(x/a)^l$  (odd or even)
- Nearly eight-fold reduction of computation time

## Modifications for Substrate with Film

Let  $\alpha = (\text{film thickness})/(\text{half edge length})$ ,  $\zeta = (\rho' - \rho)/\rho$ , and  $d_{ijkl} = c'_{ijkl} - c_{ijkl}$ .

$$\frac{1}{\rho} E_{piqj} = \delta_{ij} \left[ \int \int \int_{-1}^1 \Phi_p \Phi_q dx dy dz + \zeta \int \int_{-1}^1 \int_{(1-\alpha)}^1 \Phi_p \Phi_q dx dy dz \right]$$

$$\Gamma_{piqj} = c_{ijkl} \int \int \int_{-1}^1 \frac{\partial \Phi_p}{\partial x_k} \frac{\partial \Phi_q}{\partial x_l} dx dy dz + d_{ijkl} \int \int_{-1}^1 \int_{(1-\alpha)}^1 \frac{\partial \Phi_p}{\partial x_k} \frac{\partial \Phi_q}{\partial x_l} dx dy dz$$

### Consequences of Reduced Symmetry

- Because of the film, parity symmetry is lost in the z-direction.
- Parity symmetry remains for the x- and y-directions.
- The matrix is block-diagonal with four blocks.

$$\begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}$$

```

C.....
C      PROGRAM XYZHRQ
C.....
C      Read data file
C.....
C      OPEN(3, FILE='xyzmrq.dat', STATUS='OLD', ERR=101)
C      GO TO 102
C      WRITE(*, '(A)', ' Error opening xyzmrq.dat.')
C      GO TO 9000
C.....
C      READ(3, *, ERR=101) C11, C12, C14, E1, E2, E3
C      READ(3, *, ERR=101) (IA(I), I=1, 6)
C      READ(3, *, ERR=101) RHO, NN, NMDS
C      DO 103 I=1, NMDS
C      READ(3, *, ERR=101) FDATA(I)
C      CONTINUE
C      CLOSE(3, ERR=9000)
C.....
C      NDATA=0
C      DO 105 I=1, NMDS
C      IF (FDATA(I).LE.0.0) GO TO 104
C      NDATA=NDATA+1
C      X(NDATA)=FLOAT(I)
C      Y(NDATA)=FDATA(I)
C      GO TO 105
C      FDATA(I)=-FDATA(I)
C      CONTINUE
C.....
C      Parameter setup
C.....
C      CALL INIT
C.....
C      ALAMDA=-1.
C      SUMSQ=1.D2
C      ITER=0
C.....
C      ITER=ITER+1
C      OLAHDA=ALAMDA
C      CH10=SUMSQ
C      WRITE(*, '(A20.12)', ' Iteration number', ITER)
C.....
C      CALL WROHIN(X, Y, SIG, NDATA, A, IA, MA, COVAR, ALPHA, MHAX,
C      & SUMSQ, FREQS, ALAMDA)
C.....
C      CHANGE=ABS((SUMSQ-CH10)/CH10)
C      WRITE(*, '(5E12.4)') OLAHDA, ALAMDA, CH10, SUMSQ, CHANGE
C.....
C      IF (CHANGE.GT.TOL) GO TO 200
C.....
C      DO 8200 I=1, NMDS
C      ERR=0.0
C      IF (FDATA(I).GT.0.) ERR=100.*((FDATA(I)-MNAV(I))/MNAV(I))
C      WRITE(3, '(13.2F10.5,F8.3)') I, FDATA(I), MNAV(I), ERR
C      CONTINUE
C.....
C      STOP
C      END
C.....
C      8200
C      9000
C.....
C.....
C      SUBROUTINE XYZBLK(A, MA, WSAV)
C.....
C.....
C      Loop for eight blocks
C.....
C      DO 8000 LO=1, 2
C      DO 8000 MO=1, 2
C      DO 8000 NO=1, 2
C.....
C      IG=0
C      DO 2 I=1, 3
C      LAM(1)=LO
C      LAM(2)=MO
C      LAM(3)=NO
C      LAM(1)=LAM(1)+1
C      IF (LAM(1).EQ.3) LAM(1)=1
C      DO 2 L=LAM(1), NMPI, 2
C      DO 2 M=LAM(2), NMPI, 2
C      DO 2 N=LAM(3), NMPI, 2
C      IF (L*MIN.GT.NMPI) GO TO 2
C      IG=IG+1
C      IC(IG)=I
C      LB(IG)=L-1
C      HB(IG)=M-1
C      NB(IG)=N-1
C      CONTINUE
C      NR=IG
C.....
C      calculate Gamma and Z
C.....
C      DO 3 IG=1, NR
C      DO 3 JG=IG, NR
C      E(IG, JG)=0.D0
C      GAMMA(IG, JG)=0.D0
C      I=IC(IG)
C      J=IC(JG)
C      LS=LB(IG)+LB(JG)
C      MS=HB(IG)+HB(JG)
C      NS=NB(IG)+NB(JG)
C      GAMMA(IG, JG)=
C      & C(1, 1, J, 1)*DFLOAT(LB(IG)*LB(JG))*F(LS-2, MS, NS)*E11
C      & +C(1, 2, J, 2)*DFLOAT(HB(IG)*HB(JG))*F(LS, MS-2, NS)*E22
C      & +C(1, 3, J, 3)*DFLOAT(HB(IG)*NB(JG))*F(LS, MS, NS-2)*E33
C      & +C(1, 1, J, 2)*DFLOAT(LB(IG)*HB(JG))*C(1, 2, J, 1)*
C      & +DFLOAT(HB(IG)*LB(JG))*F(LS-1, MS-1, NS)*E12
C      & +C(1, 1, J, 3)*DFLOAT(LB(IG)*NB(JG))*C(1, 3, J, 1)*
C      & +DFLOAT(HB(IG)*LB(JG))*F(LS-1, MS, NS-1)*E13
C      & +C(1, 2, J, 3)*DFLOAT(HB(IG)*NB(JG))*C(1, 3, J, 2)*
C      & +DFLOAT(LB(IG)*HB(JG))*F(LS, MS-1, NS-1)*E23
C      GAMMA(JG, IG)=GAMMA(IG, JG)
C      IF (I.EQ.J) E(IG, JG)=F(LS, MS, NS)
C      E(JG, IG)=E(IG, JG)
C      CONTINUE
C.....
C      CALL EIGEN(GAMMA, E, WSAV)
C.....
C      DO 8020 I=1, NMDS
C      WSAV(I)=DSORT(WSAV(I)/RHO)/TWOPI
C      CONTINUE
C.....
C      8020
C      9000
C.....
C      RETURN
C      END
C.....
C      FUNCTION F(IP, IQ, IR)
C.....
C      F=0.00D+00
C      IF ((IP.LT.0).OR.(IQ.LT.0).OR.(IR.LT.0)) RETURN
C      IF ((MOD(IP, 2).NE.0).OR.(MOD(IQ, 2).NE.0).OR.(MOD(IR, 2).NE.0))
C      & RETURN
C      F=F.D0/DFLOAT((IP+1)*(IQ+1)*(IR+1))
C      RETURN
C      END

```



```

C
C
C calculate Gamma
C
NR=IG
DO 3 IG=1,NR
DO 3 JG=IG,NR
E(IG,JG)=0.D0
GAMMA(IG,JG)=0.D0
I=IC(IG)
J=IC(JG)
LS=LB(IG)+LB(JG)
MS=MB(IG)+MB(JG)
NS=NB(IG)+NB(JG)
GAMMA(IG,JG)=
& C(I,1,J,1)*DFLOAT(LB(IG)*LB(JG))*F(LS-2,MS,NS)*E11
& +C(I,2,J,2)*DFLOAT(MB(IG)*MB(JG))*F(LS,MS-2,NS)*E22
& +C(I,3,J,3)*DFLOAT(NB(IG)*NB(JG))*F(LS,MS,NS-2)*E33
& +C(I,1,J,2)*DFLOAT(LB(IG)*MB(JG))+C(I,2,J,1)*
& DFLOAT(MB(IG)*LB(JG))*F(LS-1,MS-1,NS)*E12
& +C(I,1,J,3)*DFLOAT(LB(IG)*NB(JG))+C(I,3,J,1)*
& DFLOAT(NB(IG)*LB(JG))*F(LS-1,MS,NS-1)*E13
& +C(I,2,J,3)*DFLOAT(MB(IG)*NB(JG))+C(I,3,J,2)*
& DFLOAT(NB(IG)*MB(JG))*F(LS,MS-1,NS-1)*E23
& +D(I,1,J,1)*DFLOAT(LB(IG)*LB(JG))*G(LS-2,MS,NS)*E11
& +D(I,2,J,2)*DFLOAT(MB(IG)*MB(JG))*G(LS,MS-2,NS)*E22
& +D(I,3,J,3)*DFLOAT(NB(IG)*NB(JG))*G(LS,MS,NS-2)*E33
& +D(I,1,J,2)*DFLOAT(LB(IG)*MB(JG))+D(I,2,J,1)*
& DFLOAT(MB(IG)*LB(JG))*G(LS-1,MS-1,NS)*E12
& +D(I,1,J,3)*DFLOAT(LB(IG)*NB(JG))+D(I,3,J,1)*
& DFLOAT(NB(IG)*LB(JG))*G(LS-1,MS,NS-1)*E13
& +D(I,2,J,3)*DFLOAT(MB(IG)*NB(JG))+D(I,3,J,2)*
& DFLOAT(NB(IG)*MB(JG))*G(LS,MS-1,NS-1)*E23

C
GAMMA(JG,IG)=GAMMA(IG,JG)
IF(I.EQ.J) E(IG,JG)=F(LS,MS,NS)+DRHO*G(LS,MS,NS)
E(JG,IG)=E(IG,JG)
CONTINUE
3
C

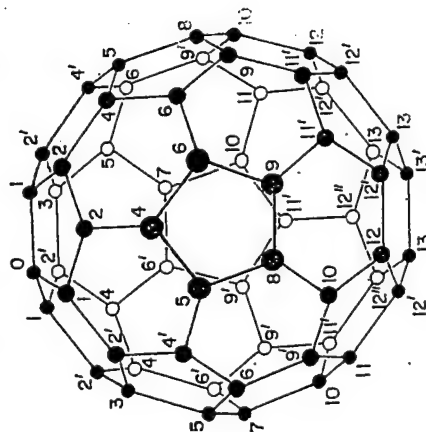
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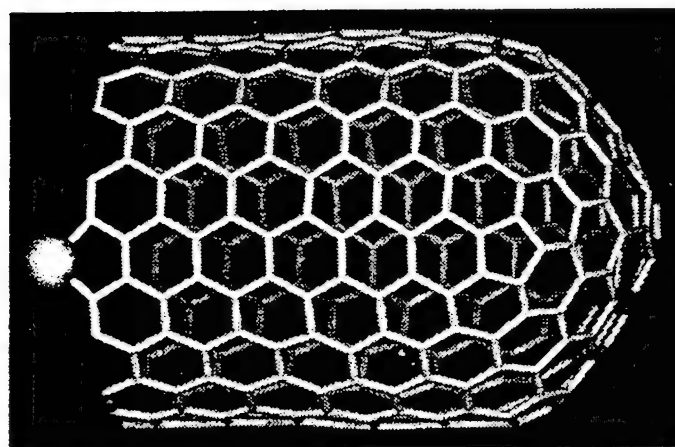
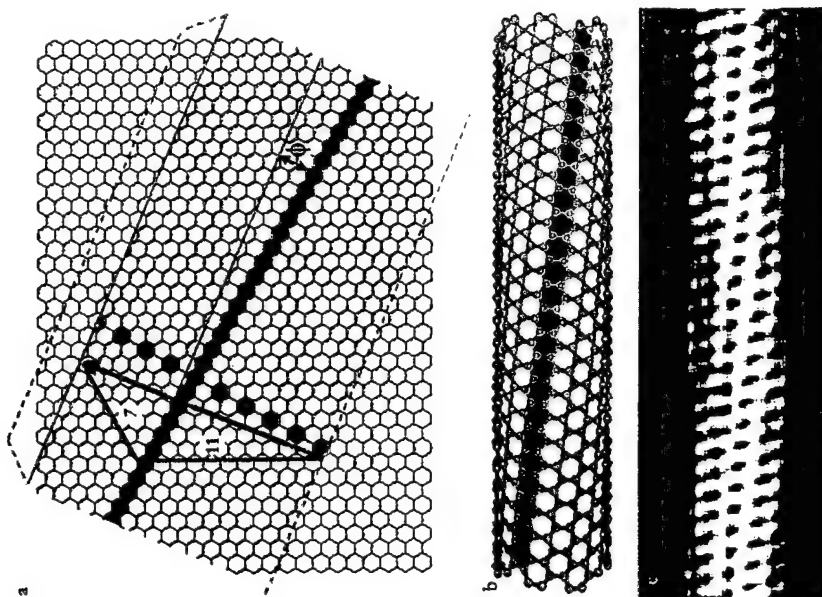
# Application: Mats of Carbon Nanotubes

Single Walled Nanotubes (SWNT):

SCIENCE NEWS of the week  
Hot Times for Buckyball

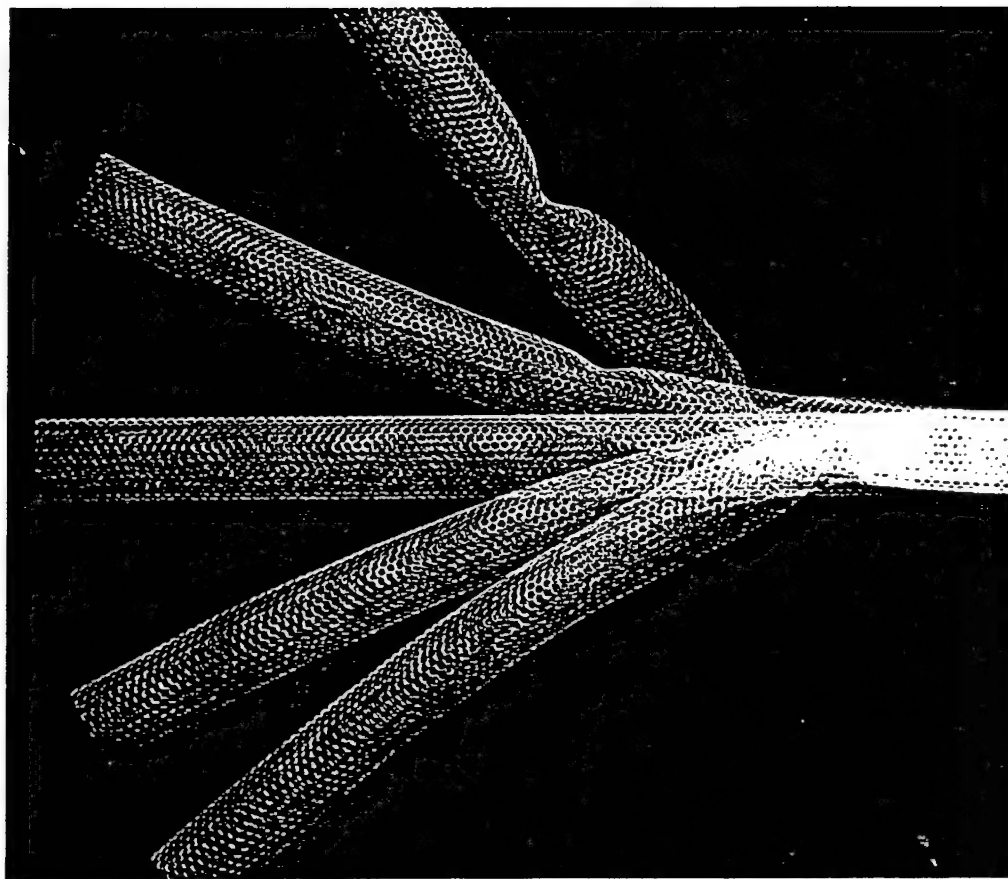


TR-23



# PHYSICS TODAY

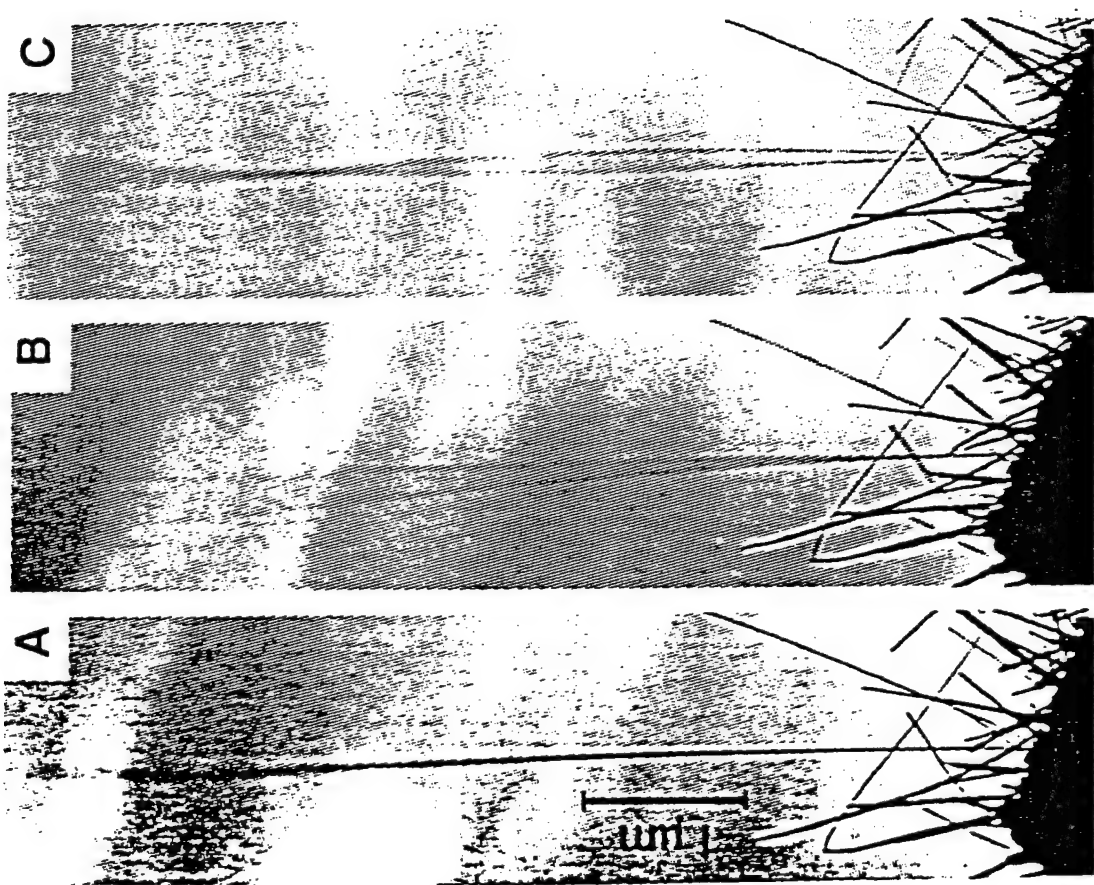
JUNE 1996



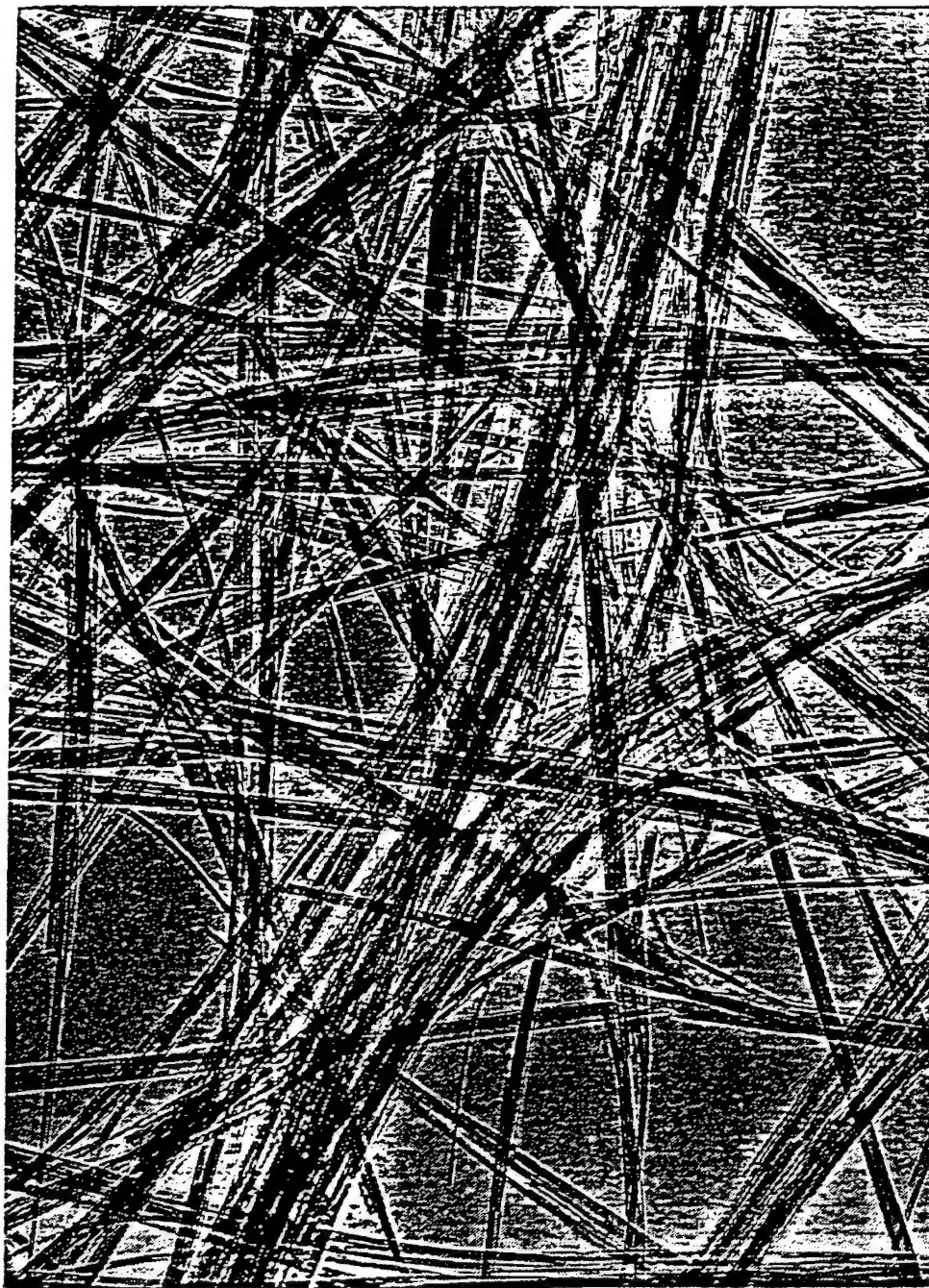
CARBON NANOTUBES

TR-24





TR-25



TR-26

# Elasticity of steel and silica glass spheres under gas pressure by RUS

I. Ohno<sup>1</sup>, M. Abe<sup>1</sup>, M. Kimura<sup>2</sup>, Y. Hanayama<sup>3</sup>, H. Oda<sup>4</sup> and I. Suzuki<sup>4</sup>

1 Department of Earth Sciences, Ehime University, Matsuyama  
790-8577, Japan

2 Department of Material Science and Engineering, Ehime University,  
Matsuyama 790-8577, Japan

3 Department of Mechanical Engineering, Ehime University, Matsuyama  
790-8577, Japan

4 Department of Earth Sciences, Okayama University, Okayama  
700-0082, Japan

## ABSTRACT

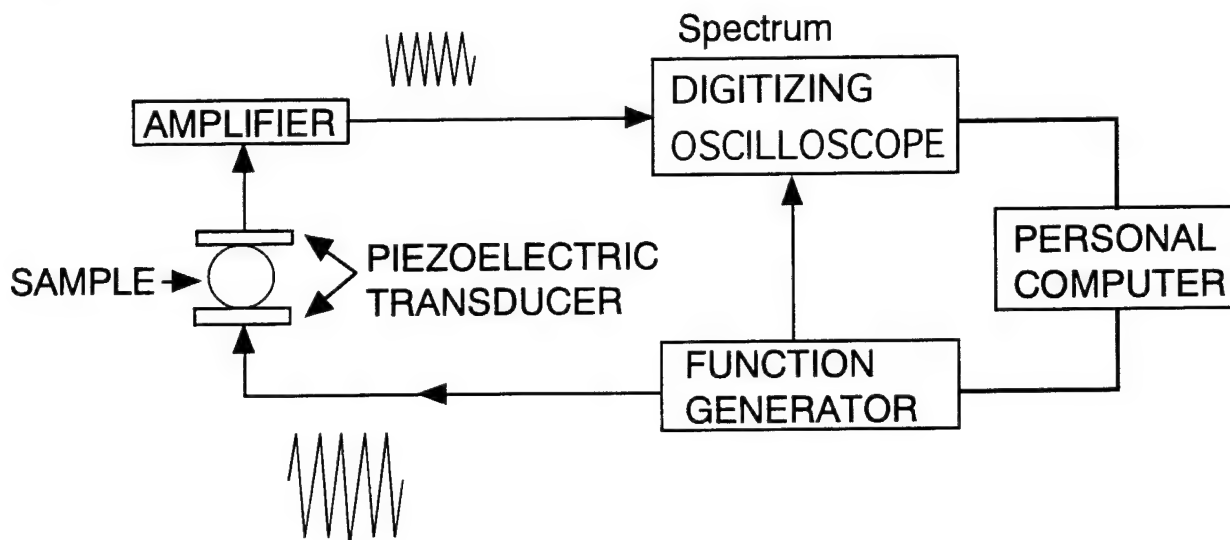
Elasticity of minerals under high pressure are of great importance from geophysical view point. In order to develop a new method to measure pressure dependence of elastic constants, resonant ultrasound spectroscopy (RUS) were applied to the condition of gas pressure. We used a three-layered spherical shell assembly of spherical sample-thin gas layer-spherical cavity container, which works to give a well-defined boundary condition in the analysis. The samples measured were spheres of steel and silica glass with diameter of 4-5 mm. Several modes, not only torsional but also spheroidal modes, were observed at least up to 200MPa (2kbar) under helium gas pressure, and pressure shifts of frequencies were obtained definitely in both samples. The data of the slope of pressure shift of frequency yield pressure derivatives of bulk modulus and shear modulus,  $dK/dP$  and  $dG/dP$ , which are in good agreement with previous data.

# 1. Method

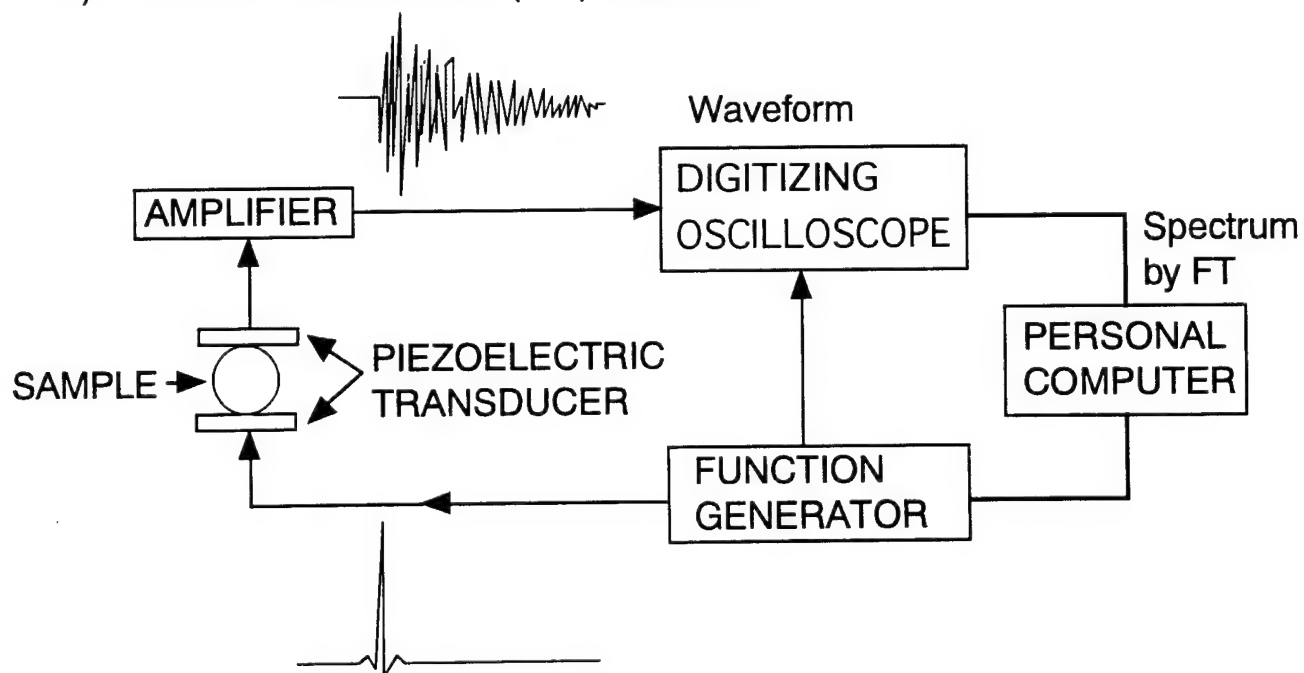
Resonant Ultrasound Spectroscopy (RUS)  
with spherical sample

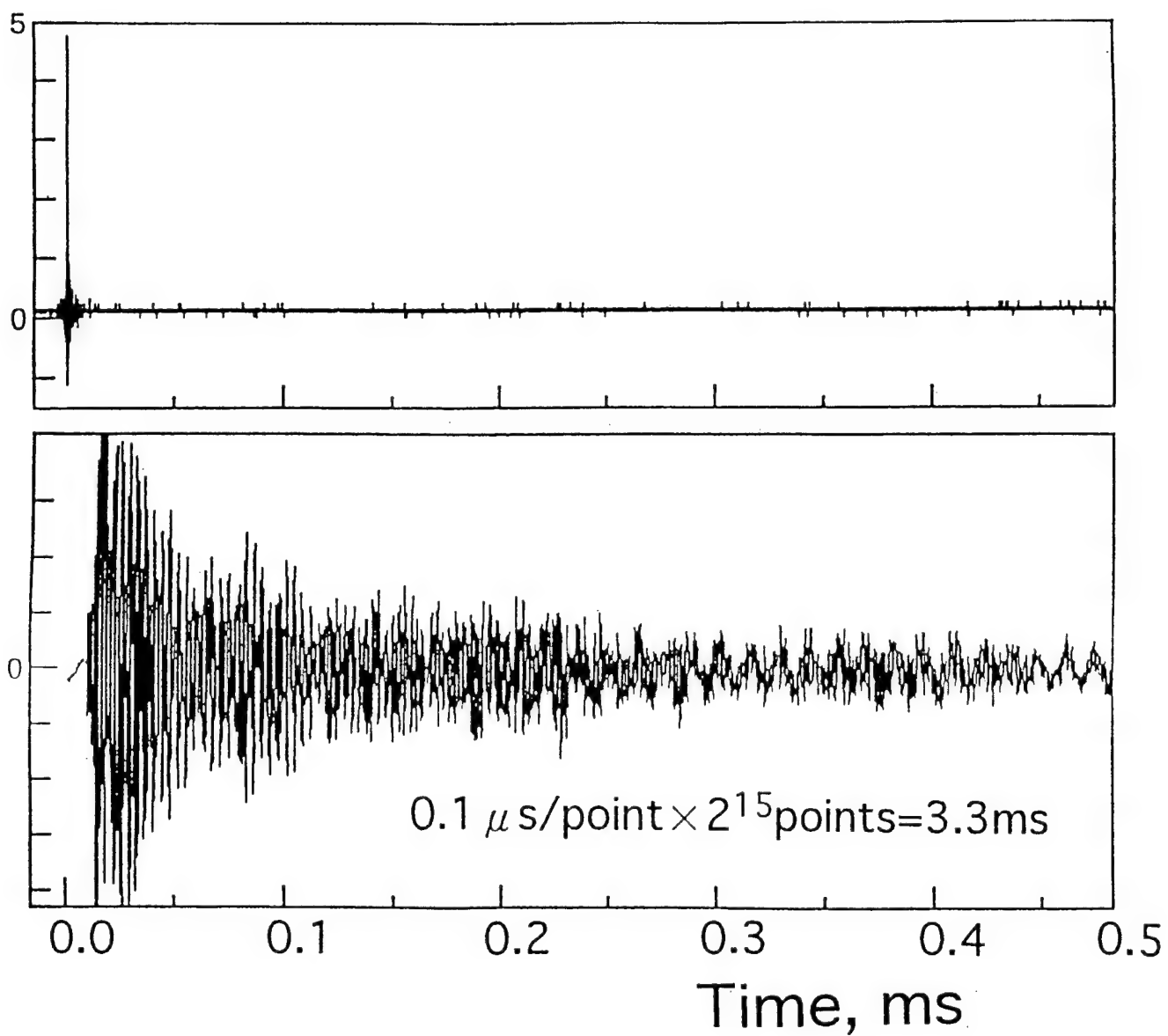
Two kinds of data acquisition

## 1) Continuous Wave (CW) method

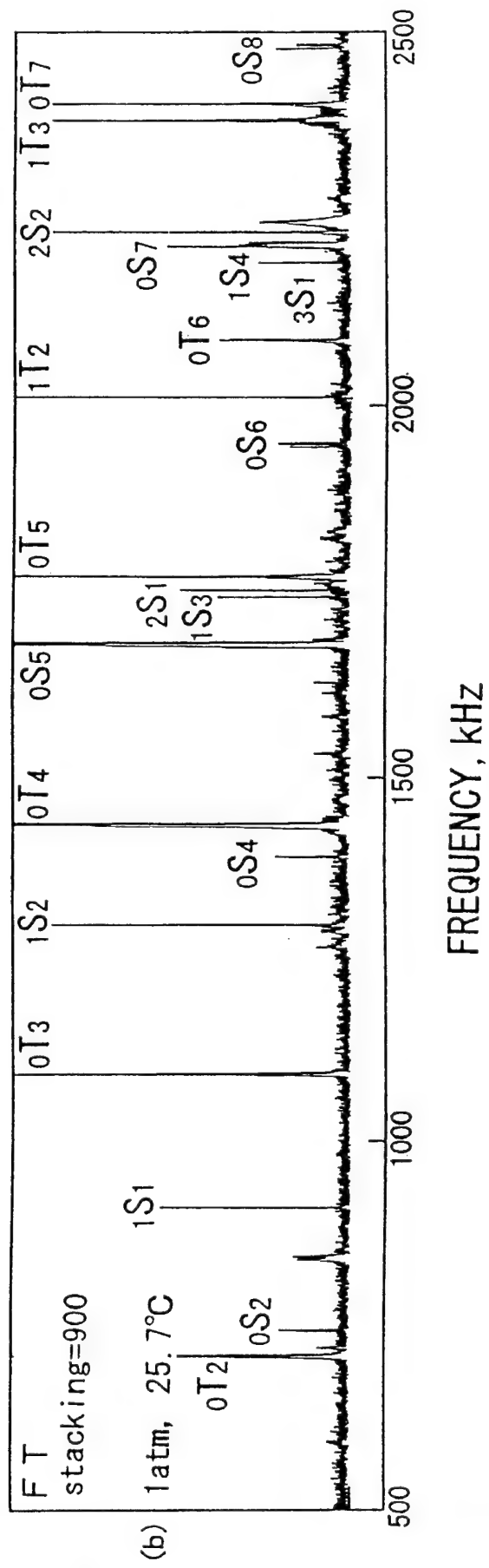
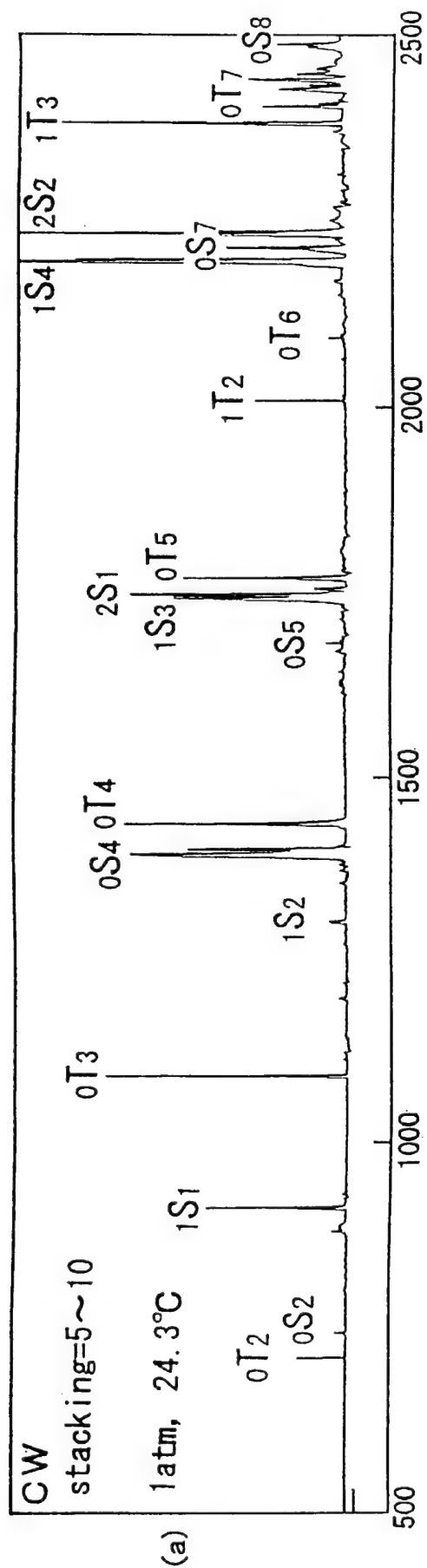


## 2) Fourier Transform (FT) method





An example of input impulse (top) and output response (bottom) in FT method.

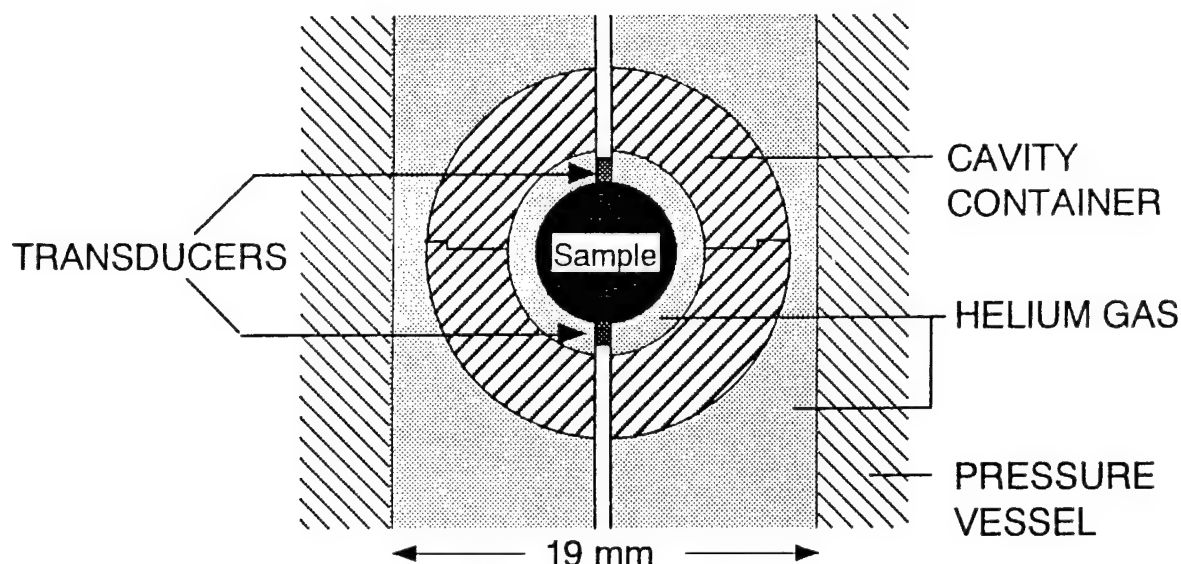


Comparison between spectra acquired by CW method (top) and FT method (bottom).  
sample: silica glass sphere.

## 2. Samples

	<u>Silica Glass Sphere</u>	<u>Steel Sphere</u>
Diameter	4.245 mm	4.766 mm
Density	2.214 g/cm <sup>3</sup>	7.798 g/cm <sup>3</sup>
Poisson's Ratio	0.164	0.293
Shear Velocity	3.768 km/s	3.194 km/s
Bulk Modulus	36.24 GPa	165.5 GPa
Rigidity	31.38 GPa	79.56 GPa

### 3. Measurement under Gas Pressure



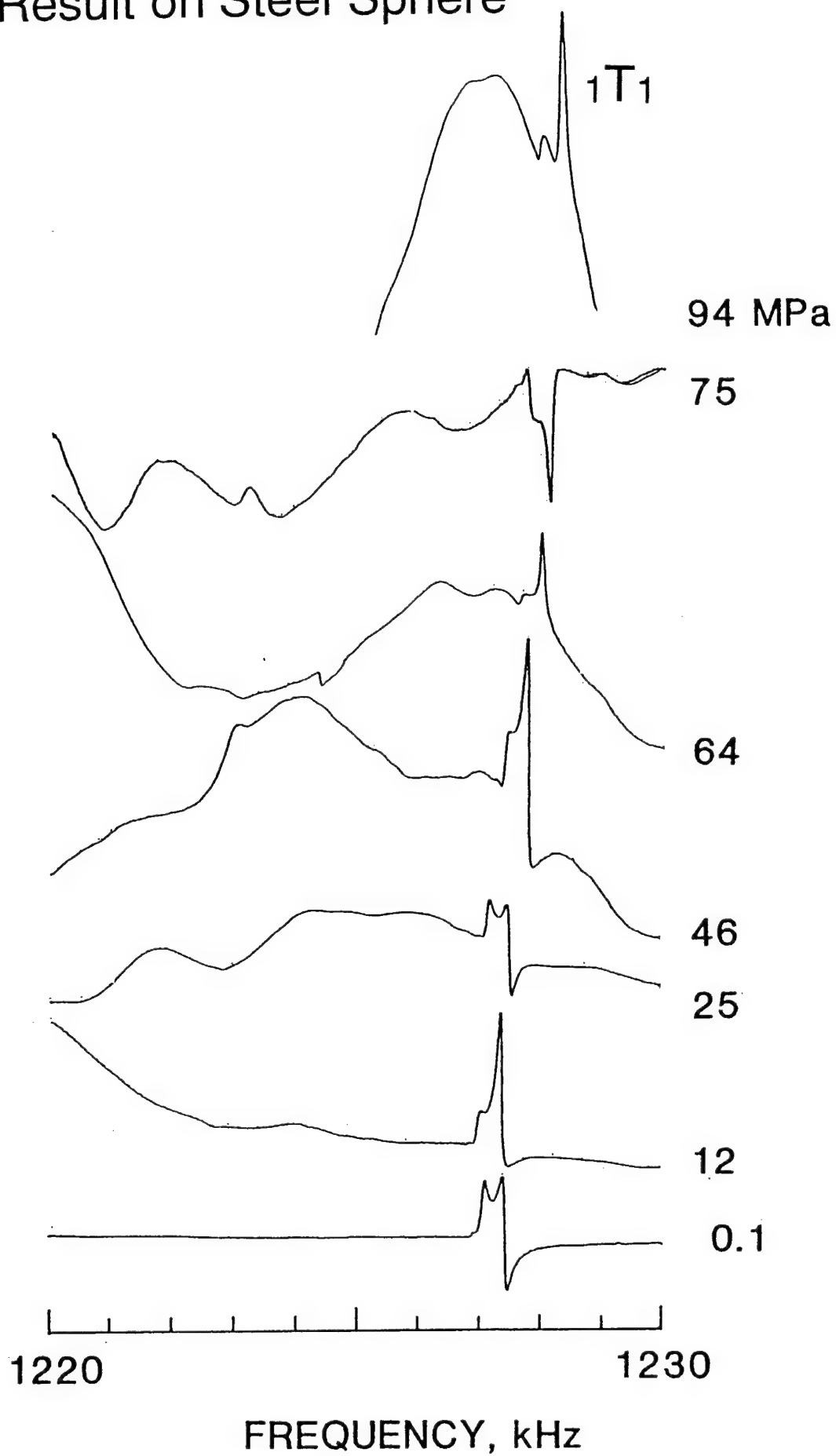
Sample assembly for measurement under pressure.

- ┌ cavity container: 5mm in inner, and 17mm in outer diameter
- └ transducer: PZT, shear type, 2.5MHz
- └ pressure range: 1atm – 200MPa (2kbar)

Eigenfrequencies of the sample-cavity(gas layer)-cavity container system were measured under gas pressure. The cavity container was used to make a well-defined boundary condition. The pressure shift of frequency of torsional mode gives  $G_0'$ , and those of spheroidal mode gives  $K_0'$ .

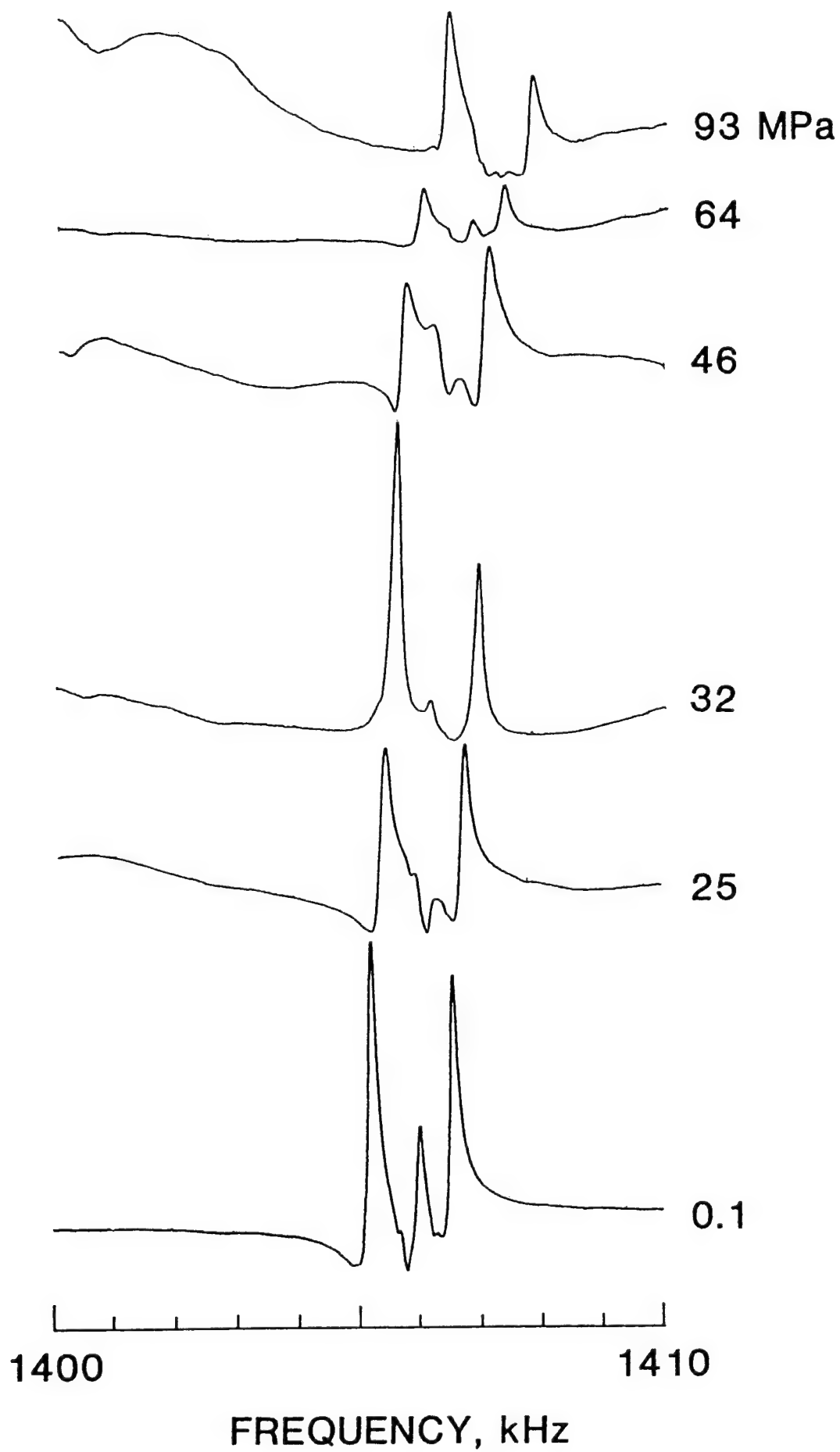


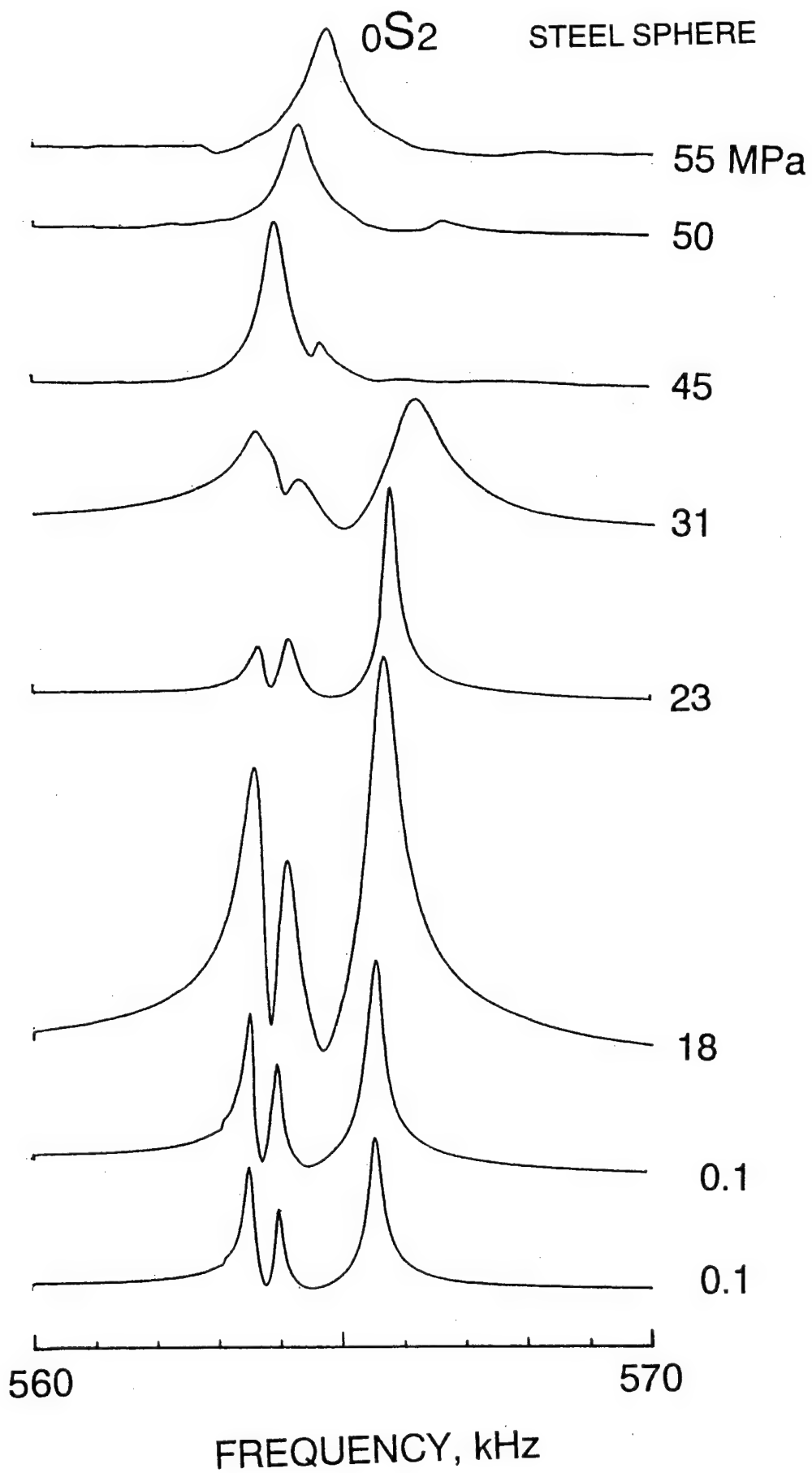
#### 4. Result on Steel Sphere

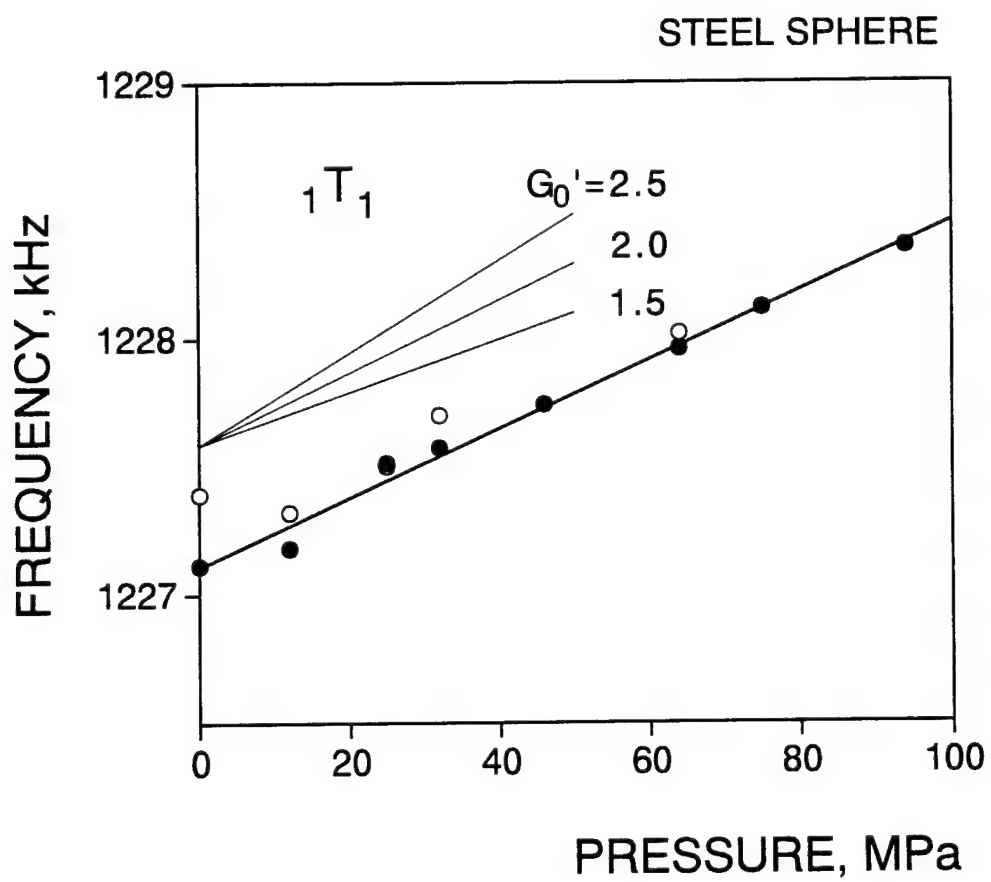


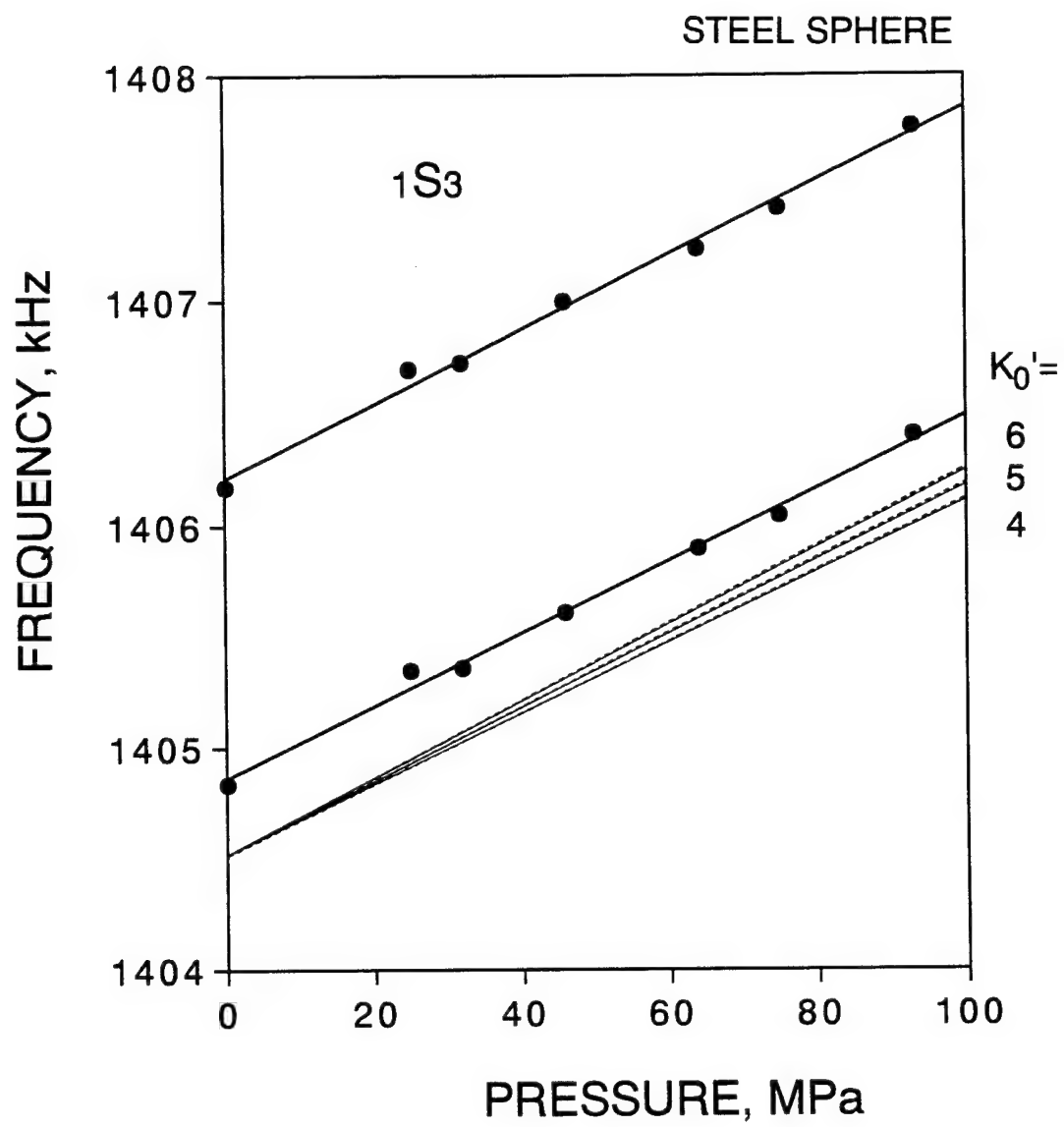
1S3

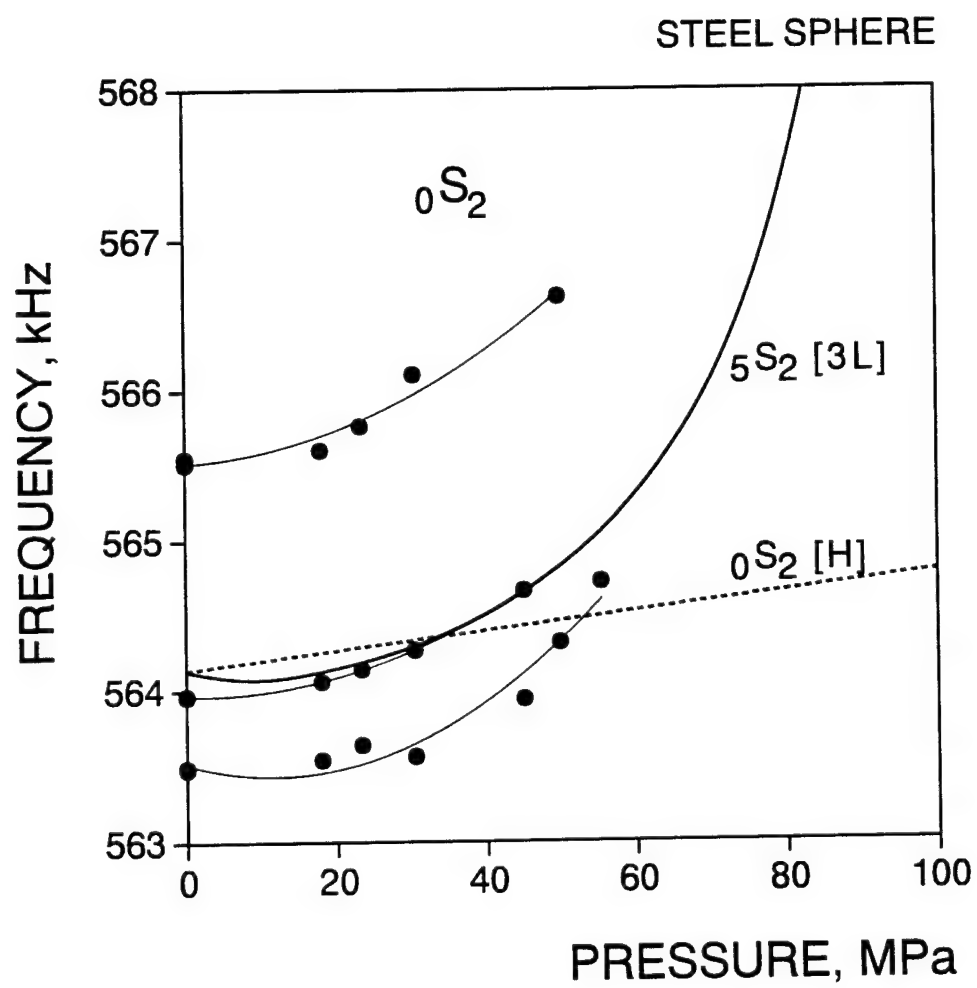
STEEL SPHERE



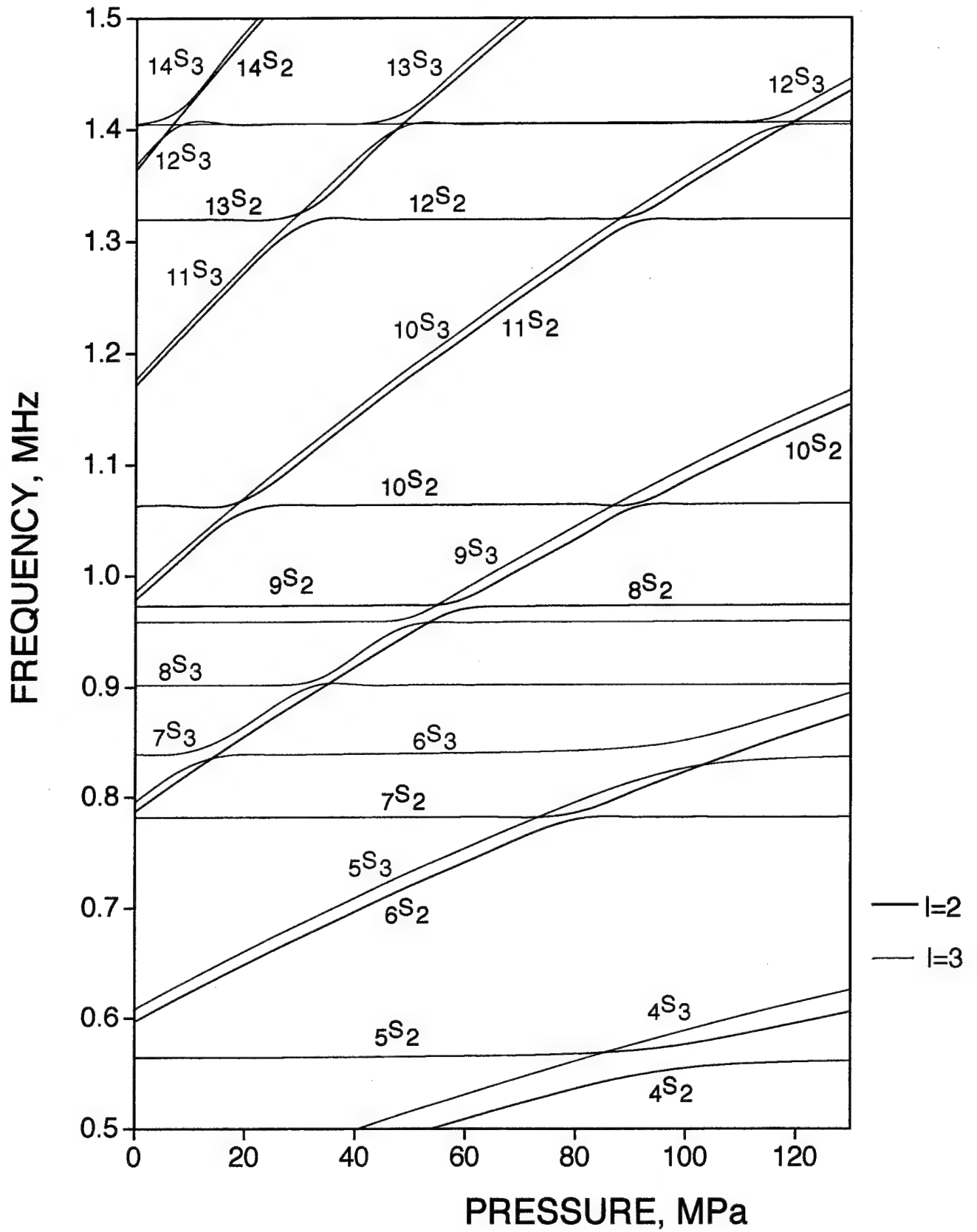






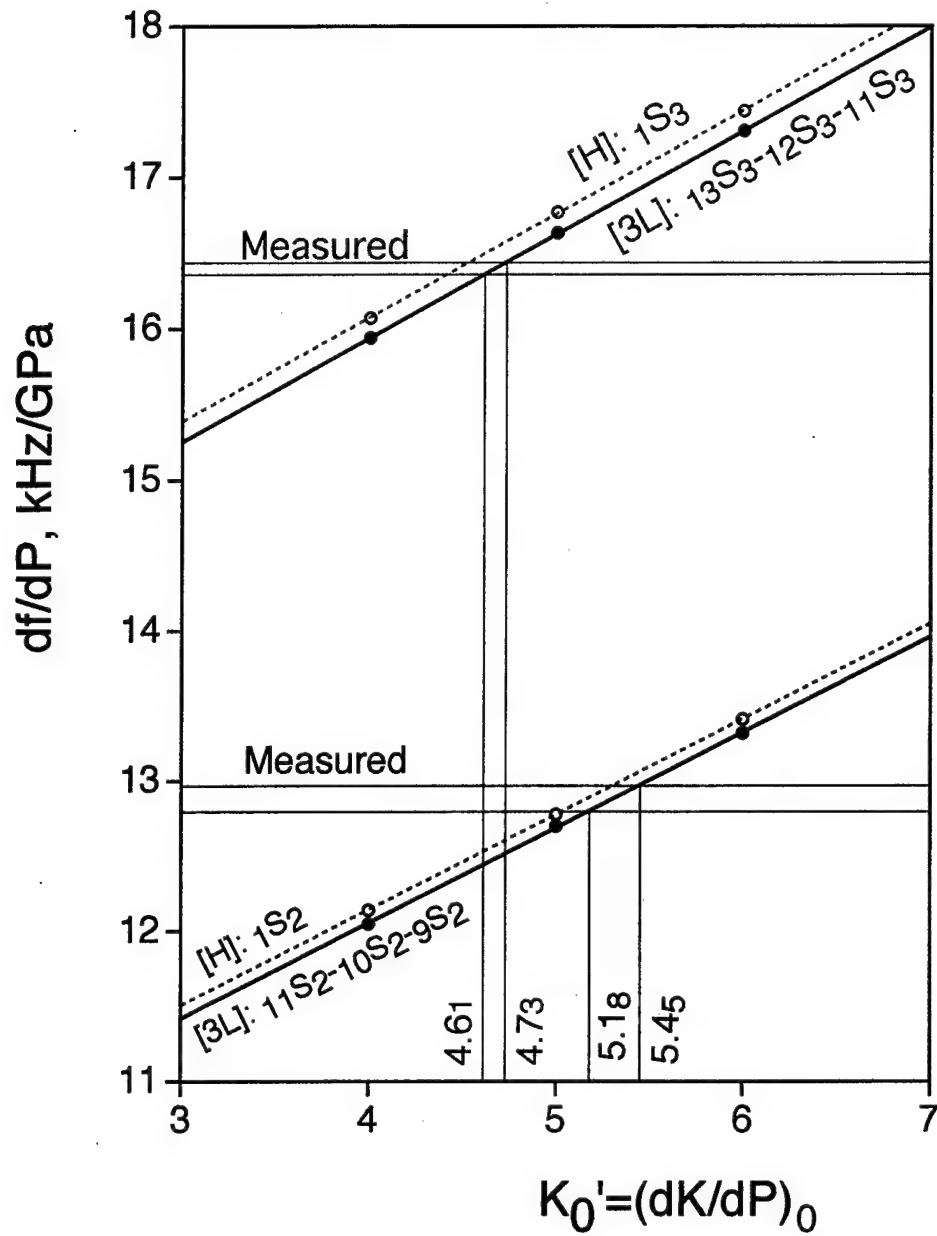


# STEEL SPHERE



Theoretical eigenfrequencies of the three-layered spherical structure. Steel sphere is 4.245 mm and the cavity is 10 mm in diameter.

# STEEL SPHERE





STEEL SPHERE

$\partial f / \partial P$  of torsional mode and  $G_0'$

Mode	Frequency kHz	$\partial f / \partial P$ kHz/GPa	$G_0' = (\partial G / \partial P)_0$
$0^T_4$	1087.0	13.4(7)	2.12
$1^T_1$	1227.1	13.5(6)	1.91
$0^T_5$	1335.9	15.1(9)	1.96
$0^T_5$	1336.4	15.7(7)	2.03
$0^T_5$	1337.0	15.8(6)	2.01
		Ave.	2.01(8)

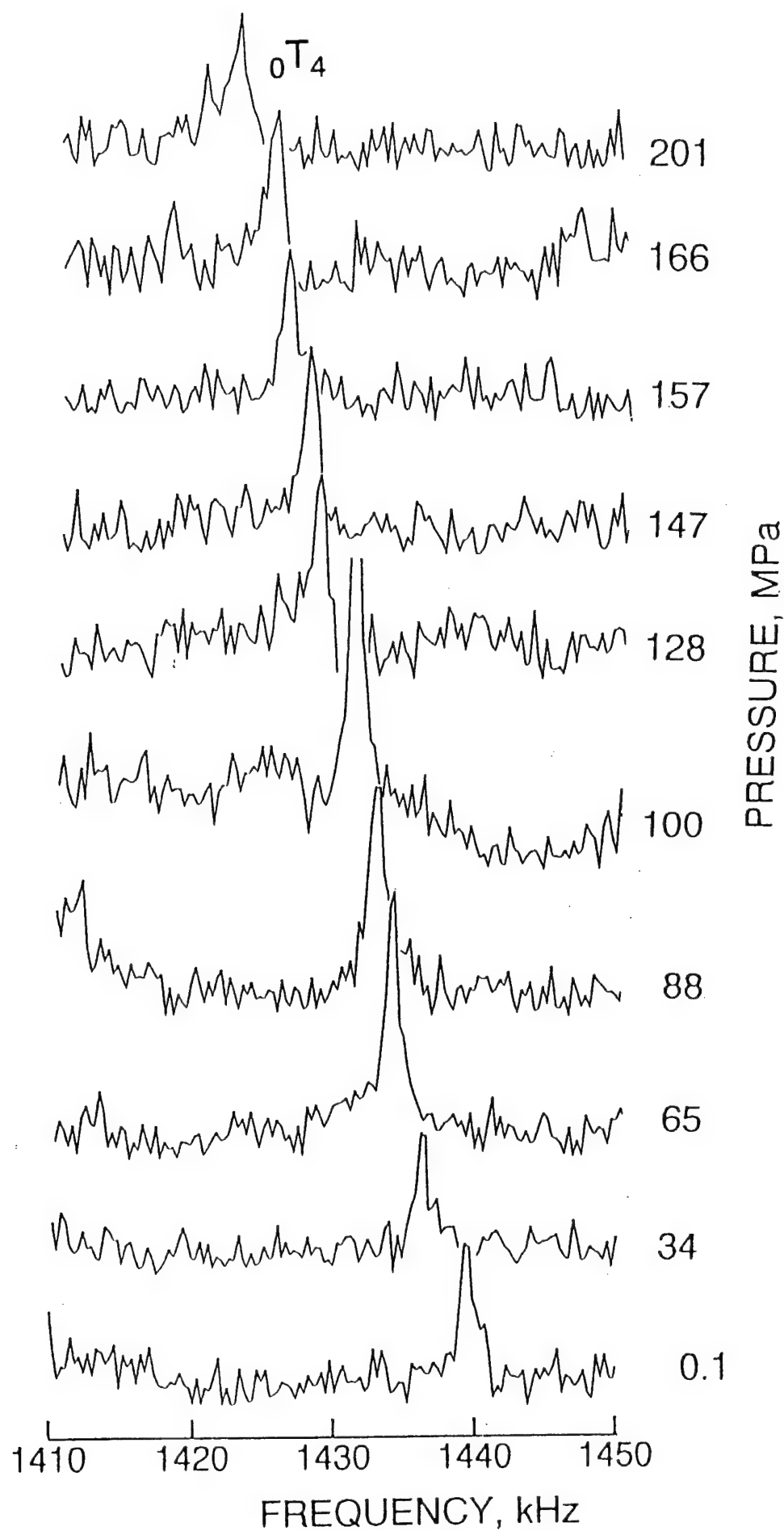
# $\partial f / \partial P$ of spheroidal mode and $K_0'$

Mode	Frequency kHz	$\partial f / \partial P$ kHz/GPa	$K_0' = (\partial K / \partial P)_0$ [H]	[3L]
$1S_2$	1063.1	12.97(4)	5.28	5.45
$1S_2$	1064.1	12.79(5)	4.88	5.18
$1S_3$	2138.7	16.35(6)	4.39	4.61
$1S_3$	2214.1	16.43(7)	4.49	4.73
		Ave.	4.76	4.99

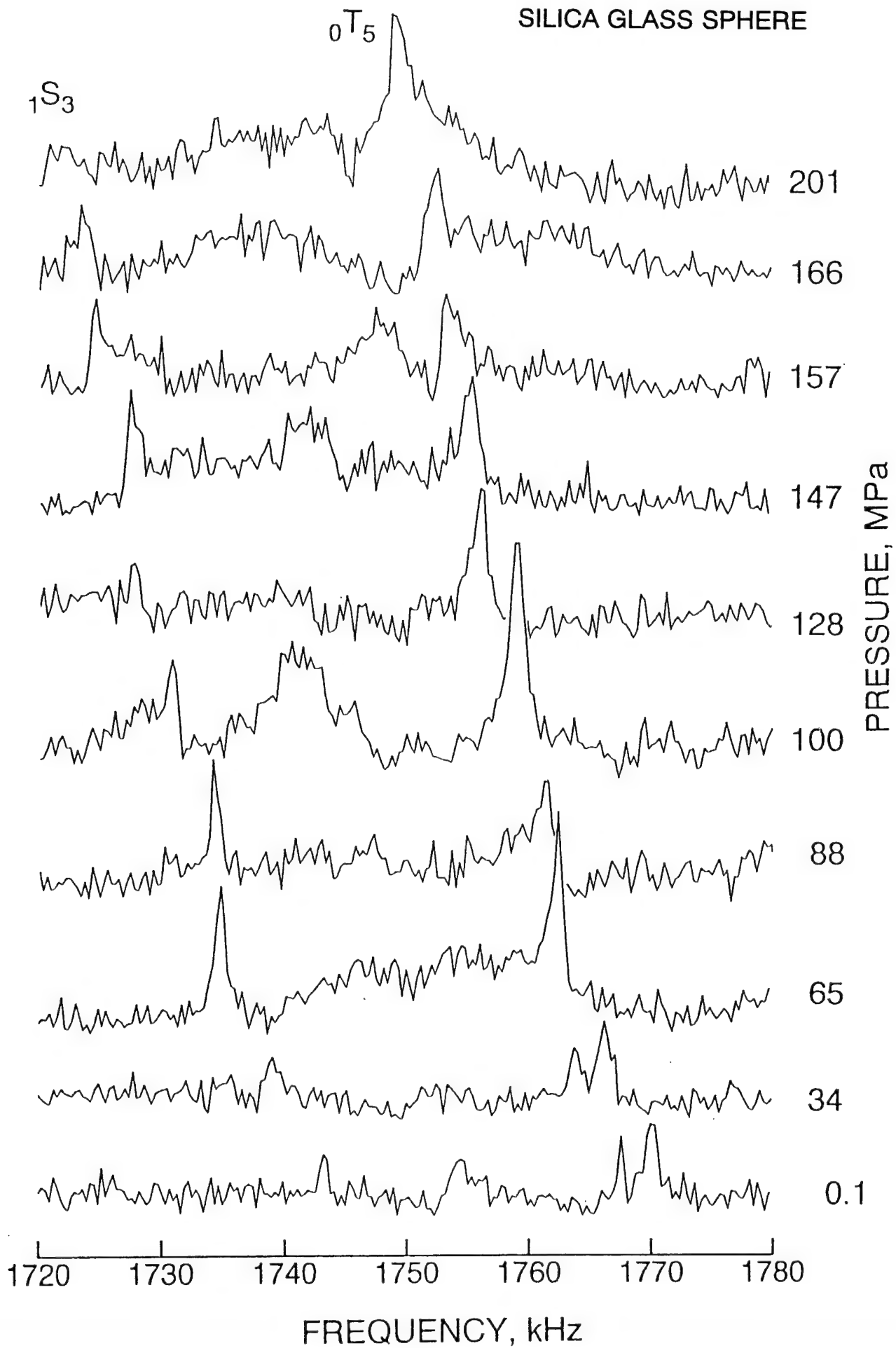
[H] : Analyzed as homogeneous sphere of sample only

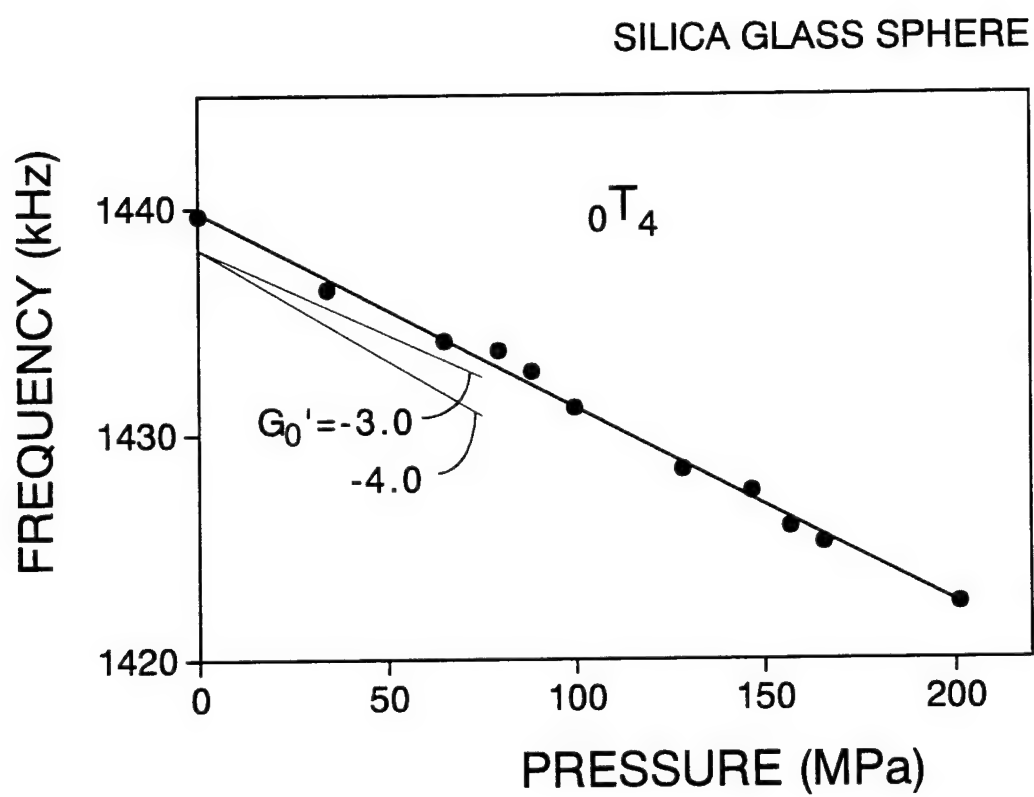
[3L] : Analyzed as three-layered spherical structure

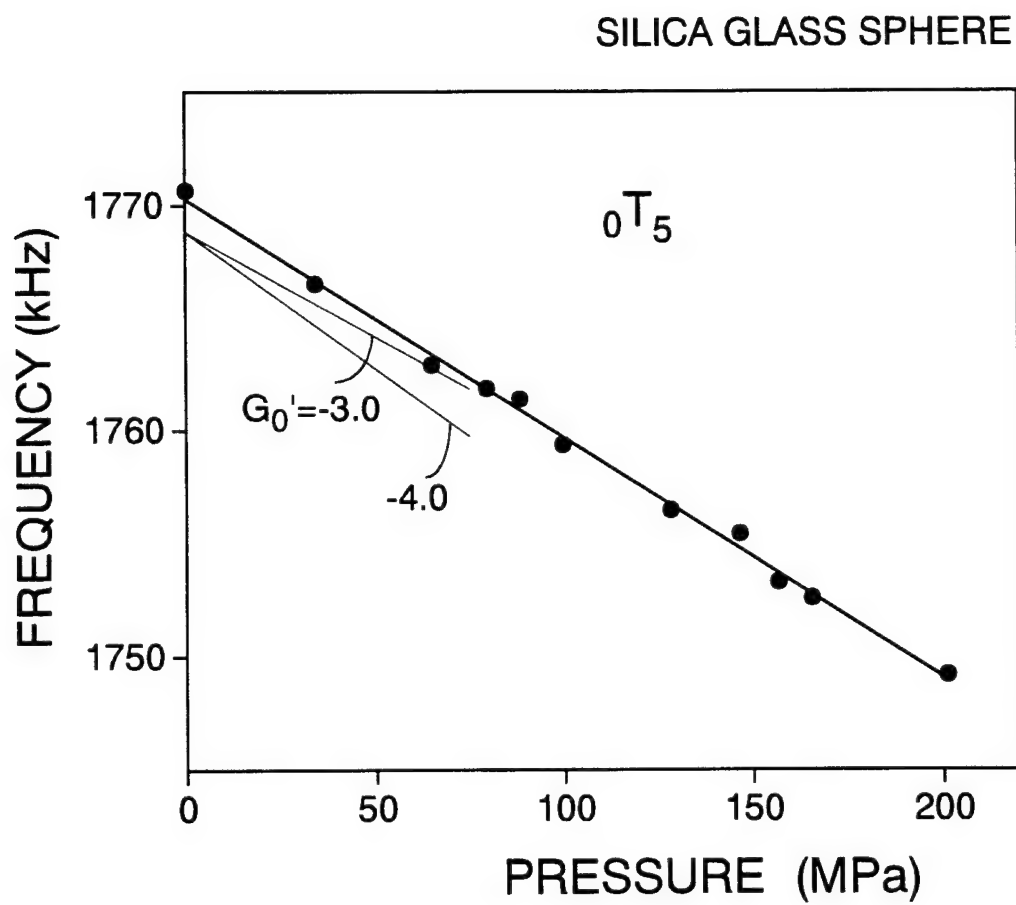
## 5. Result on Silica Glass Sphere

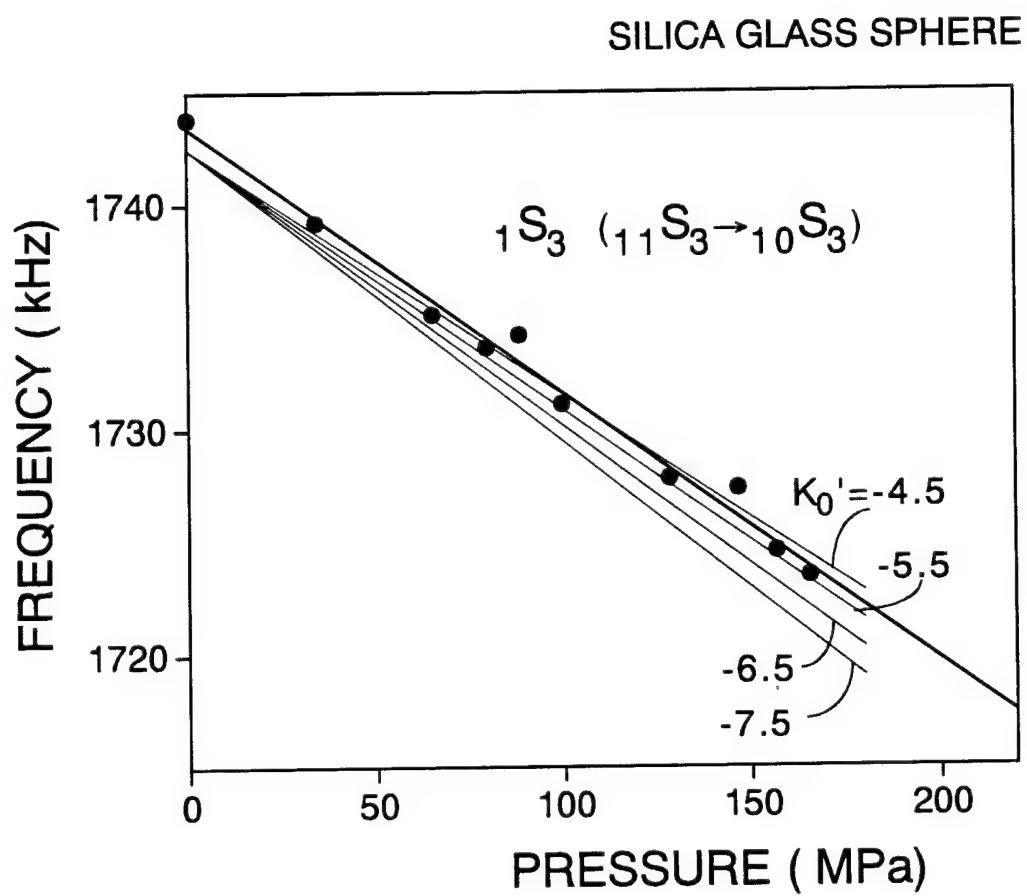


# SILICA GLASS SPHERE

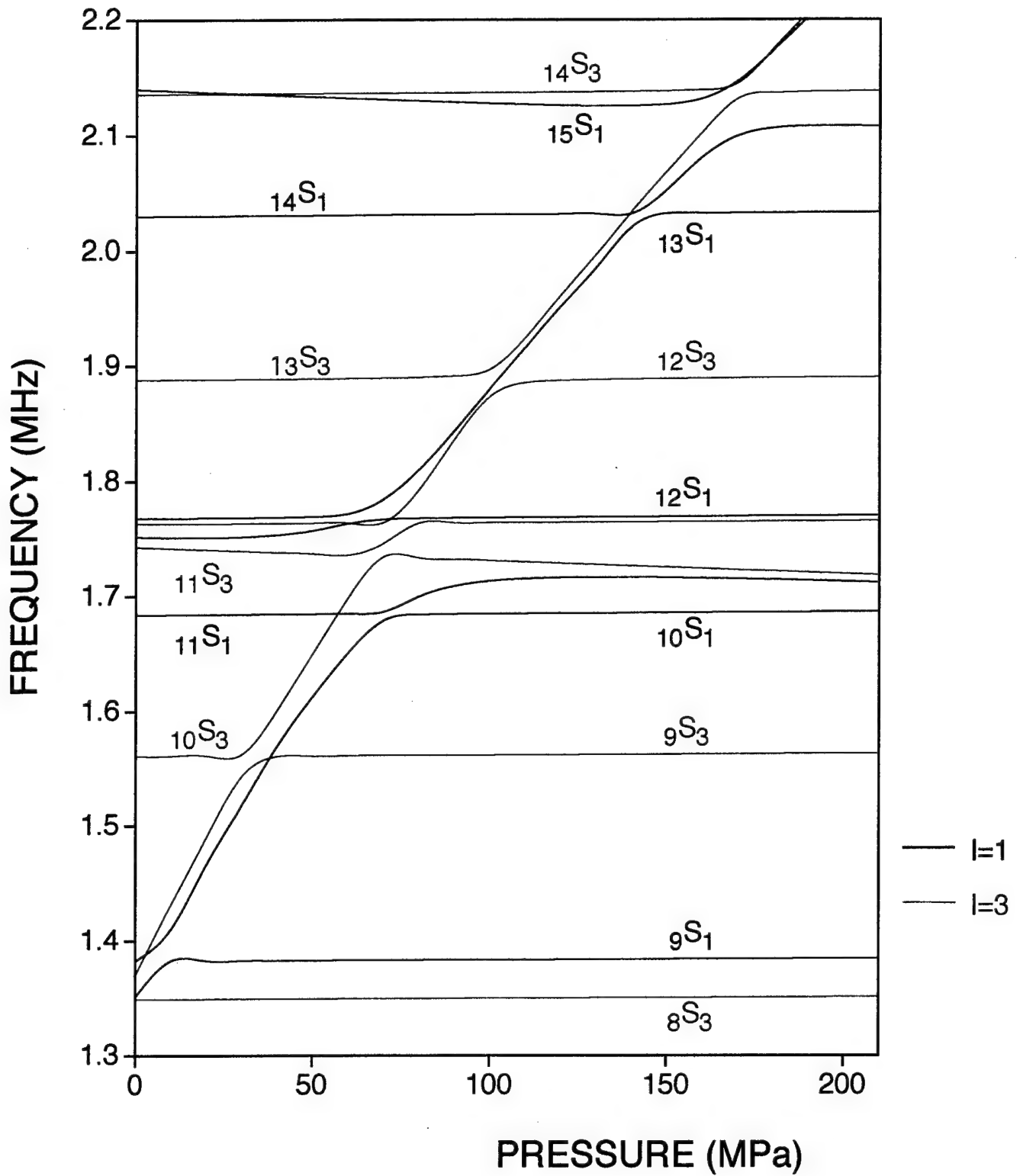








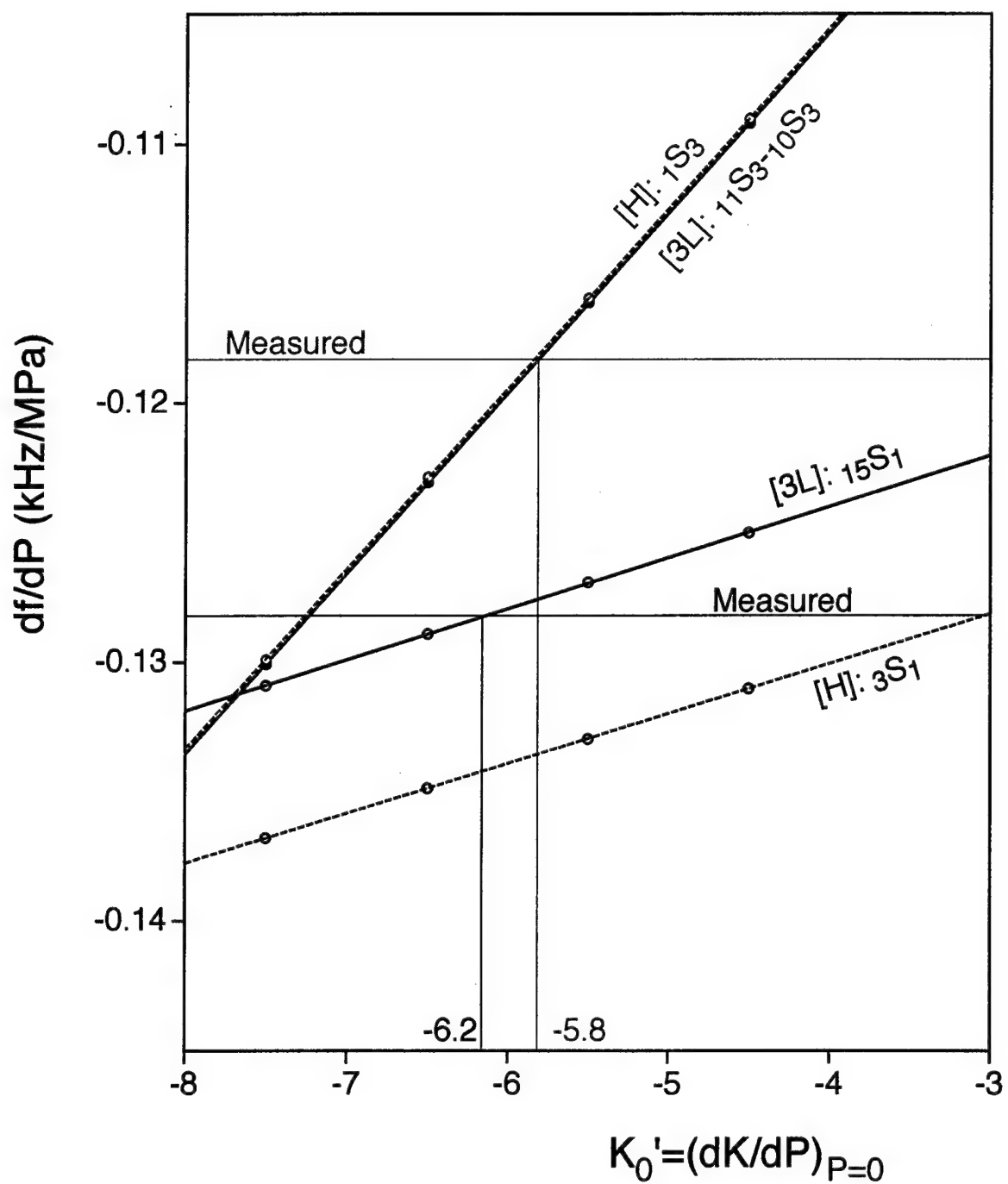
# SILICA GLASS SPHERE



Theoretical eigenfrequencies of the three-layered spherical structure.  
Silica glass sphere is 4.764 mm and the cavity is 5 mm in diameter.



# SILICA GLASS SPHERE



$\partial f / \partial P$  of torsional mode and  $G_0'$ 

Mode	Frequency kHz	$\partial f / \partial P$ kHz/GPa	$G_0' = (\partial G / \partial P)_0$
$0T_2$	709.9	-49(1)	-4.04
$0T_3$	1093.9	-69(2)	-3.67
$0T_4$	1439.7	-86(2)	-3.46
$1T_1$	1626.2	-94(2)	-3.61
$0T_5$	1770.6	-106(2)	-3.47
$0T_6$	2091.5	-119(4)	-3.28
$1T_3$	2382.3	-132(5)	-3.19
$0T_7$	2407.0	-141(4)	-3.39
Ave.			-3.51

# $\partial f / \partial P$ of spheroidal mode and $K_0'$

Mode	Frequency kHz	$\partial f / \partial P$ kHz/GPa	$K_0' = (\partial K / \partial P)_0$ [H] [3L]
$1S_3$	1742.8	-118(4)	-5.8 <sub>5</sub> -5.8 <sub>2</sub>
$3S_1$	2138.7	-128(6)	-3.1 -6.2
Ave.			-6.0

[H] : Analyzed as homogeneous sphere of sample only

[3L] : Analyzed as three-layered spherical structure

## Silica Glass

$G_0'$	$K_0'$	Press. Range GPa	Reference
-3.25	-6.15	0.1	Peselnick et al. (1967)
-3.48	-6.31	1.8	Gerlich & Kennedy (1978)
-3.4	-6.0	0.1	Meister et al. (1980)
-2.71	-3.21	3.8	Suito et al. (1994)
-3.32		0.015	Isaak et al. (1998)
-3.5	-6.0	0.2	This Study

# NONLINEAR MESOSCOPIC ELASTICITY

R. A. GUYER

UMASS

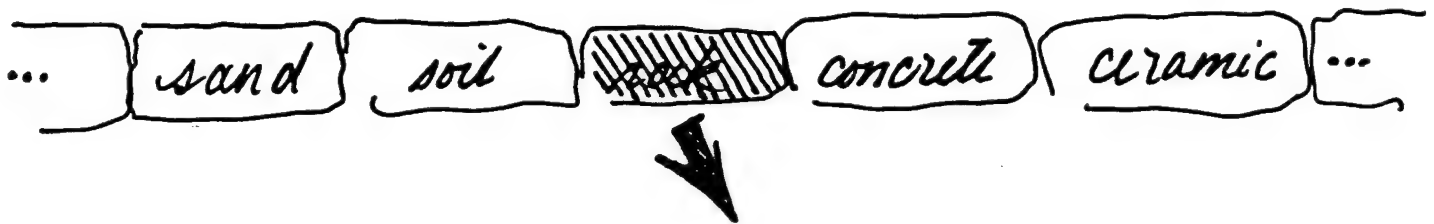
and

P. A. JOHNSON

LANL

KRM, K<sup>rd</sup>A, JT

TJS, ES, GB, LBL, ...



4. Is a generalization possible?
2. Rock as prototype
3. focus on "signatures" of unusual elastic behavior

I. DEEP BACKGROUND ; quasi-static + dynamic

## II SIGNATURES

4. NL - H - DM

2.  $\Delta f \propto |E|$

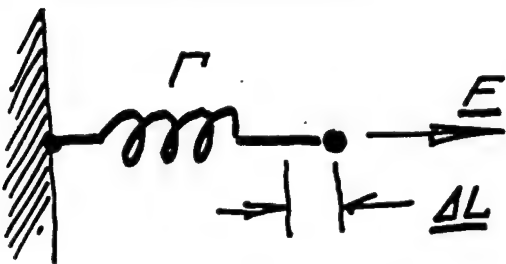
3.  $\log(t)$ ; slow dynamics

## III A BEGINNING

TR-1

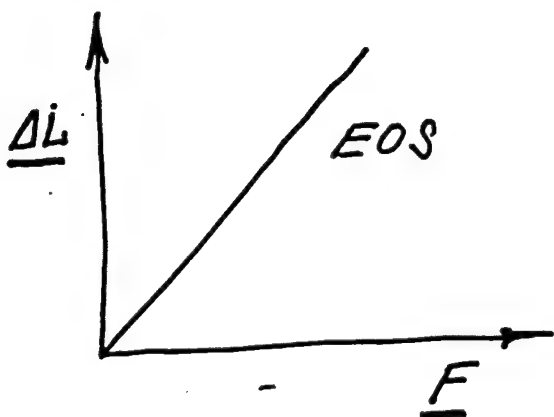
# I. DEEP BACKGROUND:

TEXTBOOK:

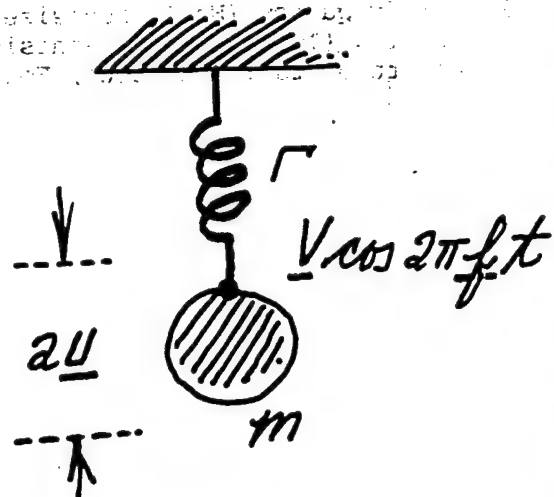


QUASI-STATIC  
MEASUREMENT

vary  $\underline{F}$ , measure  $\underline{\Delta L}$

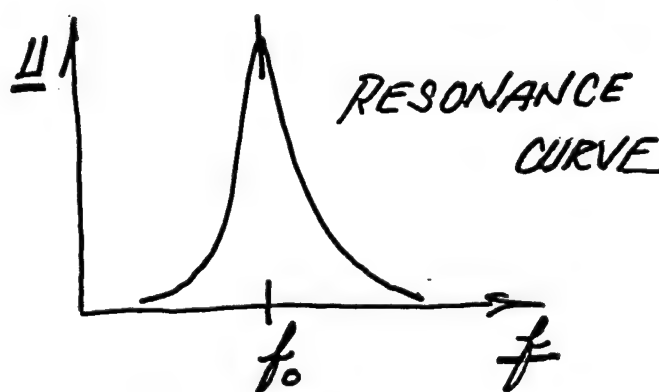


$$\underline{\Delta L} = \frac{1}{\Gamma} \underline{F}$$



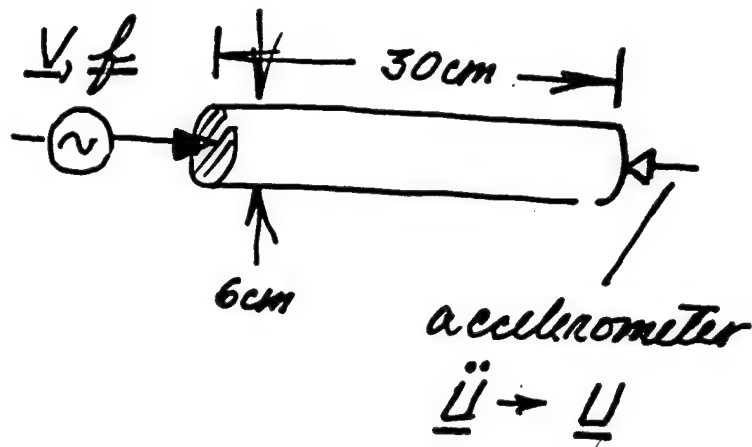
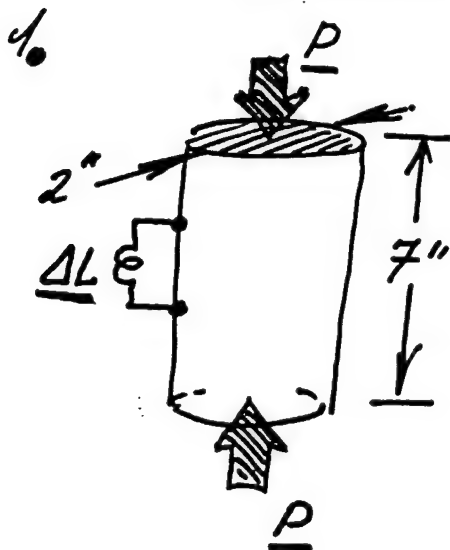
DYNAMIC  
MEASUREMENT

fix  $\underline{V}$ , vary  $f$ ,  
measure  $\underline{U}$

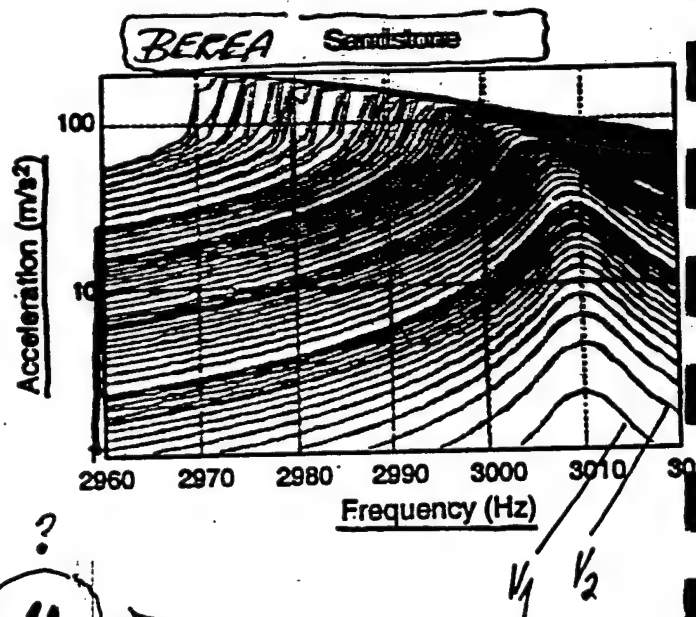
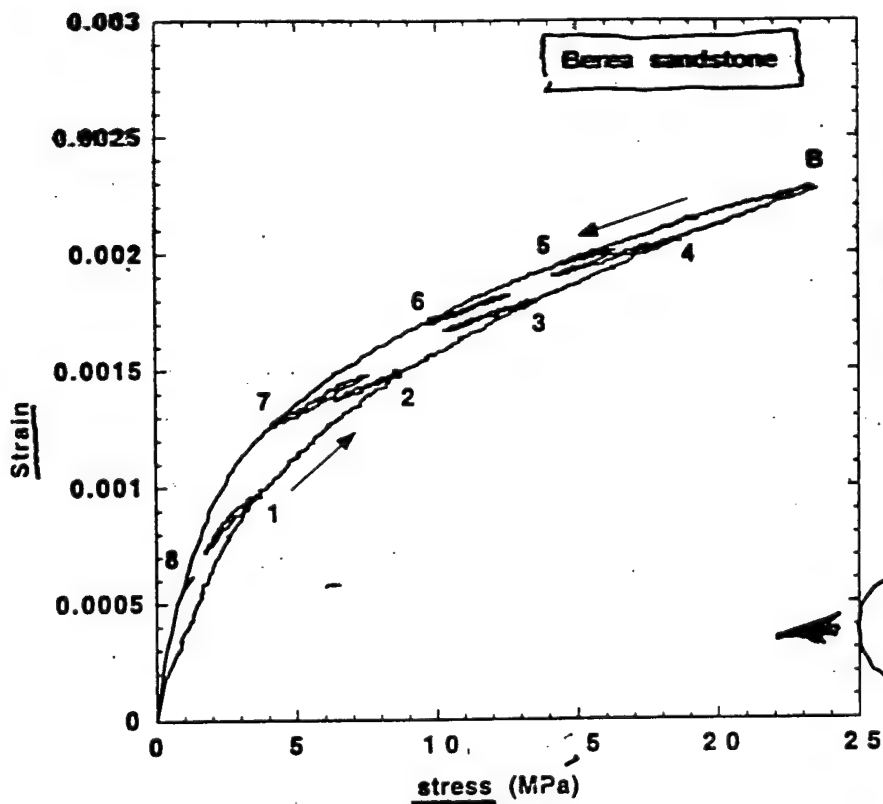
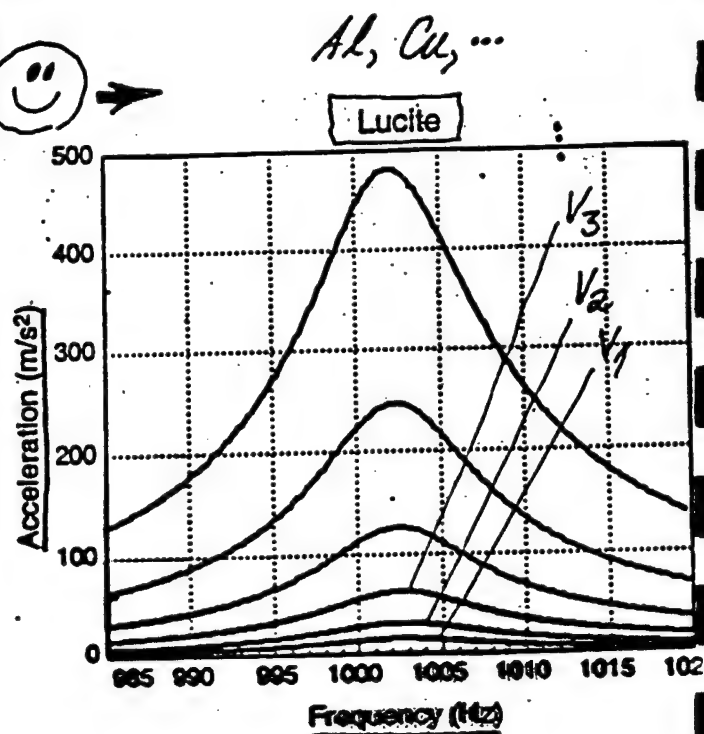
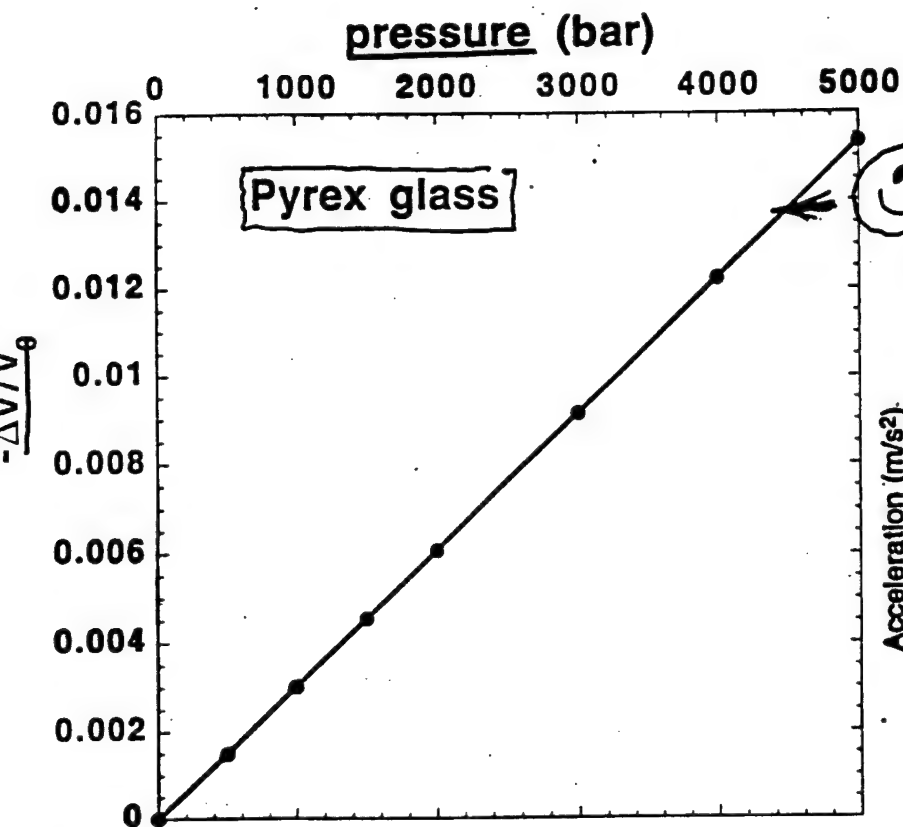


$$2\pi f_0 = \sqrt{\frac{\Gamma}{m}}$$

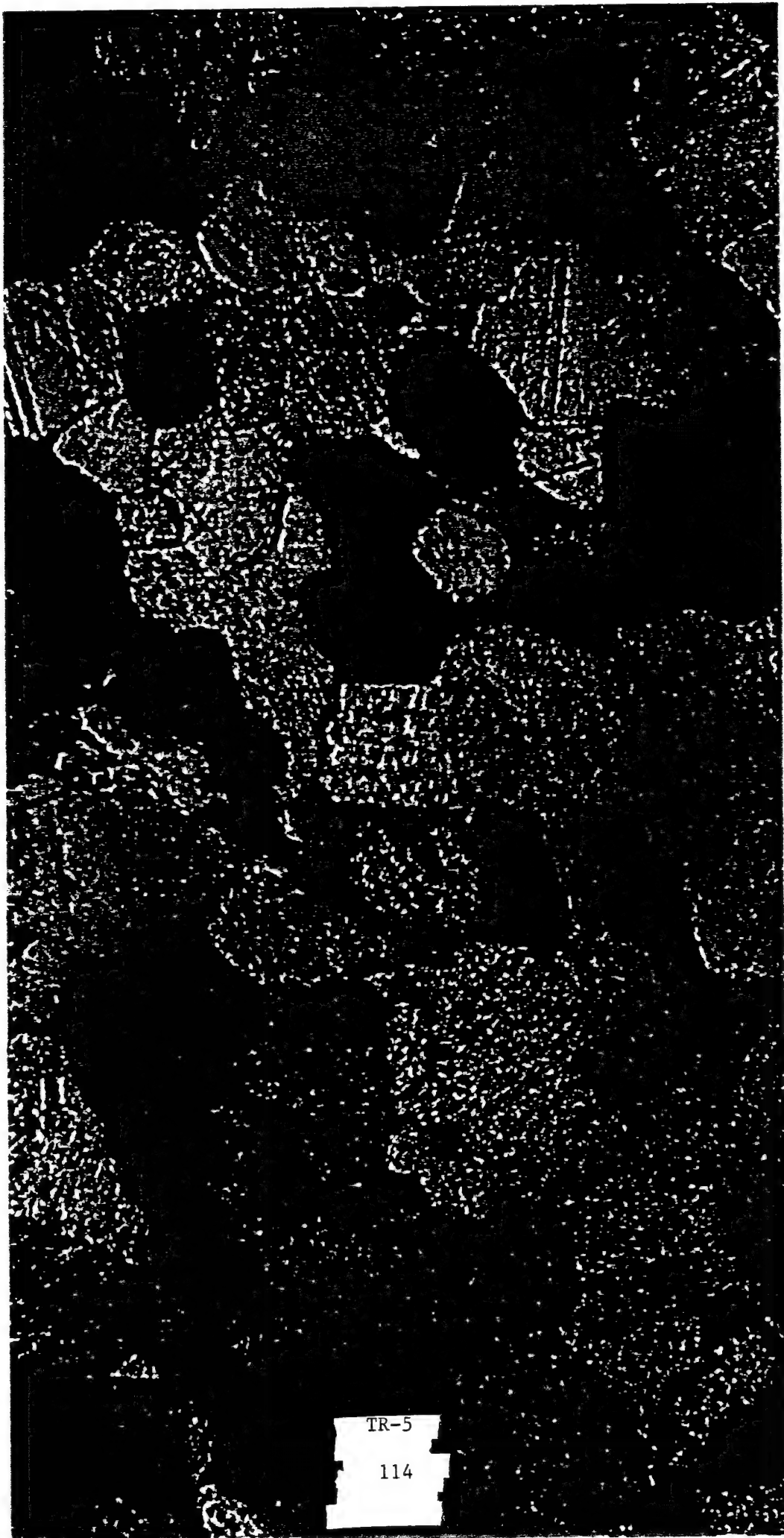
# REAL LIFE



3.



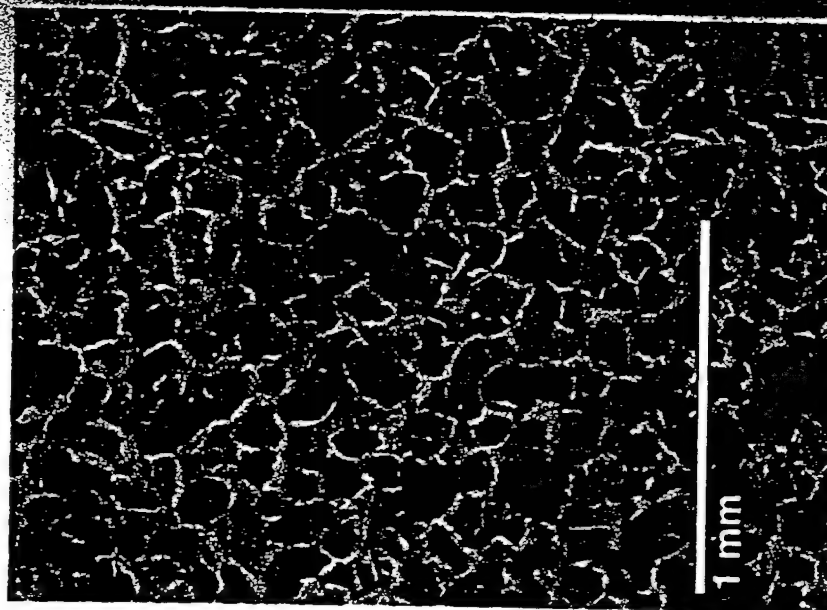




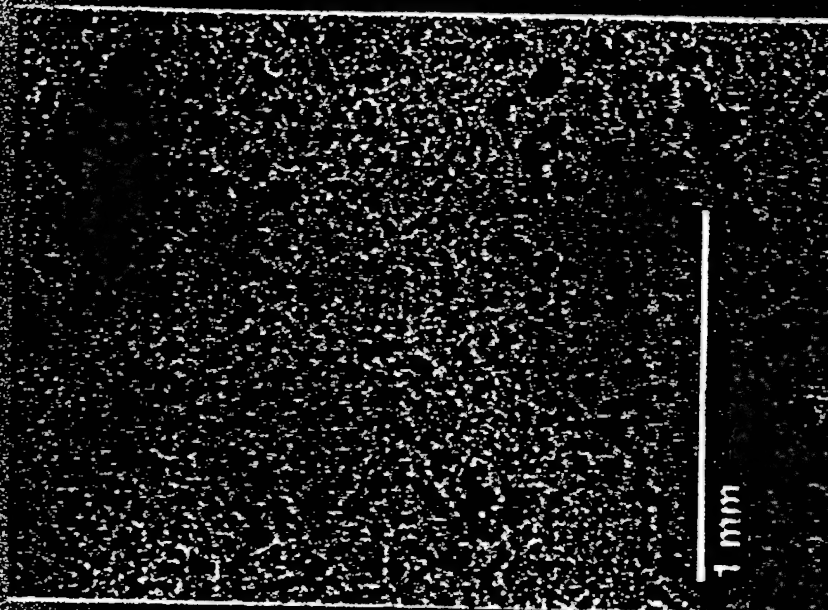
TR-5

114

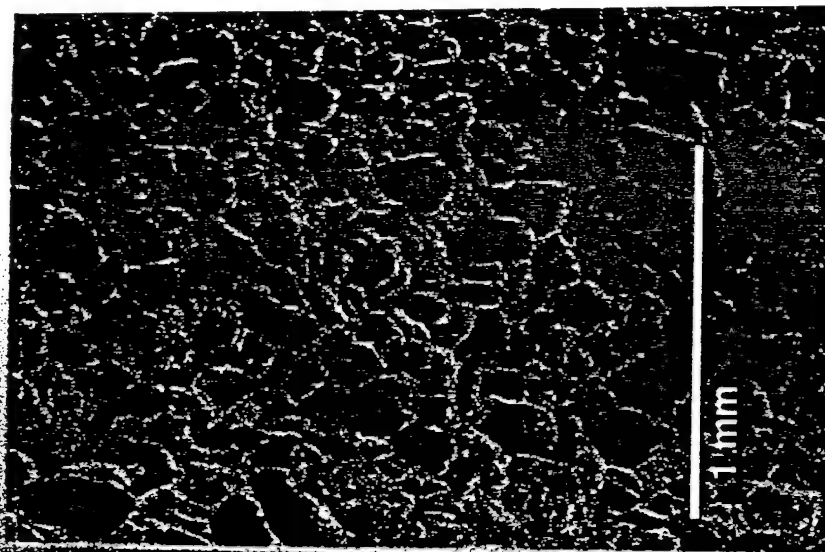
Fontainebleau  
sandstone



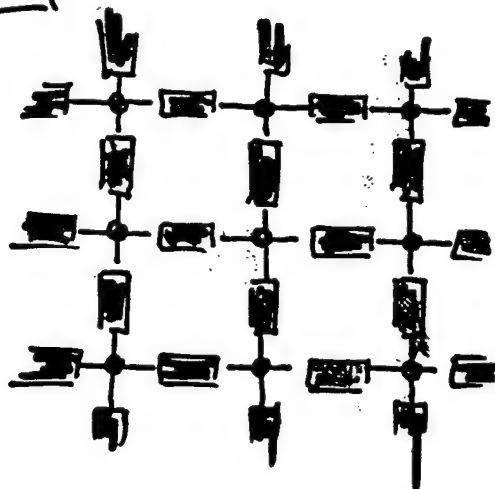
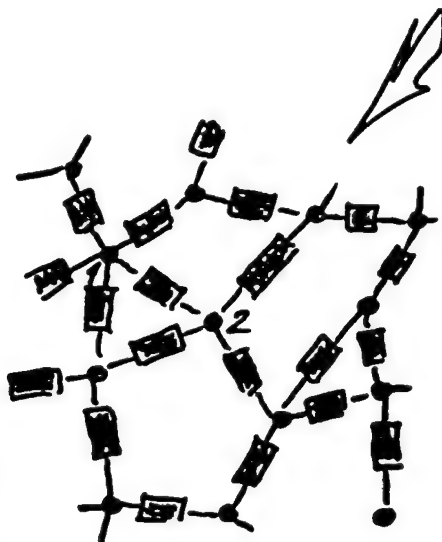
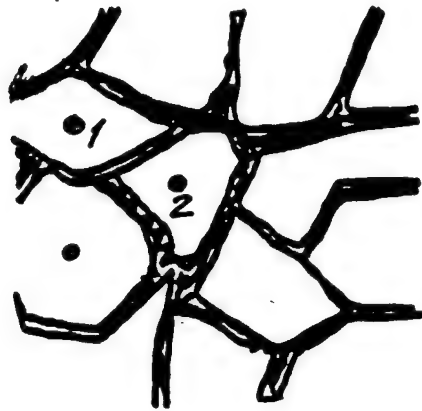
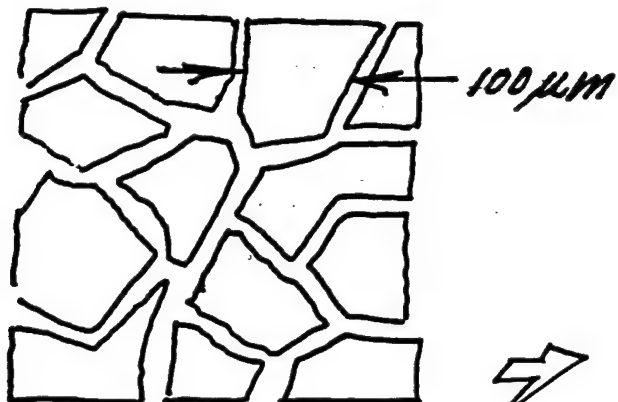
Lavoux  
limestone




Berea  
sandstone



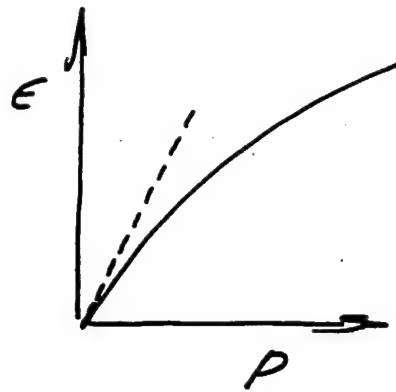
# AN ABSTRACTION:



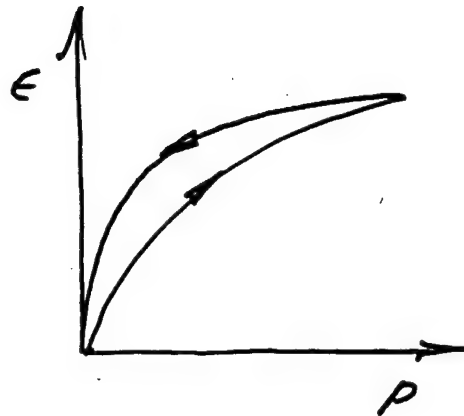
 = ?

LOOK AT  
QUASI-STATIC  
EXPERIMENTS

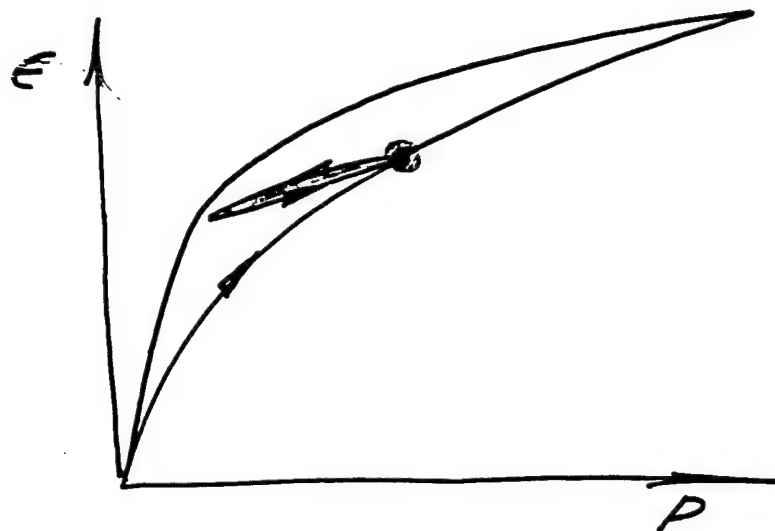
4. NON-LINEAR (VERY)

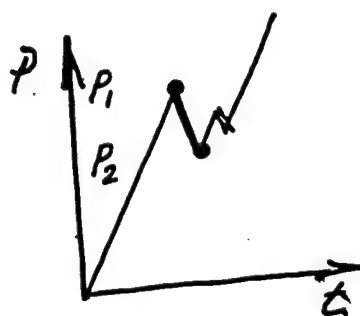
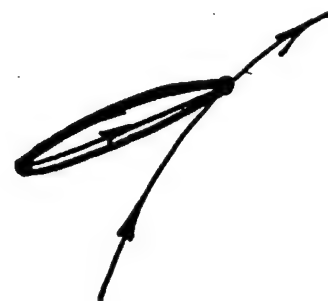
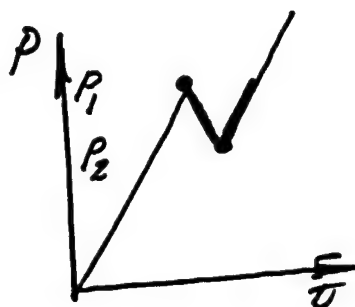
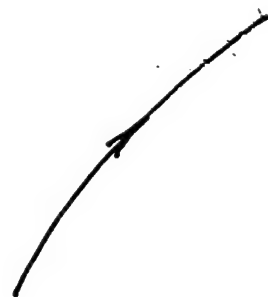
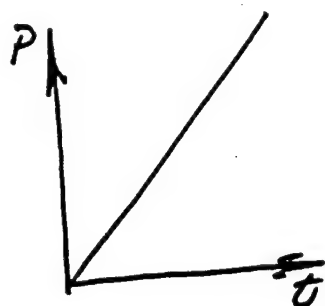
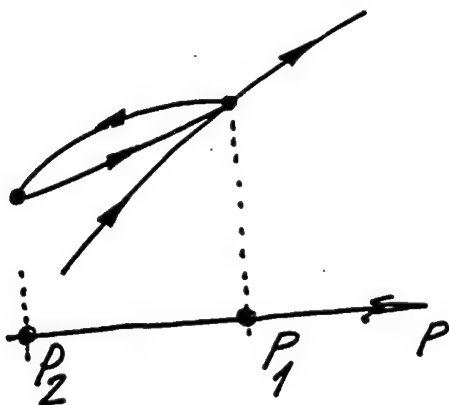


2. HYSTERETIC



3. DISCRETE MEMORY





discrete memory,  $\frac{D^1}{I}$   
[end point memory]

## SUMMARY I.:

4. THE ELASTICITY OF ROCKS EXHIBIT NL, H, DM IN QUASI-STATIC MEASUREMENTS
2. A PHENOMENOLOGY TO EXPLAIN THIS HAS BEEN DEVELOPED [GLYER and McCALL].

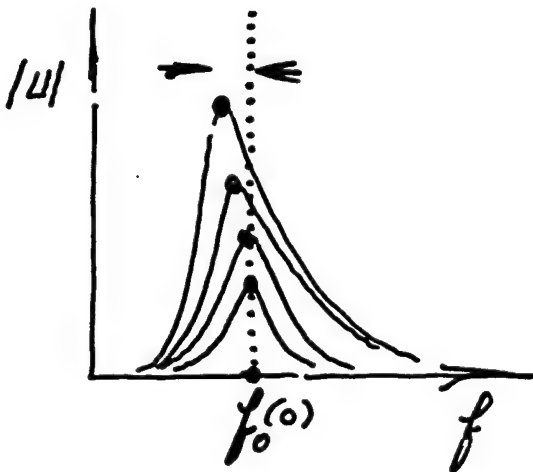
P-M SPACE MODEL, "AN ASSEMBLAGE OF HYSTERETIC ELASTIC BONDS"

~~—~~

\* BOITNOTT

3. THIS PHENOMENOLOGY PREDICTS BEHAVIOR FOR NON-LINEAR RESONANCE THAT HAS BEEN CONFIRMED IN EXPERIMENT.

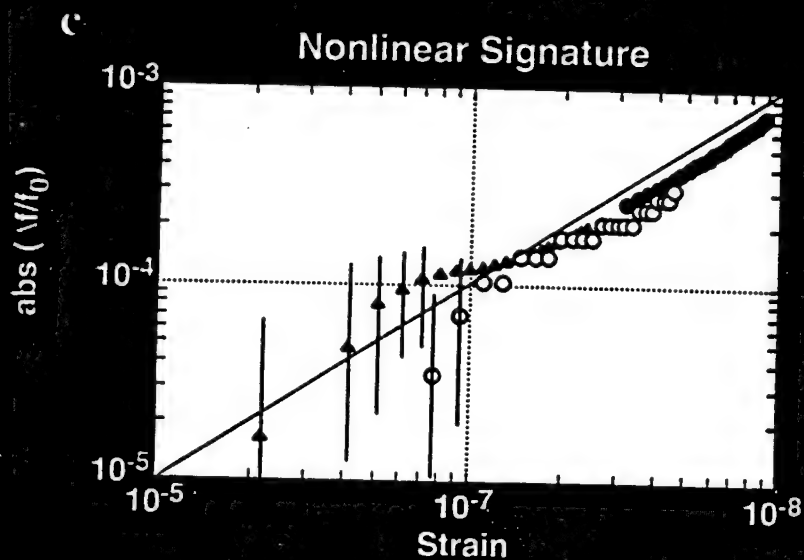
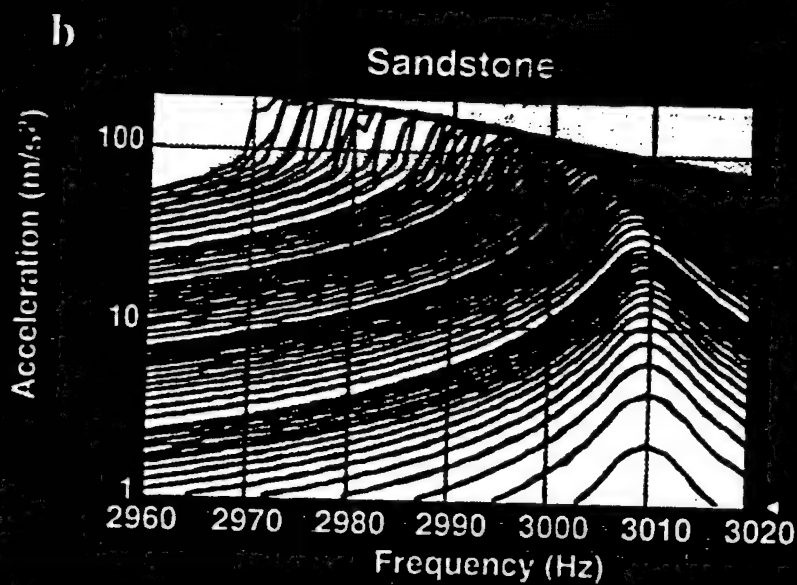
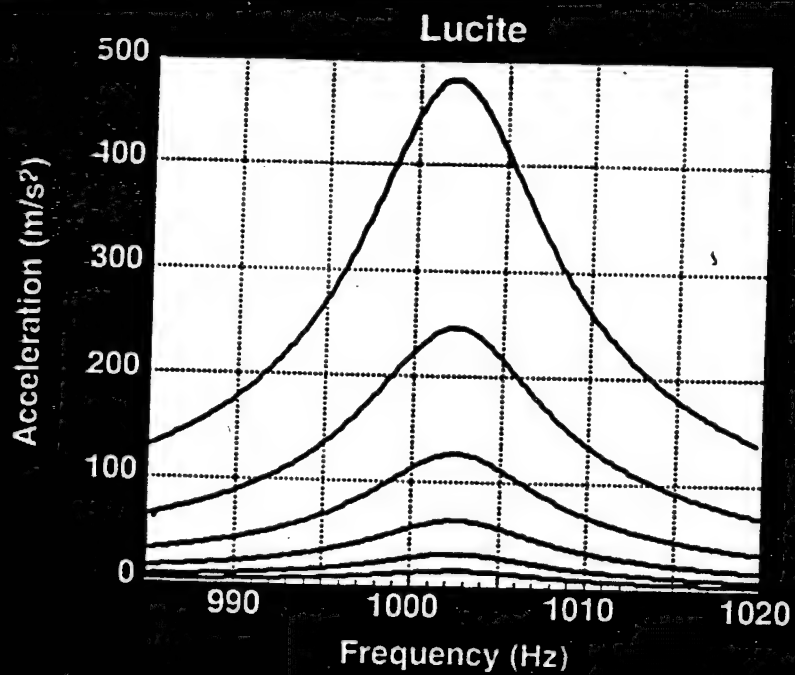
\* TENCATE



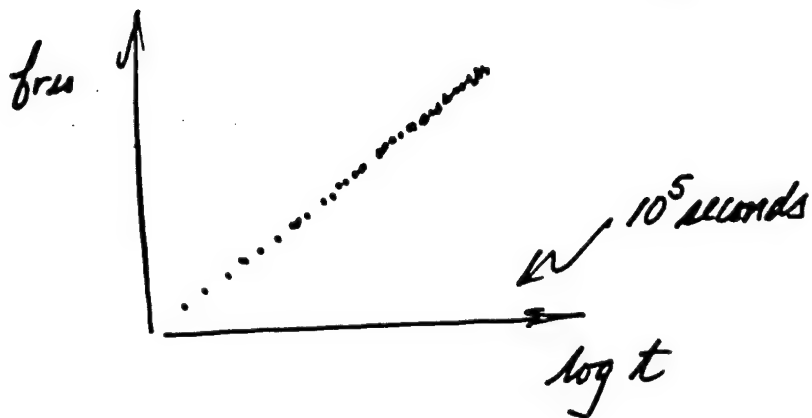
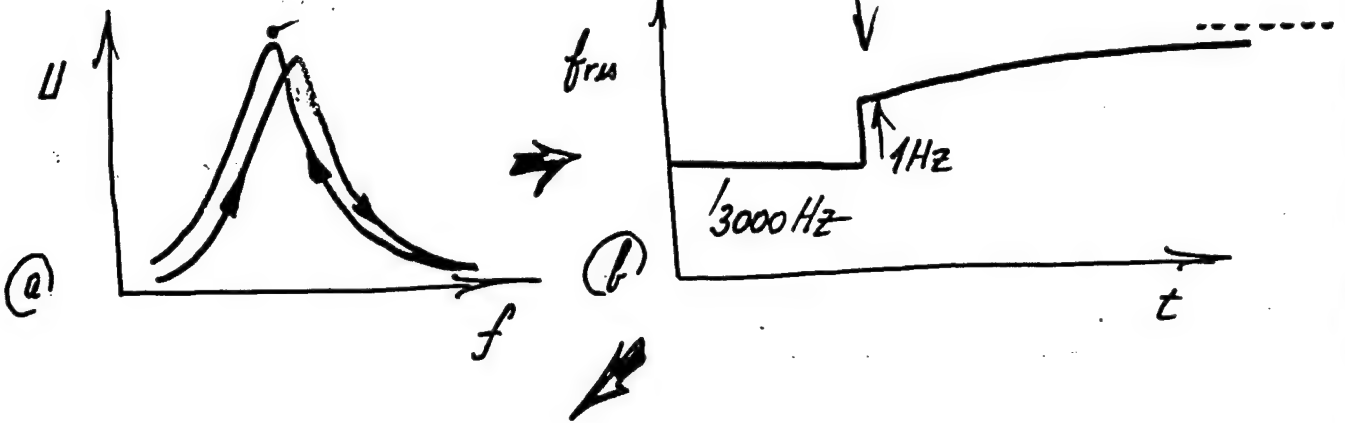
$$f_0(0) - f_0(u) = \frac{c}{F} |u|$$

GMCB

!



#### 4. SLOW DYNAMICS



#### 5. SIGNATURES



NLHDM

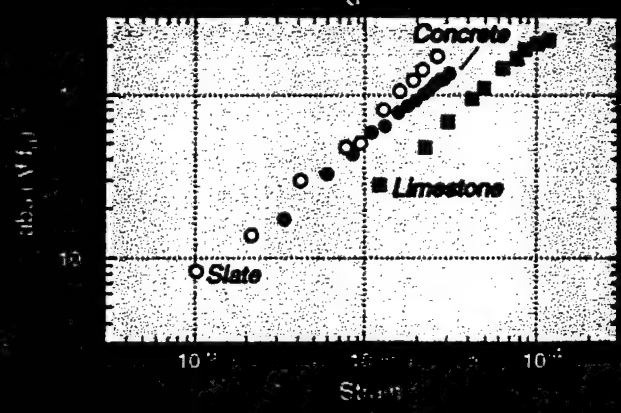
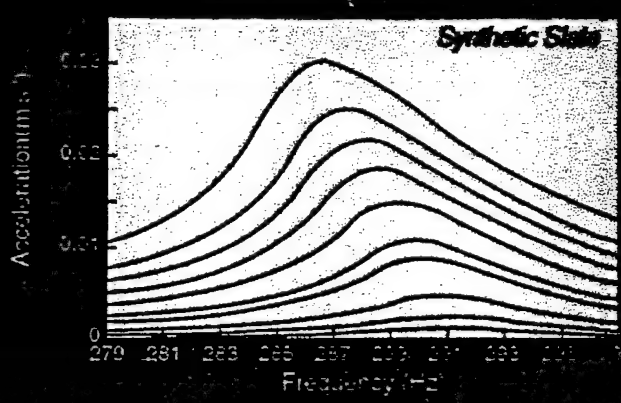
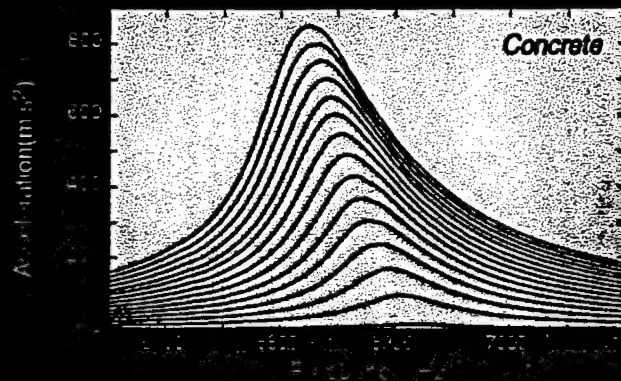
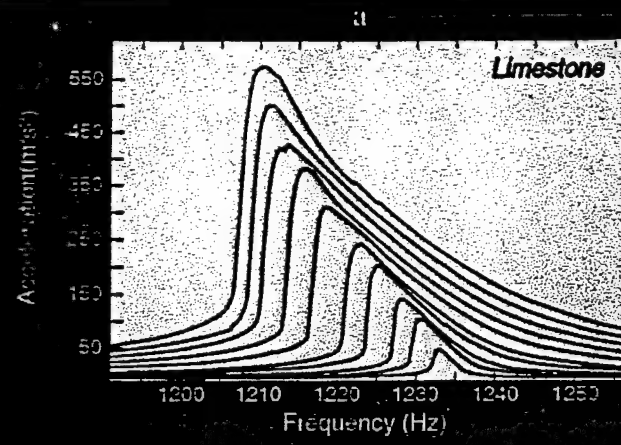
$\updownarrow$  PM

$$\Delta f \propto |E|$$

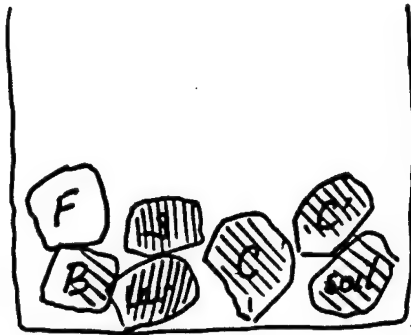
$\updownarrow$  ?

$\ln(t)$





WHO ELSE DOES IT?



soil  $\nabla$ , VEUCETIC, ...

slate, Laroux, Fontainebleau,  
concrete,

$\Delta f \propto |E|$

Berea, Laroux, Fontainebleau,  
concrete, concrete' 199

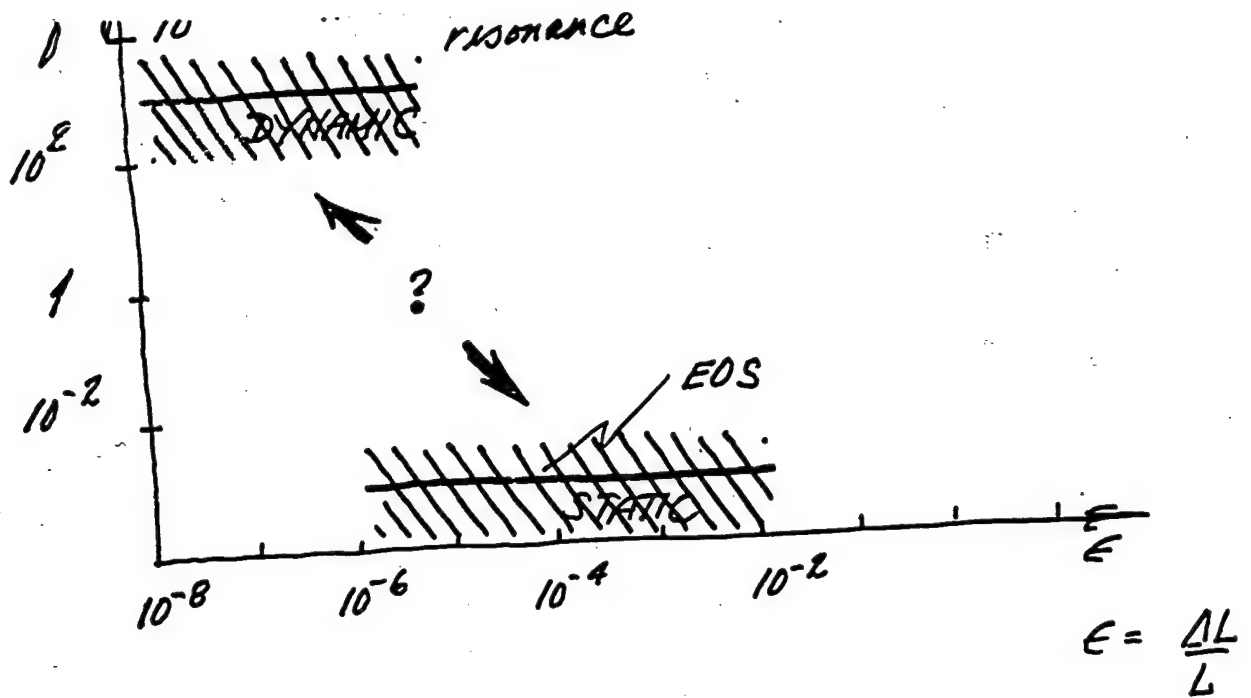


$\log(t)$

AN ELASTICITY UNIVERSALITY CLASS?

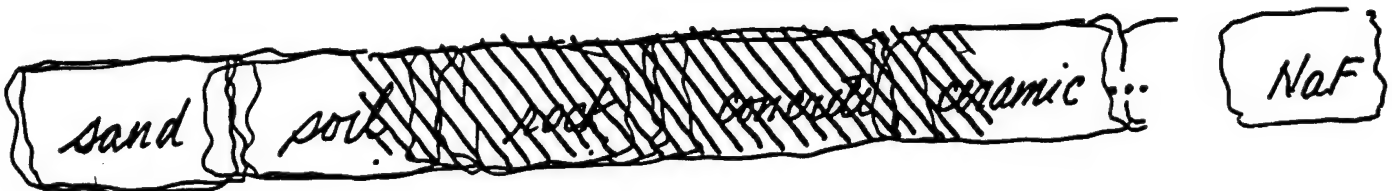
At a level where the details are blurred there is common behavior.

The description of this behavior for one should work for all.



## BROADER CONTEXT<sup>2</sup>

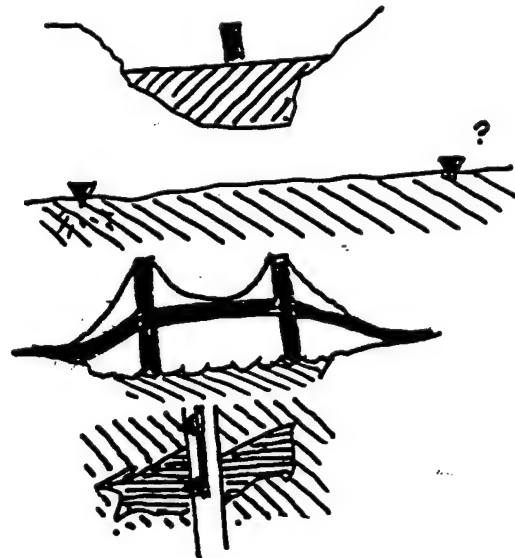
(a) kinds of materials



(b) strong motion  
verification

NDE

logging



# RUS and Rocks



K R McCall

TJ Ulrich

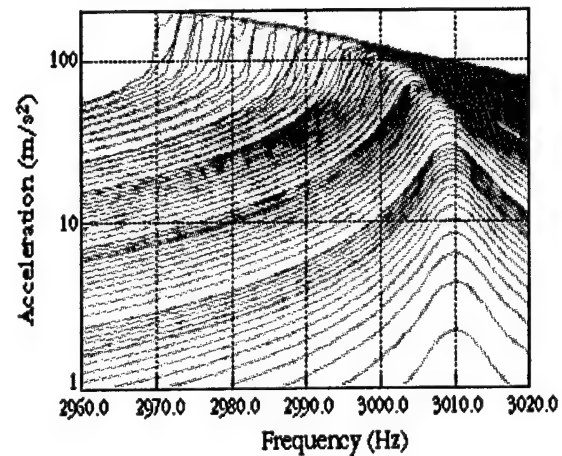
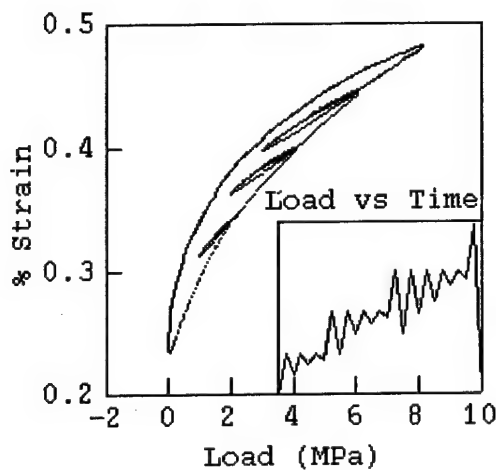
University of Nevada, Reno

# Outline

- I. History
- II. Experimental Details
- III. Numerical Modeling
- IV. Experimental Results
- V. Future Work

# I. History

A. Empirical Evidence: Macroscopically inhomogeneous materials are elastically nonlinear (R A Guyer, previous talk)



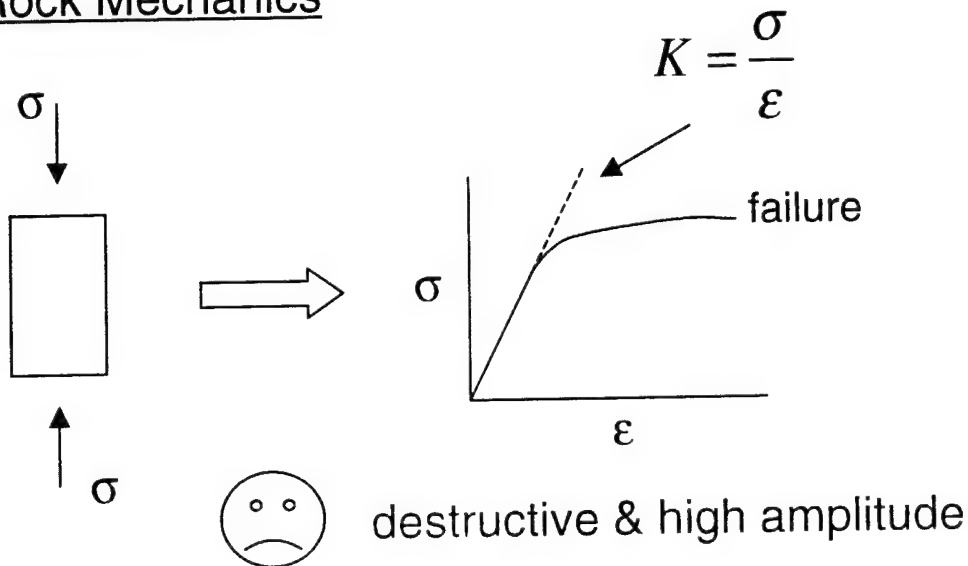
B. Theoretical Models: The elastic modulus is nonlinear and hysteretic.

$$\sigma = K\varepsilon$$

$$K = K_o[1 + \beta\varepsilon + \delta\varepsilon^2 + \alpha(|\Delta\varepsilon| \pm \varepsilon) \dots]$$

C. Hypothesis: Reliable measurements of linear moduli are the basis for quantitative study of nonlinear moduli.

1. Rock Mechanics



2. Time of flight

The diagram shows a rectangular box representing a medium, with a wavy line inside representing a pulse. Arrows indicate the direction of wave propagation. To the right of the box, the equation  $c = \frac{\Delta l}{\Delta t}$  is shown, followed by an arrow pointing to  $K = c^2 \rho$ . Below this, a sad face icon is next to the text 'path dependent (vs. average  $K$ )'.

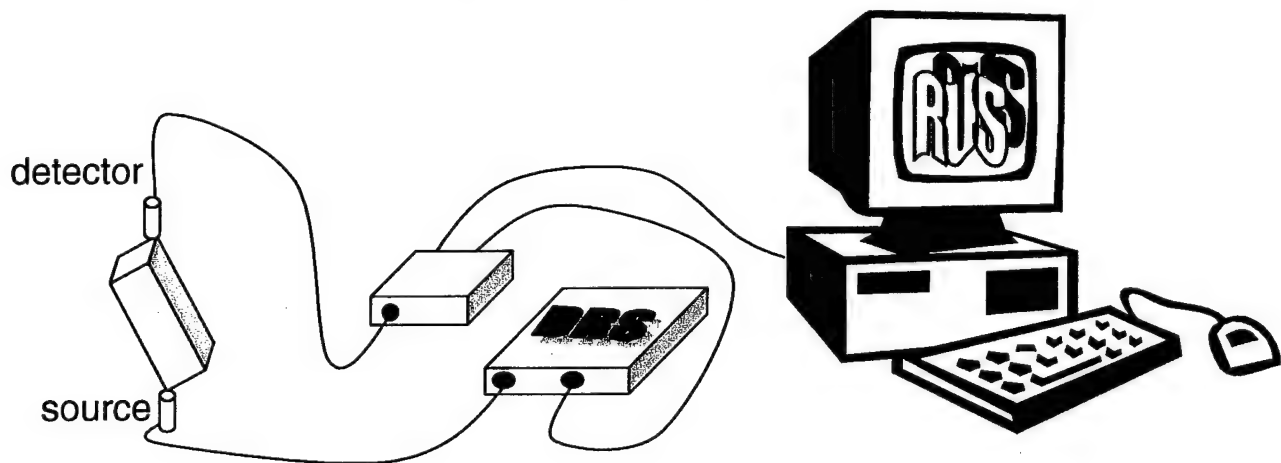
3. Resonant Ultrasound Spectroscopy



# II. Experimental Details

## A. RUS System by Dynamic Resonance Systems, Inc.

control hardware & analysis software



## B. Analysis Software Assumptions

1. Free boundaries on sample
2. Homogeneous sample
3. Parallelepiped sample

## C. Conditions for Optimal Results

4. Distinct resonance peaks
5.  $C_{11}$  dependence in low modes

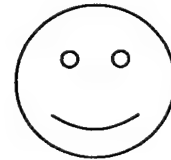
TR-5



# III. Numerical Modeling

(to explore the implications of the experimental conditions)

A. Free boundaries on sample

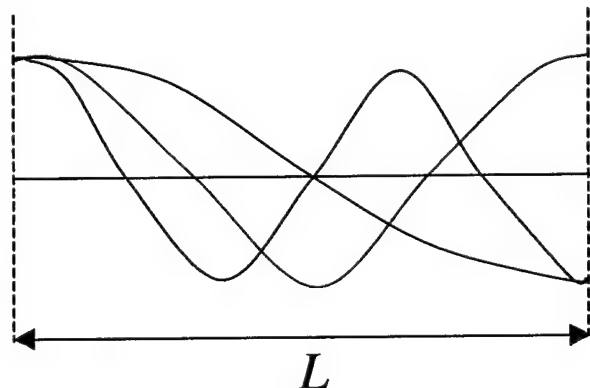


B. Homogeneous sample: Scale of inhomogeneity ( $\xi$ ) should be smaller than wavelength of resonance ( $\lambda$ ).

*Example: 1-d system*

$$\lambda = \frac{2L}{n} > \xi$$

For 10 resonances,  
 $n = 10$



$$L > 5\xi$$


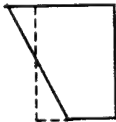
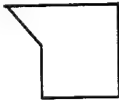
limiting sample size

C. Parallelepiped samples: Rock samples are cut on a diamond saw with limited accuracy, high probability of chips, nonparallel sides.

Perturbation Theory: 1<sup>st</sup> order evaluation of induced error (R A Guyer)

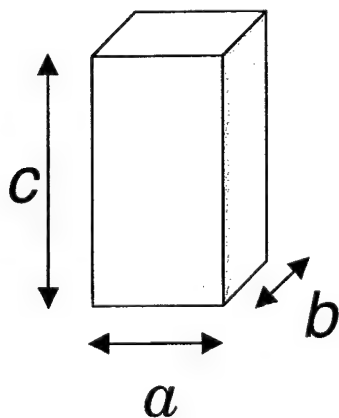
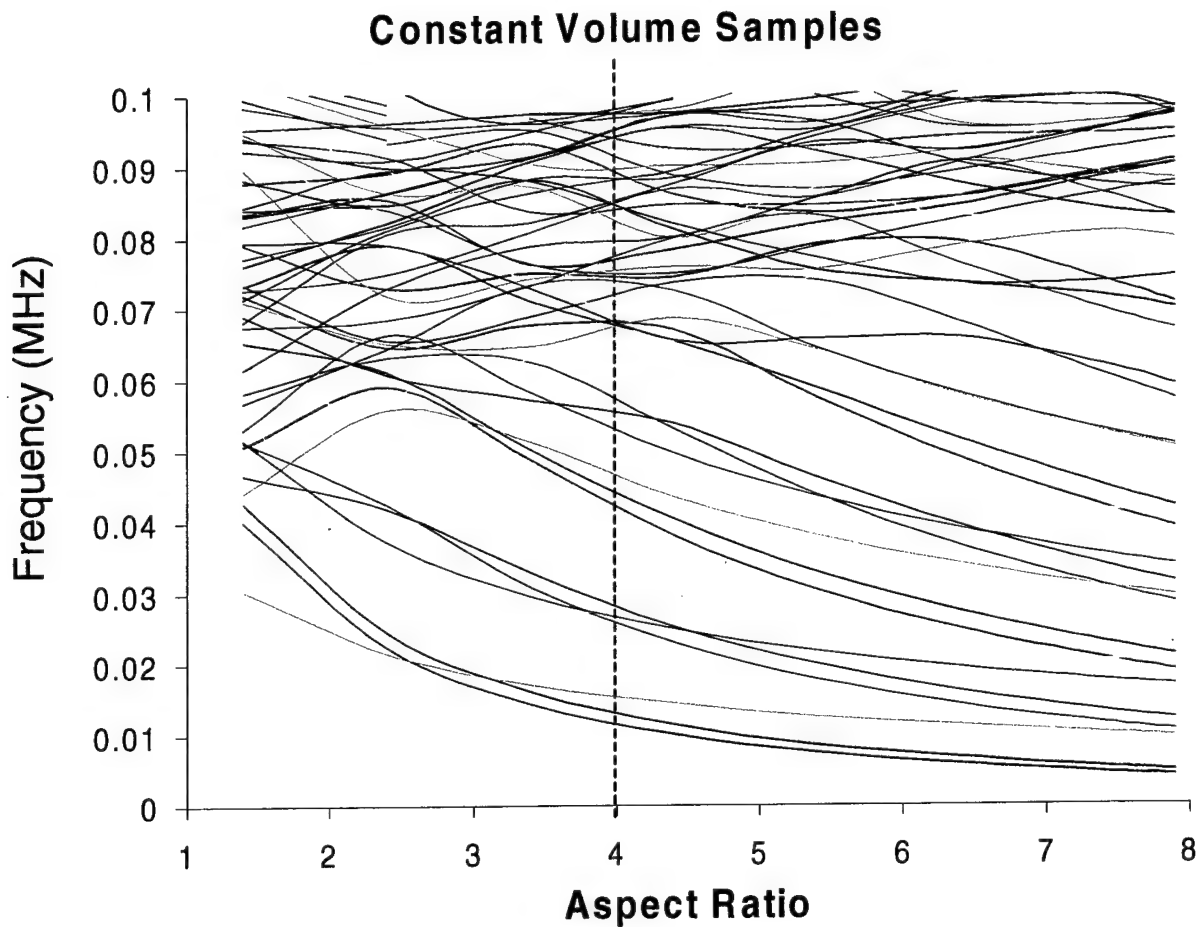
$$\frac{\omega^2 - \omega_n^2}{\omega_n^2} = \langle u_n | \Delta\rho | u_n \rangle$$

$u_n = \text{eigenmode}$   
 $\Delta\rho = \text{density change}$

	mass imperfection	amount	frequency change
1	 nick at corner	1%	0.15%
2	 symmetric distortion	5%	~ 0%
3	 asymmetric distortion	5%	<1.5%

1 & 2 most likely  $\Rightarrow$  may be minor contributor to error

D. Distinct Resonance Peaks: Low Q means broad peaks. Use aspect ratio to separate peaks.



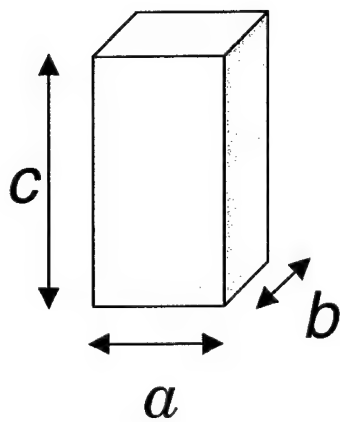
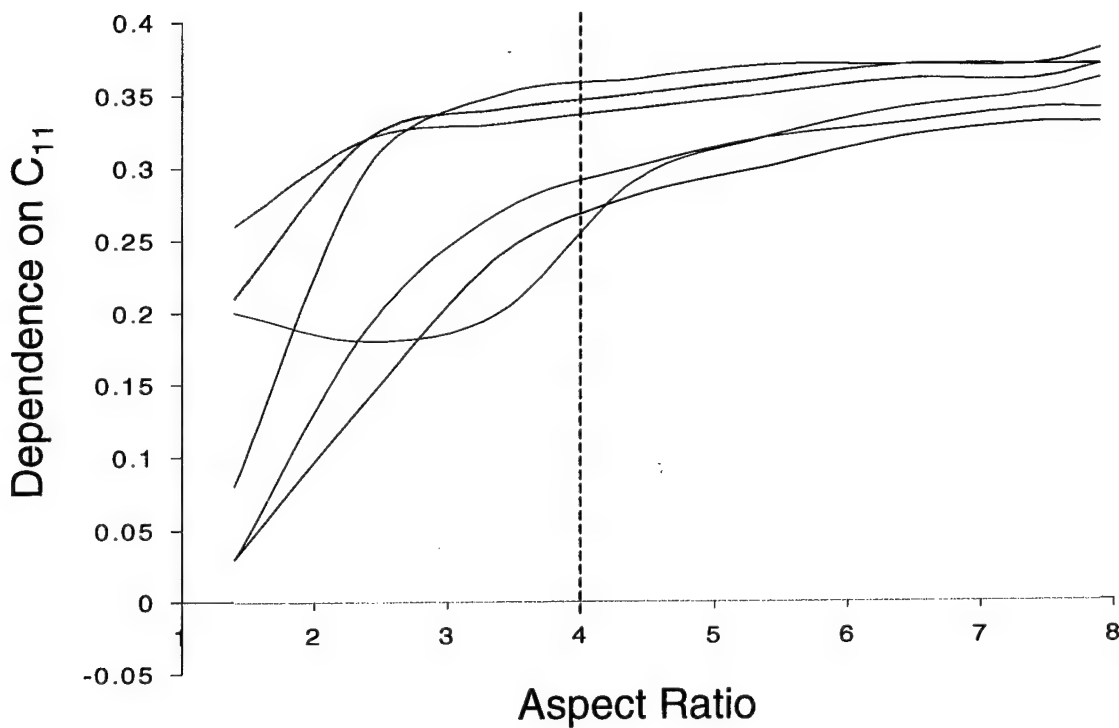
$$b = 1.2a$$

$$\text{Aspect Ratio} = \frac{c}{a}$$

(basalt  $c_{11}$  and  $c_{44}$ )

TR-8

E.  $C_{11}$  dependence in low modes: Low Q limits number of discernable resonances. Use aspect ratio to increase  $C_{11}$  dependence.

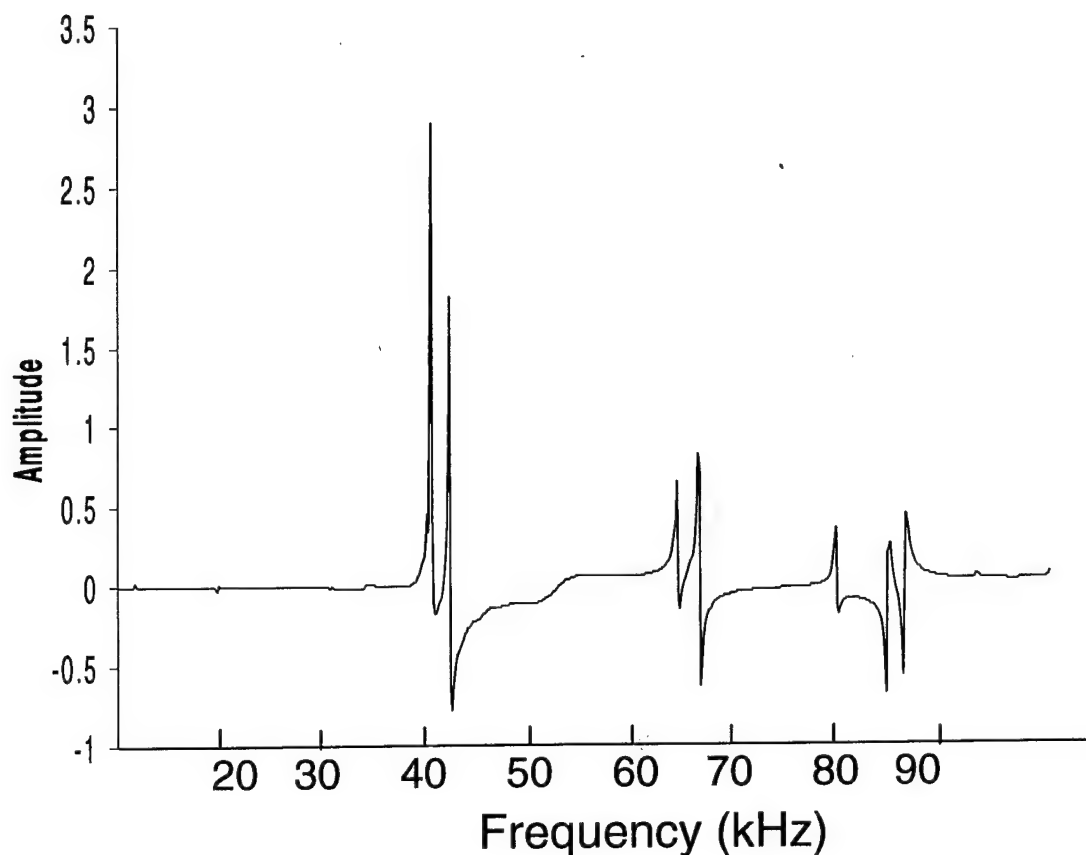


$$b = 1.2a$$

$$\text{Aspect Ratio} = \frac{c}{a}$$

(basalt  $c_{11}$  and  $c_{44}$ )

# IV. Experimental Results



$f_{\text{exp}}$ (kHz)	$f_{\text{fit}}$ (kHz)
------------------------	------------------------

17.988	17.864
--------	--------

19.586	19.649
--------	--------

23.522	23.493
--------	--------

39.786	39.692
--------	--------

40.143	40.161
--------	--------

41.877	42.071
--------	--------

46.944	46.917
--------	--------

63.858	63.810
--------	--------

65.925	66.142
--------	--------

Basalt Sample

Aspect Ratio = 4

% error = 0.313

# of modes = 10

Basalt       $C_{11} = 87 \text{ GPa}$      $C_{44} = 32 \text{ GPa}$

$\frac{c}{a}$	$a$	$b$	$c$	$\xi$	modes	$C_{11}$	$C_{44}$	% err	<Q>
	(cm)					(GPa)			
1.5	2.13	2.44	3.04	0.61	12	88.1	31.3	0.632	276
1.7	2.72	2.74	4.70	0.39	10	88.4	31.7	1.967	242
2.0	2.78	3.04	5.43	0.11	11	87.3	31.6	0.709	256
4.0	1.73	1.98	6.69	0.11	10	87.2	31.5	0.313	336

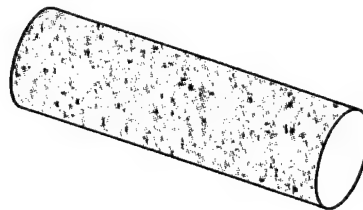
Sierra White Granite       $C_{11} = 43 \text{ GPa}$      $C_{44} = 19 \text{ GPa}$

$\frac{c}{a}$	$a$	$b$	$c$	$\xi$	modes	$C_{11}$	$C_{44}$	% err	<Q>
	(cm)					(GPa)			
1.5	3.68	4.16	5.64	0.66	11	37.6	18.7	1.238	146
1.6	2.77	3.11	4.51	0.66	10	43.2	18.6	2.023	140
3.3	0.96	1.20	3.17	0.44	11	20.8	21.3	17.29	149
4.0	1.69	2.02	6.82	0.28	12	37.5	19.1	0.630	142

# V. Future Work

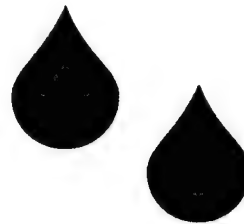
A. Same analysis for cylindrical samples.

(Izabela Santos, Senior Thesis)



B. Temperature and Saturation Control.

(TJ Ulrich, PhD Thesis)



C. Provide baseline for nonlinear analysis.

# CONSTANT STRAIN ANALYSIS

R. A. GUYER      UMass / LANL

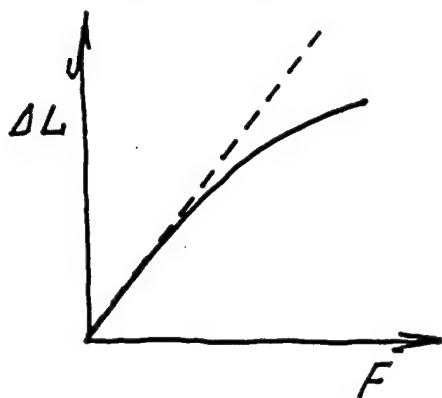
JIM TENCATE LANL

ERIC SMITH LANL

4. a rock
2. traditional theory  $\rightarrow L$
3. PM-space "model"  $\rightarrow R$
4. to confirm quasi-static  $\leftrightarrow$  dynamic  
connection
5. an upt and analysis



# TRADITIONAL THEORY OF NONLINEAR ELASTICITY, THE STRAW MAN.



EOS

$$F = \Gamma(\Delta L) \Delta L$$

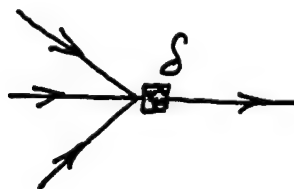
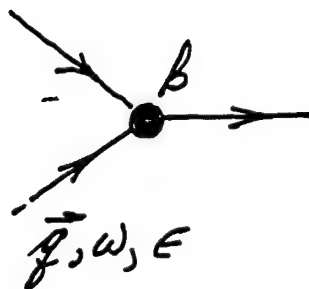
$$F = \Gamma_0 \left[ 1 + \beta \frac{\Delta L}{L_0} + \delta \left( \frac{\Delta L}{L_0} \right)^2 + \dots \right] \Delta L$$



same springs work here as here\*\*

## WAVE EQUATION

$$\rho \frac{\partial^2 u}{\partial t^2} = K \frac{\partial}{\partial x} \left[ \underbrace{\frac{\partial u}{\partial x}}_{\text{LINEAR}} + \underbrace{\beta \left( \frac{\partial u}{\partial x} \right)^2}_{\substack{\text{CUBIC} \\ \text{NL}}} + \underbrace{\delta \left( \frac{\partial u}{\partial x} \right)^3}_{\substack{\text{QUARTIC} \\ \text{NL}}} + \dots \right]$$



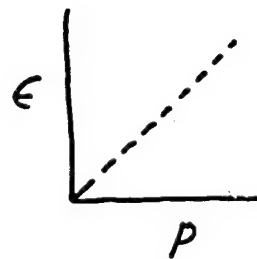
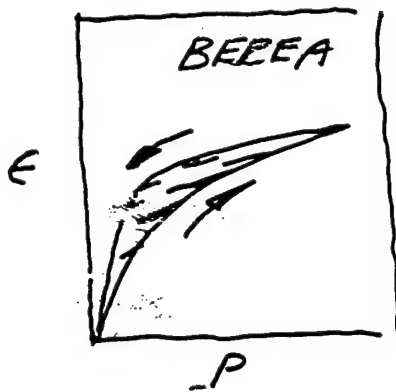
basic physical process is  
due to  $\beta, \delta, \dots$   
"coalescence" of strain fields

TRADITIONAL THEORY OF LINEAR and NONLINEAR ELASTICITY IS VERY ELABORATE, I.G. LANDAU & LIFSHITZ, BECAUSE THE BASIC FIELD, the strain field, IS A TENSOR

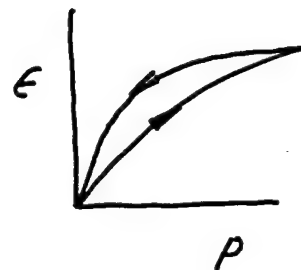
$$\epsilon = \frac{\Delta L}{L} \leftrightarrow \frac{\Delta L_x}{L_y} = \epsilon_{xy}, \dots$$

OTHERWISE: T T N L E = TAYLOR SERIES EXPANSION

BUT:

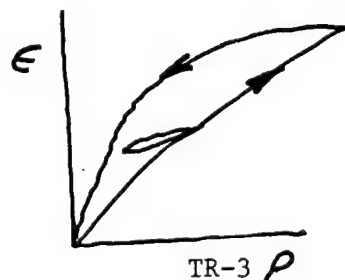


nonlinear, NL

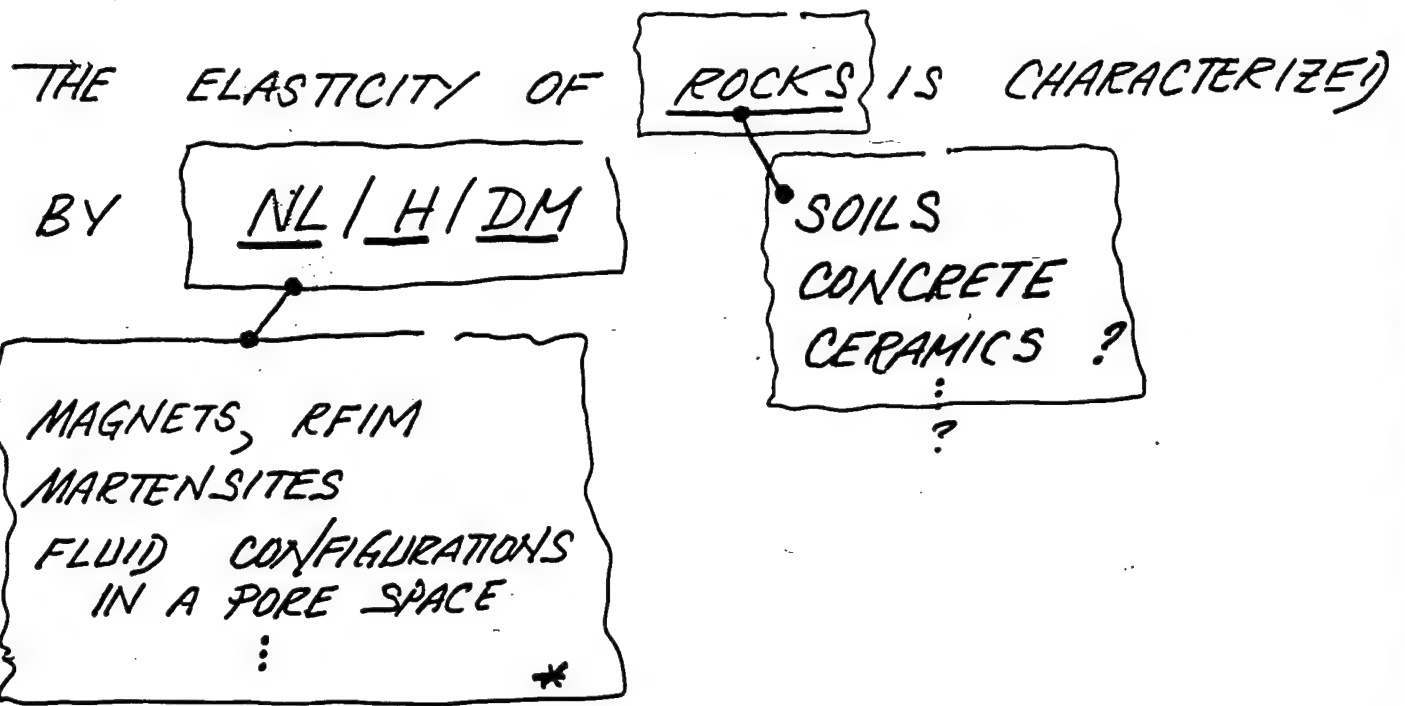


hysteresis, H

NL/H/DM



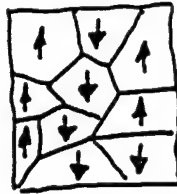
discrete memory, DM



There is a generic\* explanation of the elasticity of rocks; details are specific to rock elasticity; essential features shared by ;

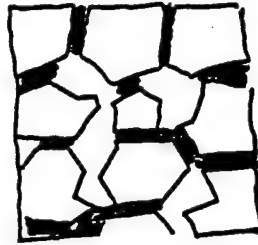
\* ALL OF THESE SYSTEMS HAVE MACROSCOPIC BEHAVIOR THAT IS DUE TO AN ASSEMBLY OF HYSTERETIC MESOSCOPIC MICROSCOPIC UNITS.


magnet -



domains 

fluid in a pore space -



pore   
full  
empty

rock elasticity -



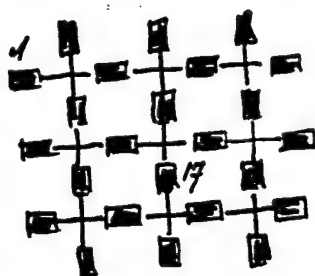
elastic bonds   
?  
?

# A PHENOMENOLOGY FOR DESCRIBING THE ELASTICITY OF ROCKS: PM-SPACE.

PREISACH '35 M-H  
MAYERGOYZ '85

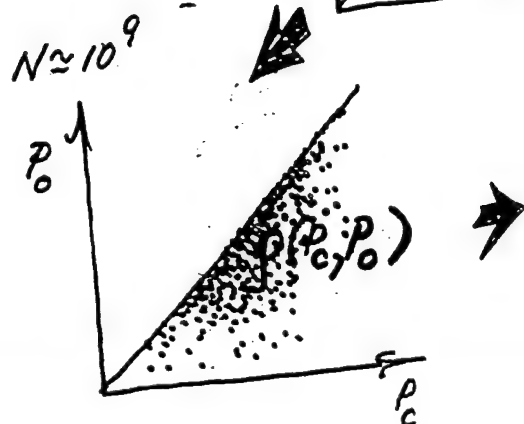
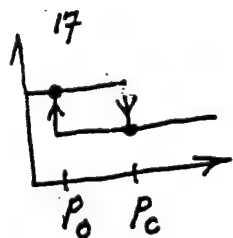
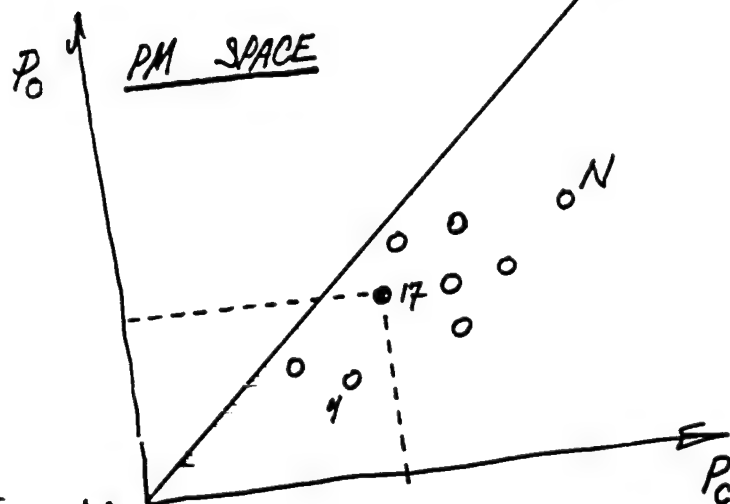


≡



assembly of hysteretic elastic bonds

central construct:  
PM space, a space  
where you keep  
track of the  
state of each  
elastic bond



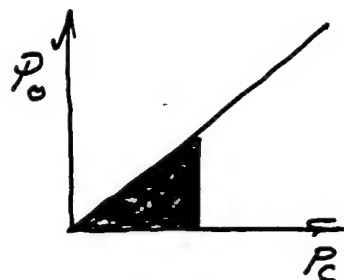
$\rho(P_c, P_0)$  = DENSITY OF ELASTIC  
BONDS IN PM SPACE  
IS THE MOST IMPORTANT  
QUANTITY.

# FEATURES OF THE PM-SPACE MODEL:

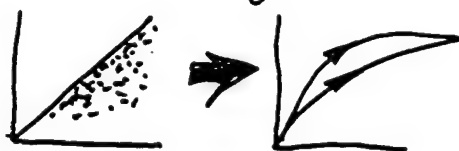
1.

$$\epsilon(P) = \epsilon_0 F(P)$$

← "fraction" of elastic bonds in state (closed, b).



2. When  $p(P_c, P_0)$  is known you can find the response to any pressure protocol.



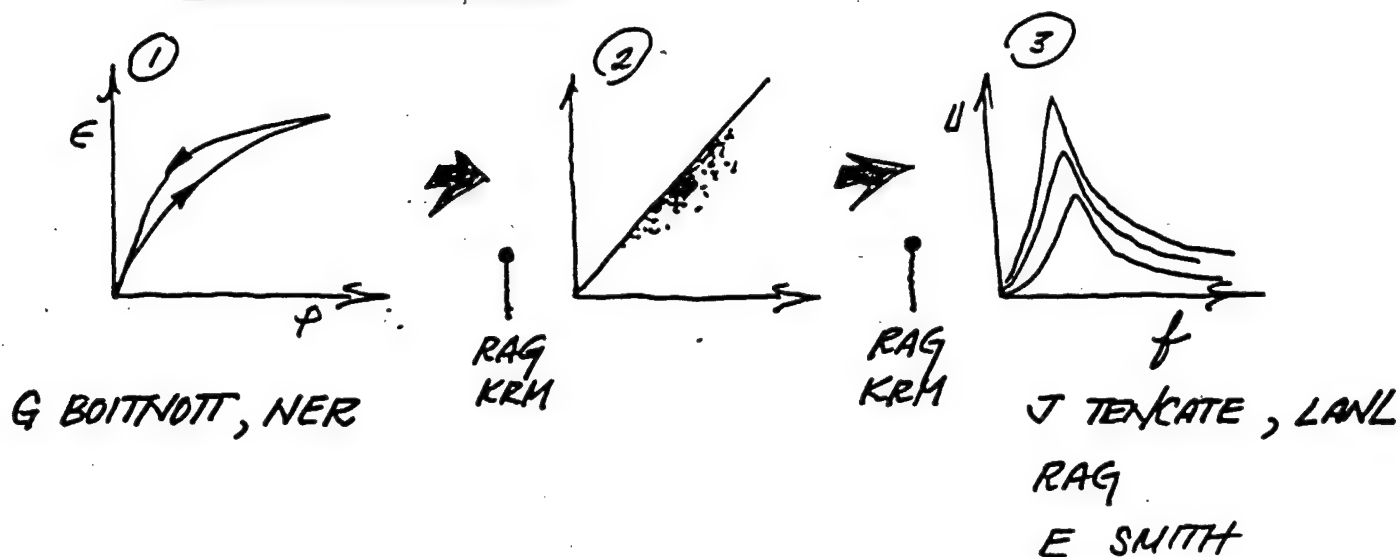
3. There is an inverse problem: from suitable  $\epsilon$ - $P$  data you can find  $p(P_c, P_0)$



4. A sound wave is a pressure protocol.



# THE PROGRAM



WHAT DO YOU EXPECT FOR ③?

TRADITIONAL THEORY

$\Delta f$  = frequency shift

$$\Delta f \sim \delta |U|_{\max}^2$$

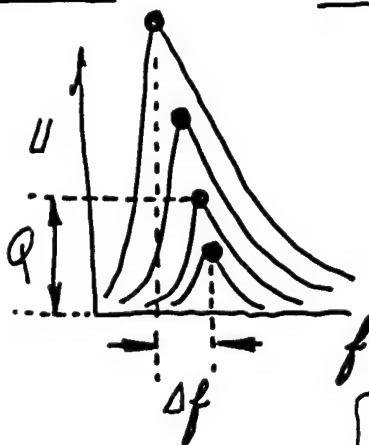
$$\frac{1}{Q} - \frac{1}{Q(0)} \sim \delta |U|_{\max}^2$$

$$\delta \sim 10^6$$

quartic anharmonicity

PM-SPACE MODEL

$\frac{1}{Q}$  = attenuation



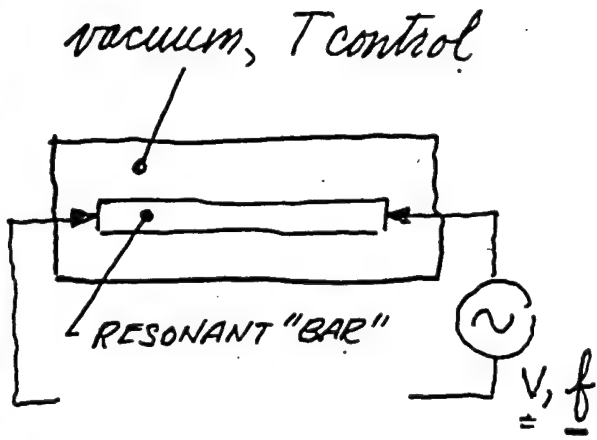
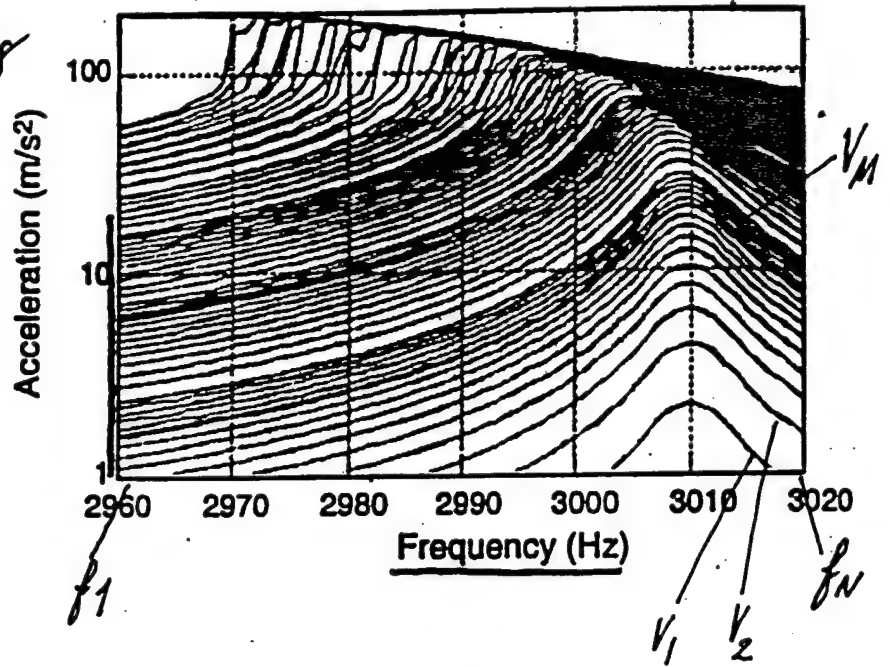
$$\Delta f = C_1 |U|_{\max}$$

$$\frac{1}{Q} - \frac{1}{Q(0)} = C_2 |U|_{\max}$$

$$C_1 \sim C_2 \sim \text{[scribble]}$$

②

J. TENCATE, LANL '98



detect  $a(t)$ ,  
Report  $a(f)$ ; amplitude of motion  
at drive frequency

2. STUDY THE RESONANCE CURVES:

(a) rigorous acceptance criterion

$V_1, f_1, f_2, \dots, f_N, \dots, f_1$

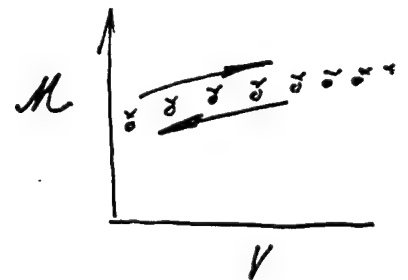
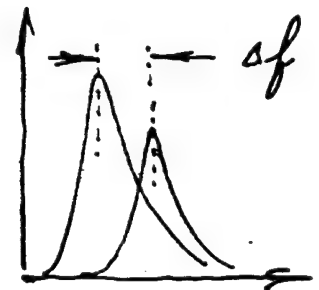
$V_2, f_1, f_2, \dots, f_N, \dots, f_1$

$\vdots$

$V_M, f_1, f_2, \dots, f_N, \dots, f_1$

$\vdots$

$V_1, f_1, \dots, f_N, \dots, f_1$



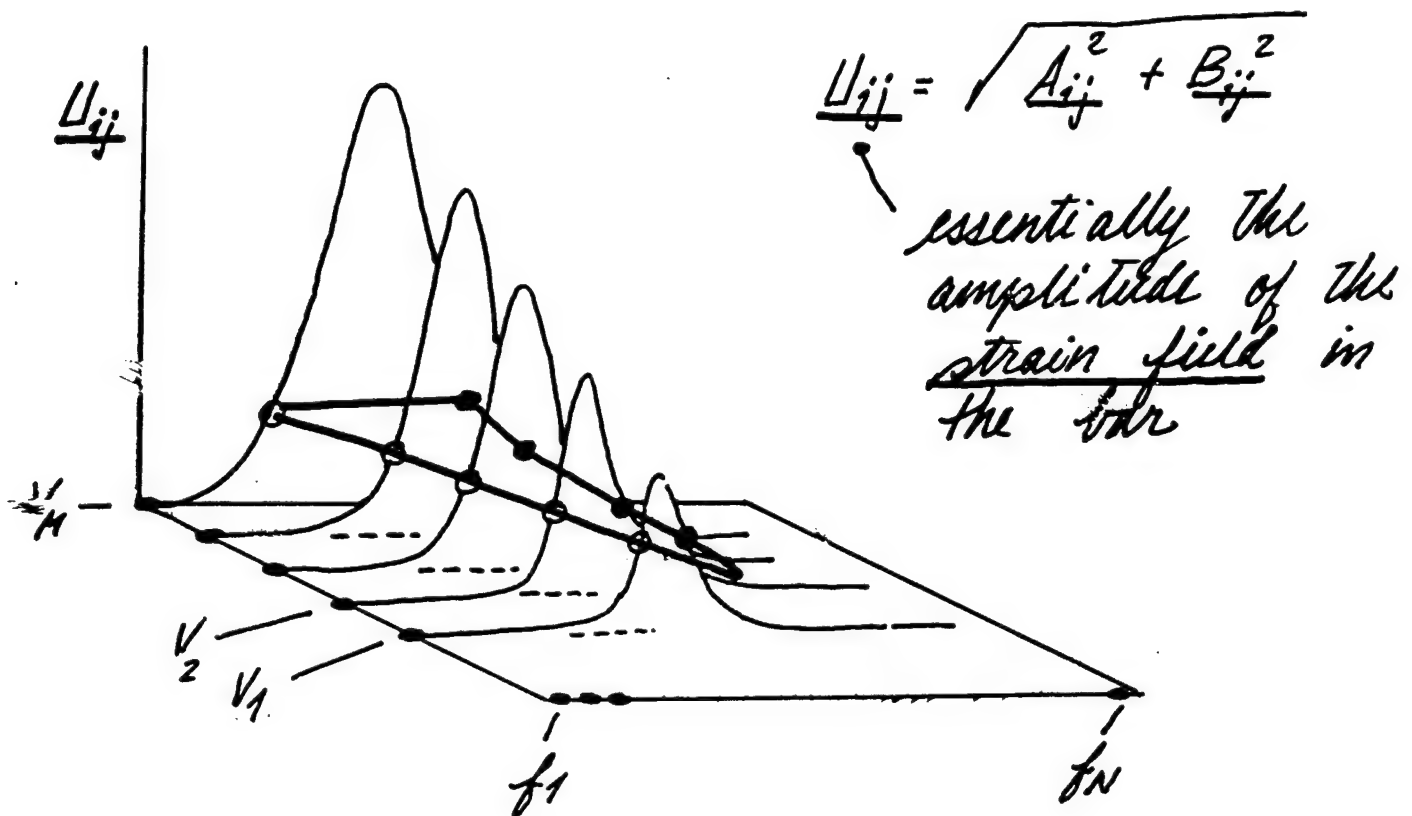


at  $V_i, f_j$

(b) measure  $\underline{A_{ij}}$ ,  $\underline{B_{ij}}$   
                     in phase                      out of phase

$$i = 1 \dots M \leftrightarrow V_1, V_2, \dots V_M$$

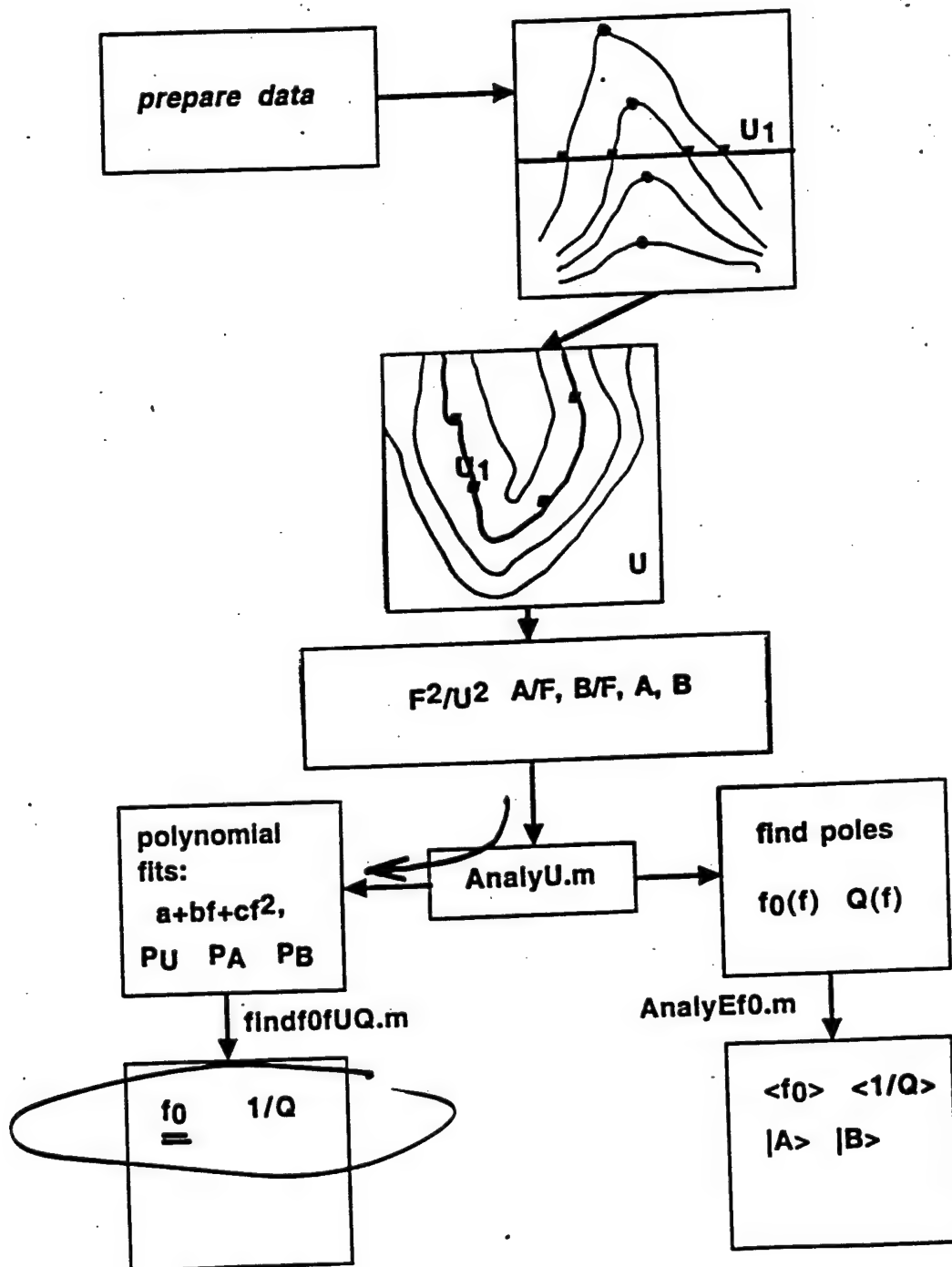
$$j = 1 \dots N \leftrightarrow f_1, f_2, \dots f_N$$



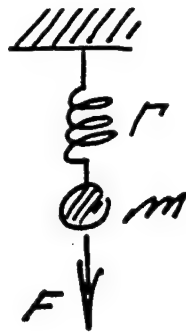
physical state of rock is set by the  
strain field in the rock - rock is in the  
 same physical state along ———

when nonlinear - state  $\leftrightarrow$  strain : it makes  
no sense to analyze curves of constant  $V$ !

DO THIS :



Some algebra;



$$\omega_0^2 = \gamma/m,$$

$$\Omega = \omega/\omega_0,$$

$$f = \frac{F}{m\omega_0^2}.$$

$$A = \frac{1 - \Omega^2}{(1 - \Omega^2)^2 + \frac{\Omega^2}{Q^2}} f,$$

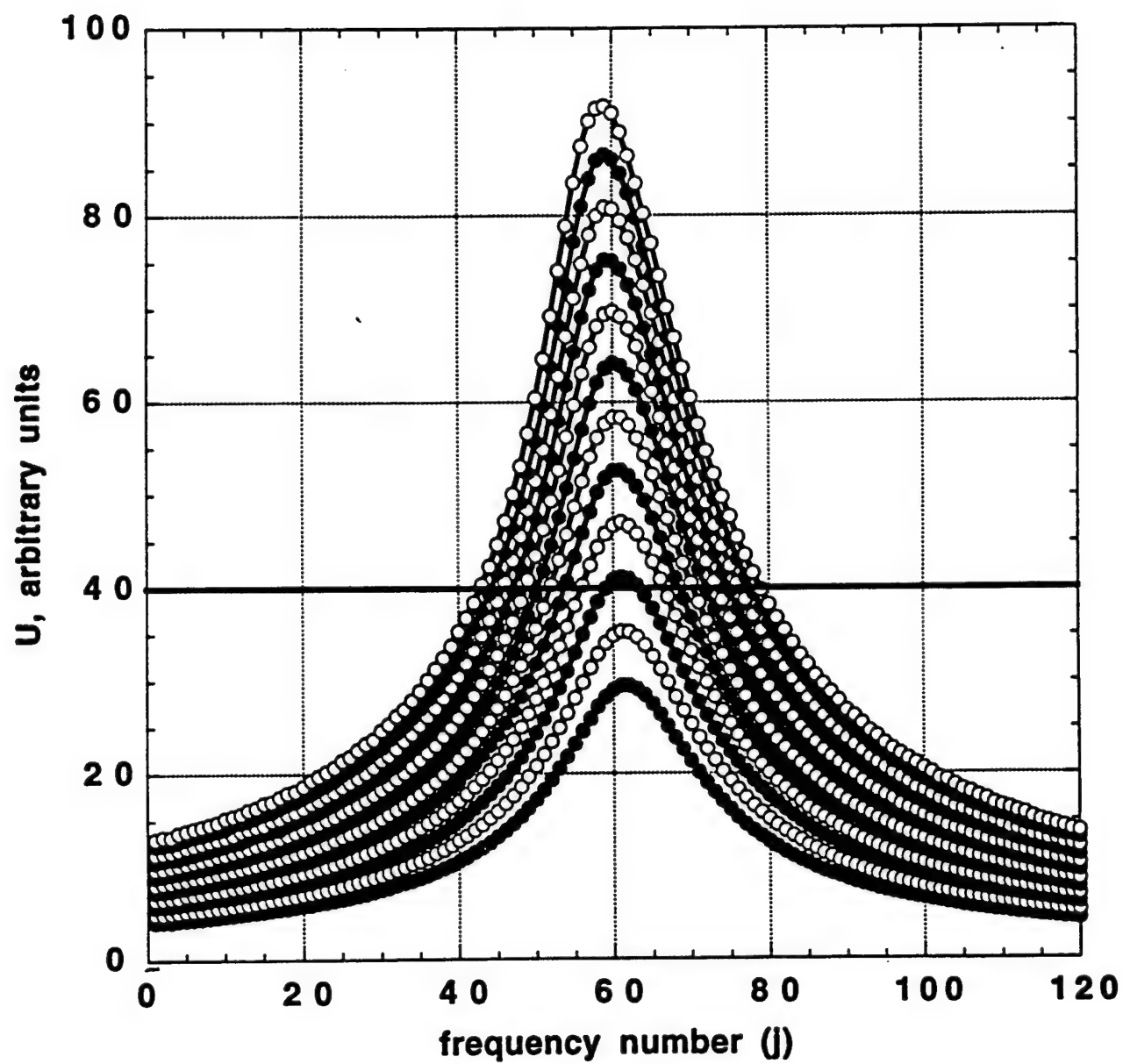
$$B = -\frac{\Omega}{Q} \frac{1}{(1 - \Omega^2)^2 + \frac{\Omega^2}{Q^2}} f,$$

$$|U| = \sqrt{A^2 + B^2} = \frac{1}{\sqrt{(1 - \Omega^2)^2 + \frac{\Omega^2}{Q^2}}} f.$$



$$\frac{1}{|U|^2} A \propto 1 - \Omega^2$$

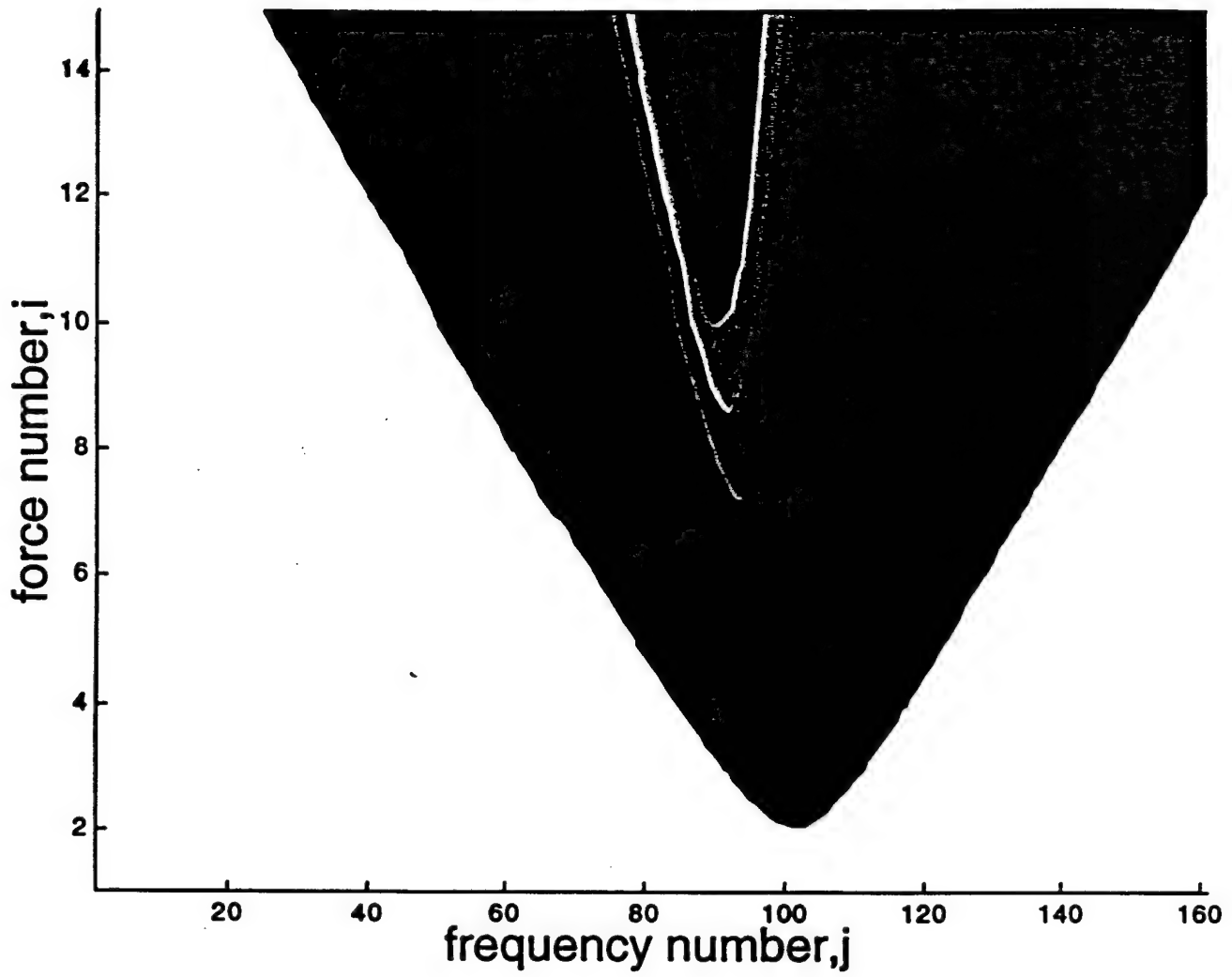
$$\frac{1}{|U|^2} B \propto -\frac{\Omega}{Q}$$



TR-13

148a

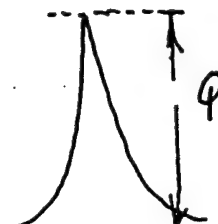
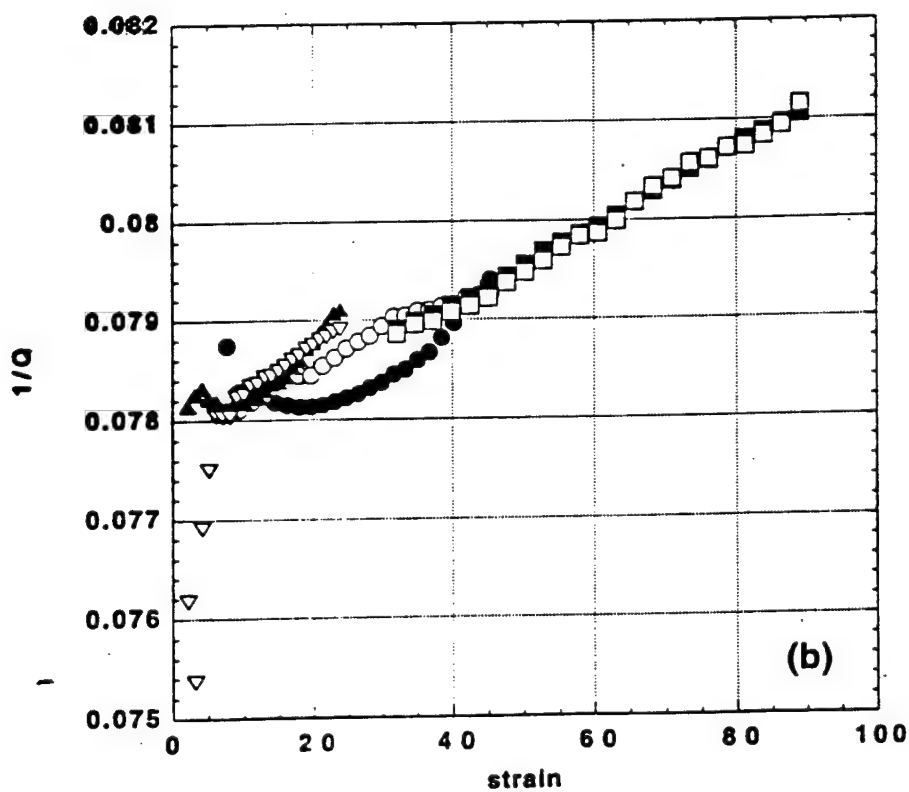
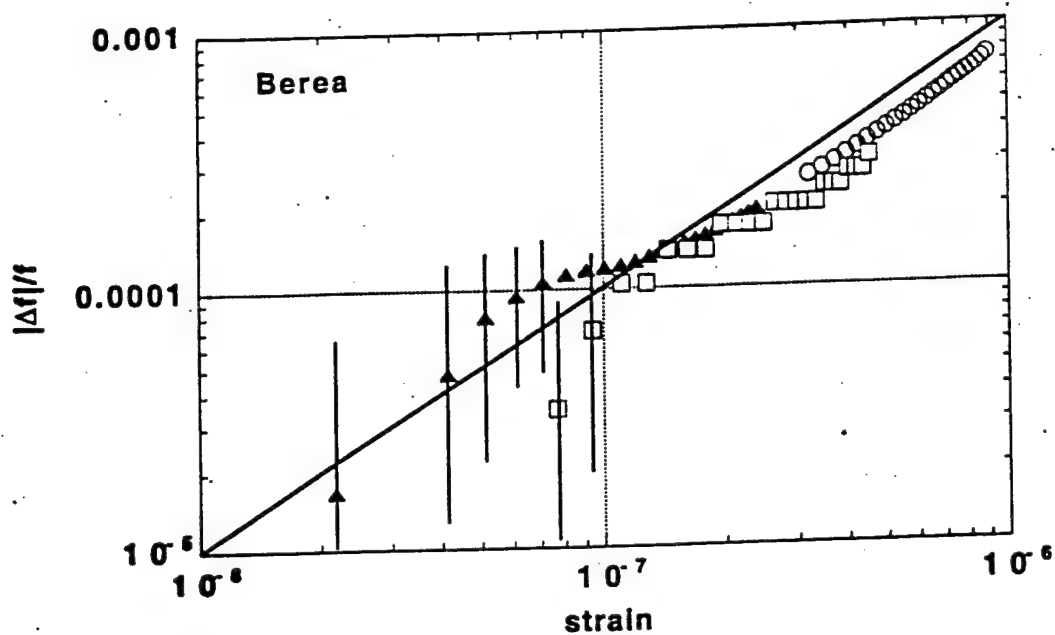
contours of constant U



TR-14

148b

# RESULTS :



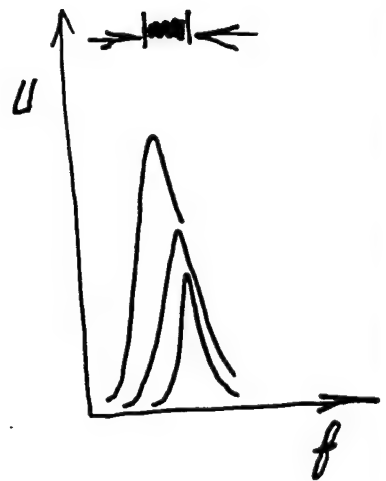
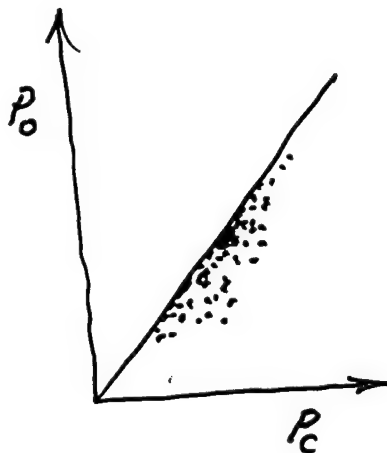
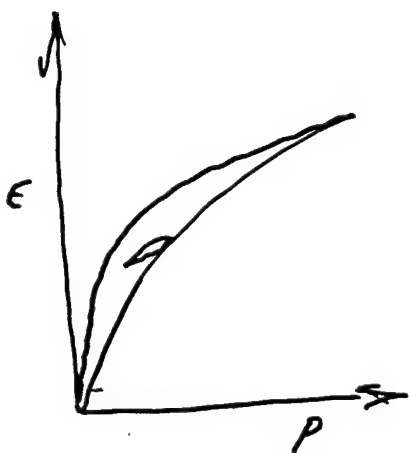
$$\frac{\Delta f}{f_0} \approx C_f \underline{|\epsilon|} \approx \frac{3 \text{ Hz}}{3000 \text{ Hz}}$$

$$\frac{1}{Q(\epsilon)} - \frac{1}{Q(0)} \approx C_Q \underline{|\epsilon|}$$

$$C_f \approx C_Q \approx 2 C_1 \approx \underline{2000}$$

$$2 \times 10^{-8} \lesssim |\epsilon| \lesssim 10^{-6}$$

excellent agreement with PM-space picture.



# **Global Symmetry, the Phonon Density of States, and RUS**

**Tim Darling and Albert Migliori**

**LANL**

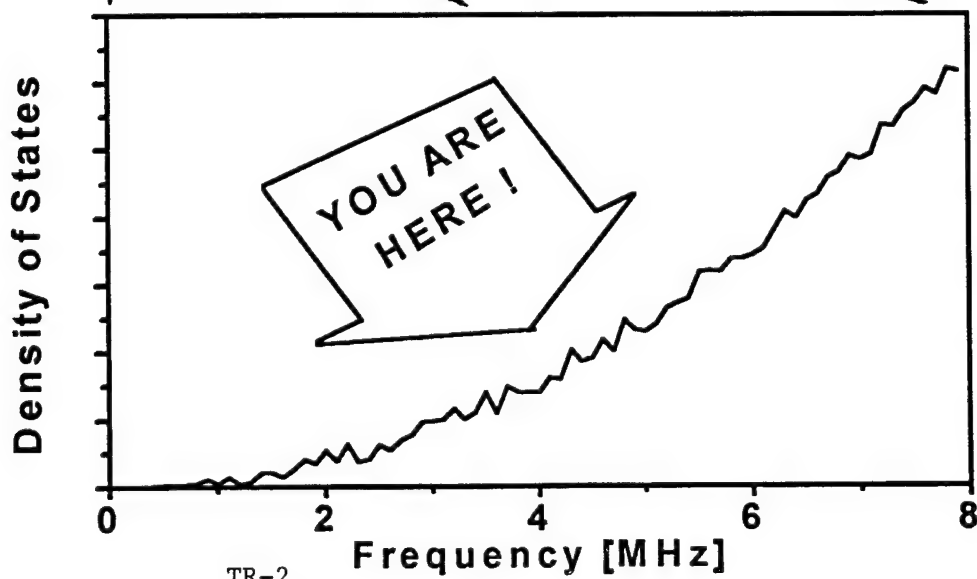
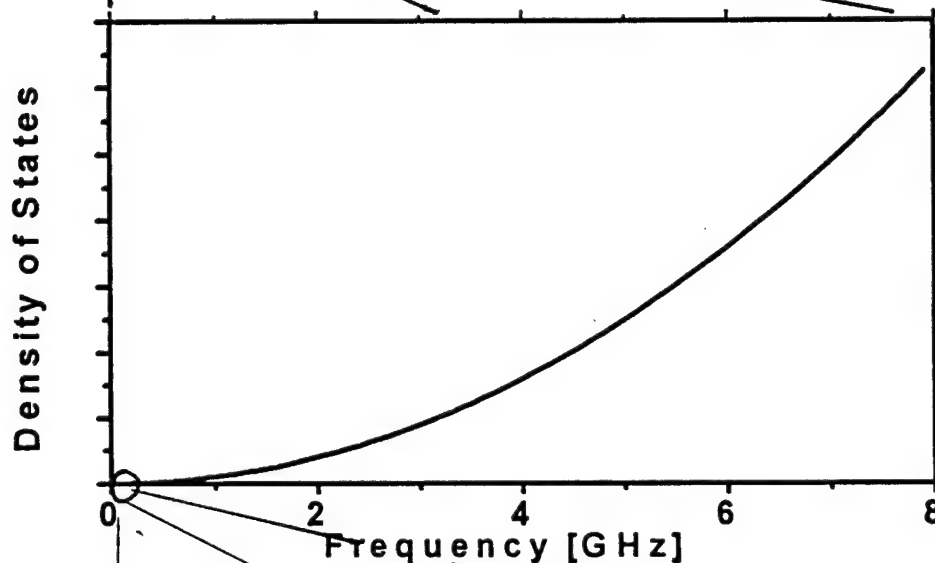
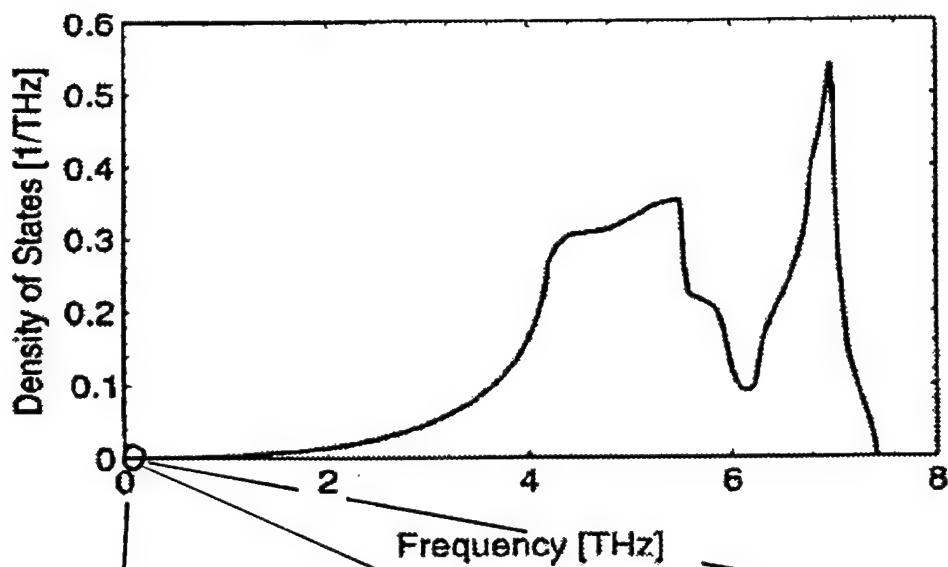
**RUS measures a tiny piece of the phonon  
DOS and can determine a global elastic  
symmetry.**

**Many physical properties depend on the  
entire DOS and the meso- or microscopic  
symmetry.**

**How good is the connection?**

**For when you are put into the Vortex you are given just  
one momentary glimpse of the entire unimaginable infinity  
of creation, and somewhere in it a tiny little marker, a micro-  
scopic dot on a microscopic dot, which says 'You are here.'**





## Harmonic approximation:

Replace  $3N$  degrees of freedom by  $3N$  harmonic oscillators (normal modes)

$E = (n+1/2)hf$ ,  $n \Rightarrow$  boson statistics

Born-Von Karman (cyclic) boundary conditions in an infinite solid – “travelling wave” solutions

No dissipative processes

No thermal expansion

No temperature dependence of moduli

Isothermal and adiabatic moduli equal

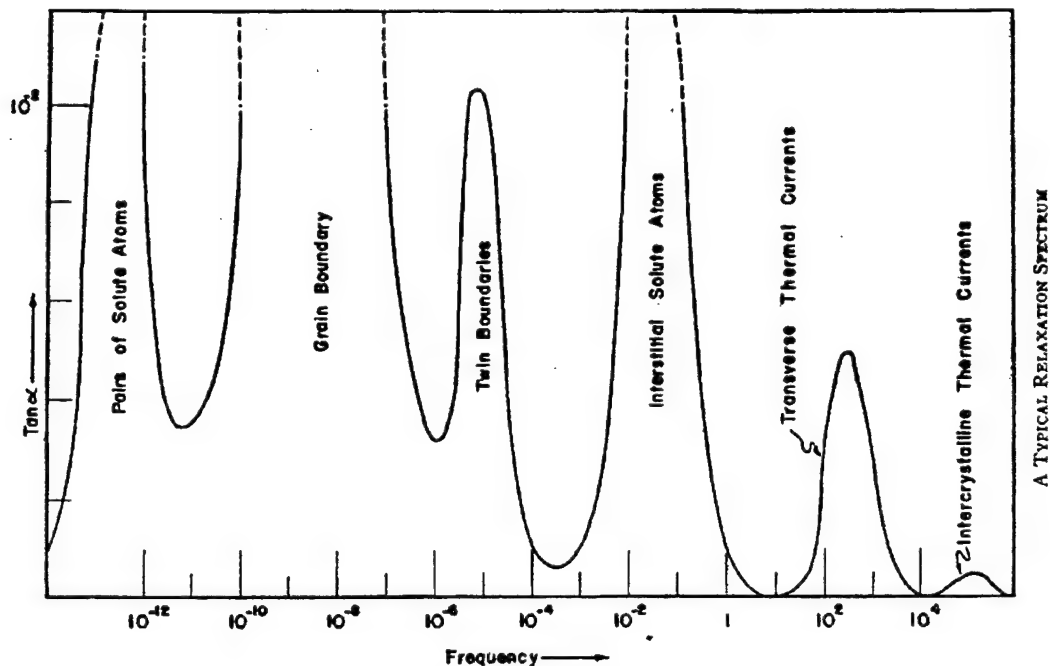
## Debye Model

Continuum (isotropic)

Frequency cutoff by requiring  $3N$  modes

Linear dispersion in all branches

$DOS \propto f^2$  (and  $T^3$ )



PLOT OF  $\tau^{-1}$  - ZENER, 1948, LOW TEMPERATURES

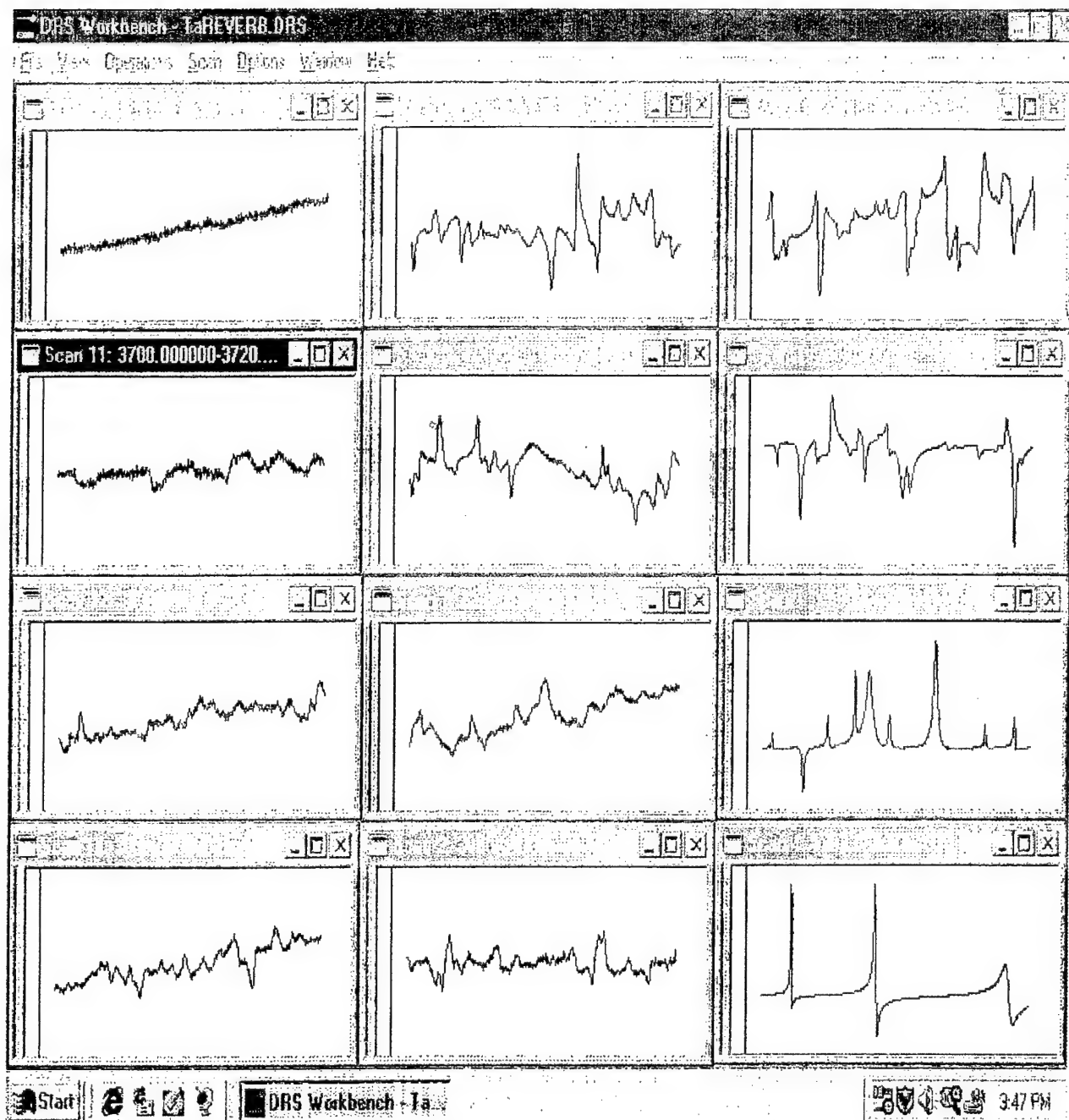
RUS HAS A TRICKY TIME HERE, BECAUSE

$\omega$  IS GENERALLY HIGH ( $\gg$  kHz)

WE SCAN  $\omega$  OVER A WIDE RANGE

SEPARATING OUT CONTRIBUTIONS FROM DISTINCT MODES IS TRICKY BECAUSE OF THE  $\omega$  VARIATION

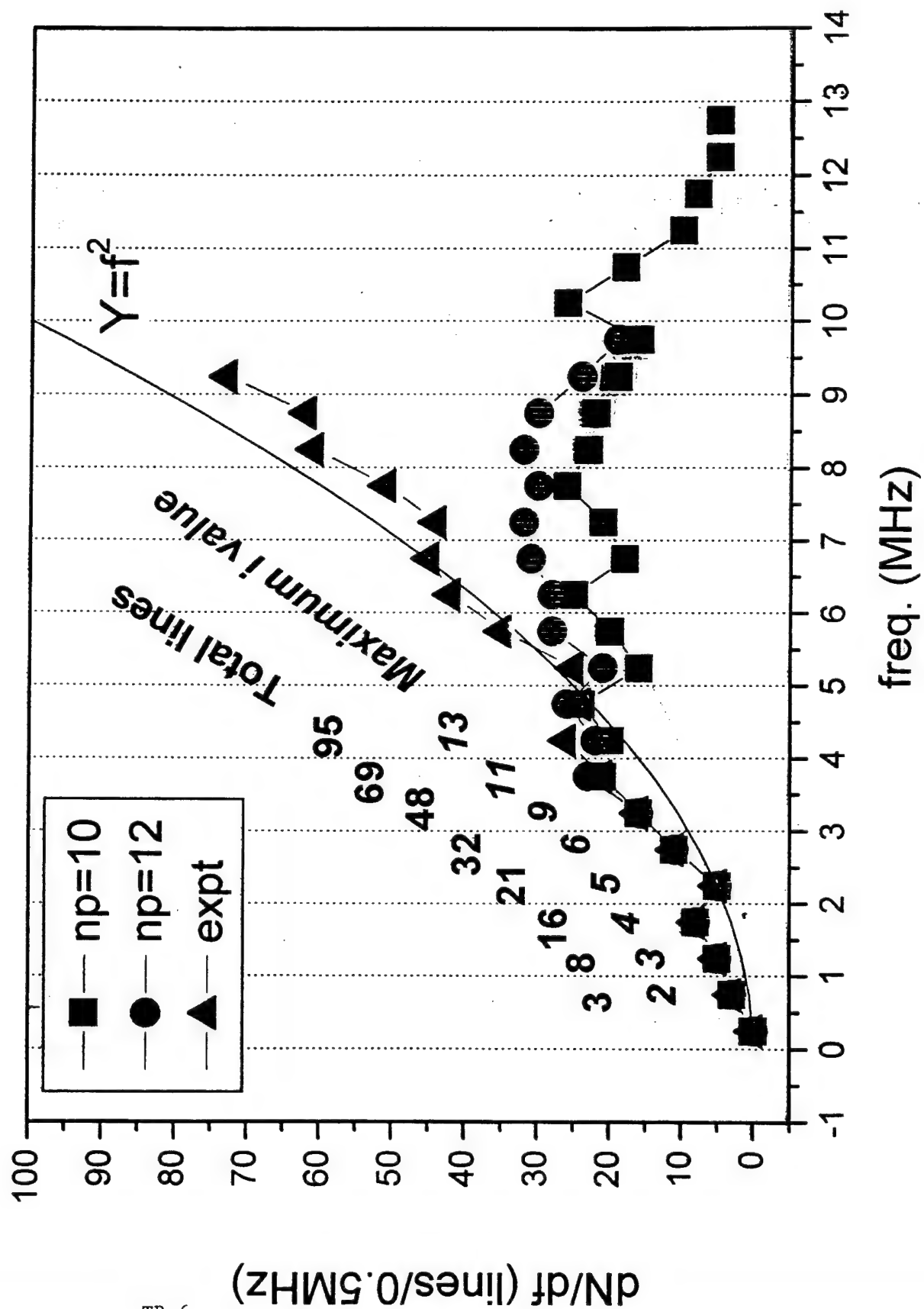
BUT, WE ARE IN RANGE OF CERTAIN MOBILE INTERSTITIALS (H), AND DOMAIN WALLS AND TWIN AND PHASE BOUNDARIES NEAR PHASE TRANSITIONS WHERE THE MOBILITY IS HIGH



Ta polycrystal data

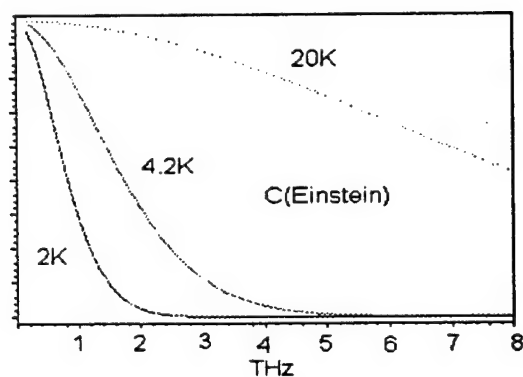
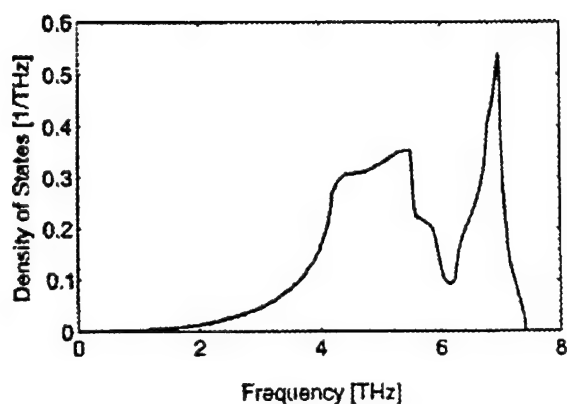
TR-5

# SrTiO<sub>3</sub> Sample C1, $\Gamma \sim 3\text{kHz}$



## Specific heat

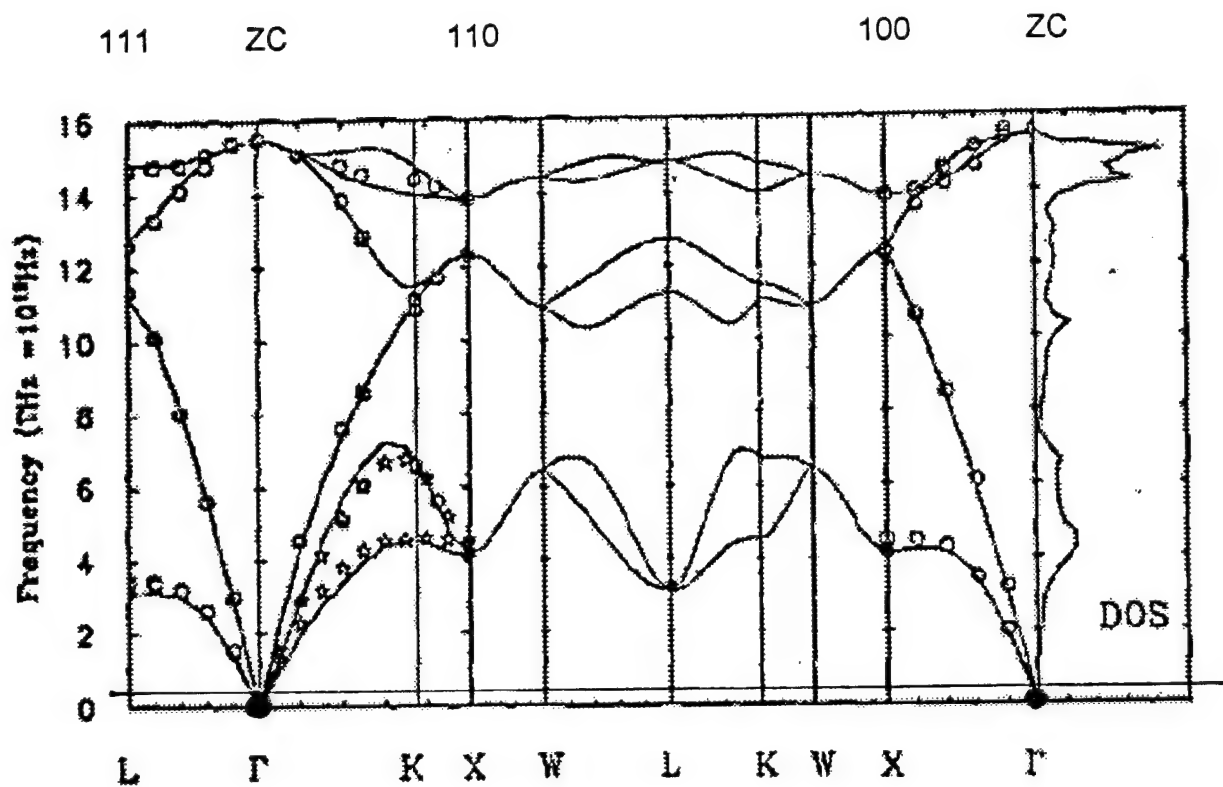
$$C(T) = \int_0^{\infty} g(\omega) C(\omega, T) d\omega$$



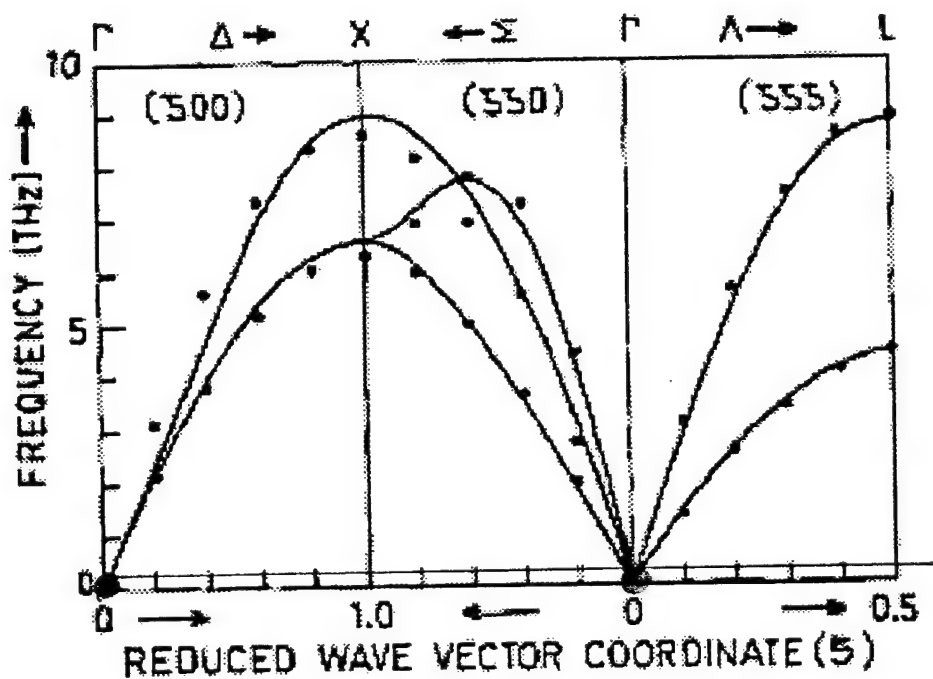
RUS DOS – Relevant for  $T \sim \text{few } \mu\text{K}$ ,  
otherwise, every material we measure,  
looks like a Debye solid,

$$g(\omega) \propto \omega^2$$

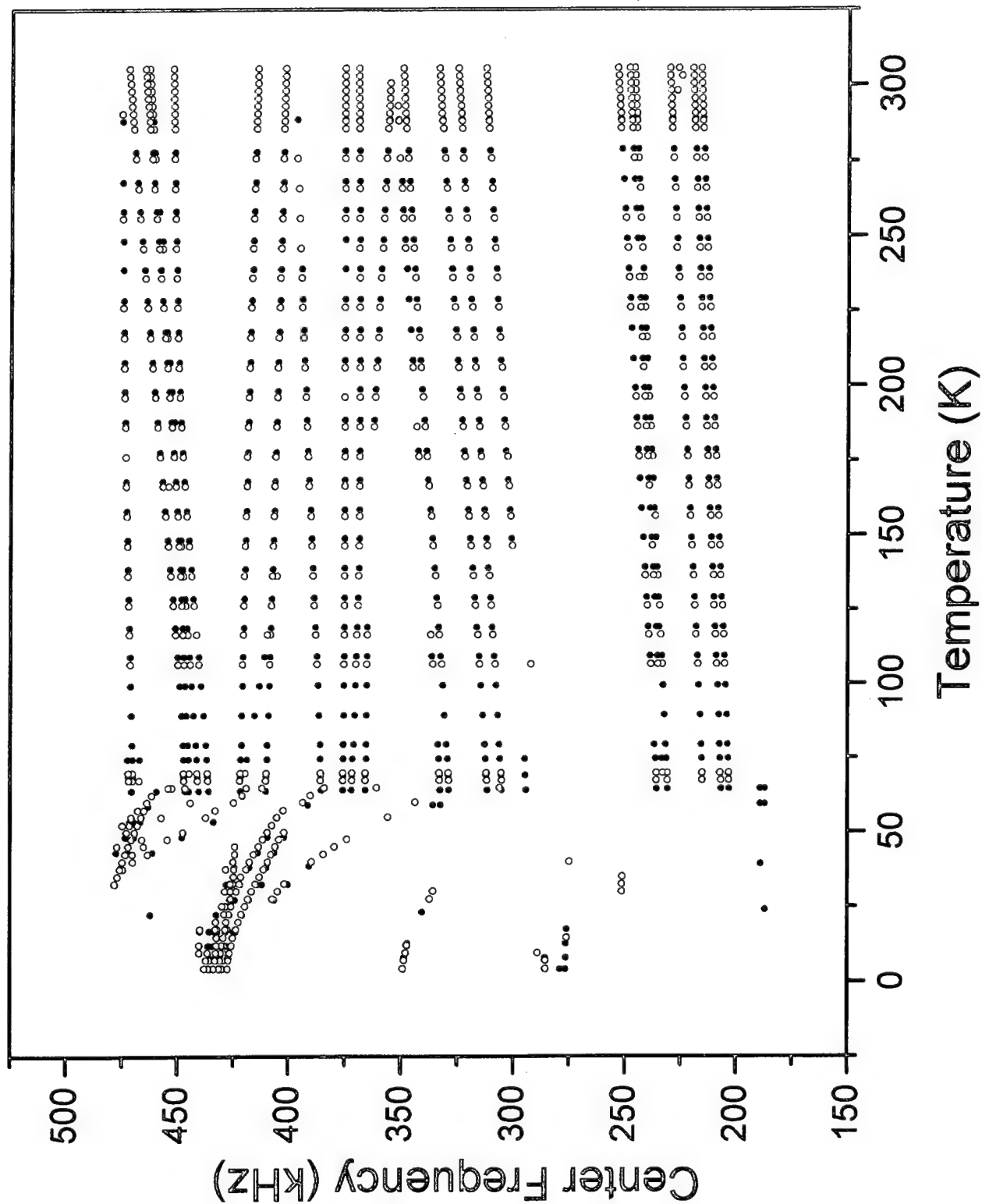
- Don't see optical branches
- Statistics too poor to see anomalies or contributing acoustic branches



# DISPERSION CURVES FOR Si, Ni



# AuZn single crystal - resonance frequencies



TR-9



Applicability of RUS results is only as good as the Debye model.

$$\omega = c k \quad \text{linear dispersion}$$

$c$  = wavespeed per branch (1 longitudinal)  
2 transverse

Thermodynamic results in terms of  $\bar{c}$

$$\frac{3}{\bar{c}^3} = \frac{1}{c_{t1}} + \frac{1}{c_{t2}} + \frac{1}{c_l}$$

eg.  $\Theta_D \propto \omega_D \propto \bar{c}$ , Free energy derivatives...

Isotropic :  $c_{t1} = c_{t2}$

+ cubic :  $c_{t1} \neq c_{t2}$ , and direction dependent  
+ lower symm.

RUS DETERMINES  $c_t, c_l$  ... VERY ACCURATELY - PROVIDED WE KNOW HOW MANY BRANCHES (independent  $c_{ij}$ 's) THERE ARE :

$$c \propto \sqrt{\frac{c_{ij}}{\rho}}$$

→ GLOBAL SYMMETRY

# SrTiO<sub>3</sub> sample, room temperature, 32 lines

	C11	C33	C23	C12	C44	C66	Bulk	$\chi^2/\text{nd}$	RMS
ISO	2.7845				1.2041		1.179	45.39	1.08
ISO*	2.7202				1.2241		1.088		0.098
HEX 1	3.1171	1.2310	1.2310	0.7778	1.1697	1.1697	1.810	25.68	0.87
HEX 2	2.8813	3.0530	0.8137	0.4203	1.2200	1.2305	1.423	8.30	0.35
HEX 3	2.8483	3.0274	0.7780	0.3551	1.2207	1.2466	1.381	3.34	0.24
CUF	3.1692			1.0391	1.2225		1.749	0.75	0.11
TEL	3.0139			0.8204	1.2336		1.265	0.47	0.093
HEX2	3.1538	3.1617	2.0264	1.0177	1.2215	1.2241	1.735	0.92	0.12
HEX3	3.1436	3.1590	2.0214	1.0060	1.2230	1.2302	1.727	0.67	0.10
ORTH	3.0088	2.1877	3.5529	0.8878	1.2021	1.2200	1.356	0.42	0.083

## 2% $\chi^2$ – increase errors

ISO	1.16				0.42				
HEX 1		18.1	10.85	10.0	0.30	0.5			
HEX 2		0.54	2.81	6.84	0.08	0.43			
HEX 3		0.39	1.87	5.49	0.06	0.37			
CUF	0.28		1.05	0.64					
TEL	0.43	4.42	6.2	1.65	0.9	0.01			
HEX2	0.28	0.27	1.03	0.61	0.01	0.06			
HEX3	0.22	0.27	1.02	0.61	0.01	0.06			
ORTH	0.43	3.93	5.74	1.42	0.08	0.02			

## **“Magic numbers” for RUS fits**

**Number of lines  $< np^*8$   
(80 or 96)**

**(could be much lower  
depending on  $c_{ij}$  and dim.'s)**

**Number of lines  $\geq nc^*5$   
(10,15,25,30,45)**

**rms%  $< 0.5\%$  AND all  $\Delta c_{ij} \leq 2\%$**

**rms% changes of  $< 20\%$  may  
not be meaningful**

## Conclusions:

1. RUS = Debye model
2. Dispersion relations from  $C_{ij}$ 's are essential to keep models honest
3. Determining symmetry ( $\#$  of  $C_{ij}$ 's) must be done very carefully
4. It seems we miss some interesting length scales, but sometimes the results of averaging procedures can be enlightening

## Future:

Temperature dependence of  $C_{ij}$ :

measurements 4K  $\rightarrow$  800K

correlation with thermal expansion

models for anomalous behavior

**CONTACTLESS MODE-SELECTIVE RESONANCE ULTRASOUND  
SPECTROSCOPY: ELECTROMAGNETIC ACOUSTIC RESONANCE**

HIROTSUGU OGI<sup>1,2</sup> AND HASSEL LEDBETTER<sup>2</sup>

<sup>1</sup>OSAKA UNIVERSITY

<sup>2</sup>NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY

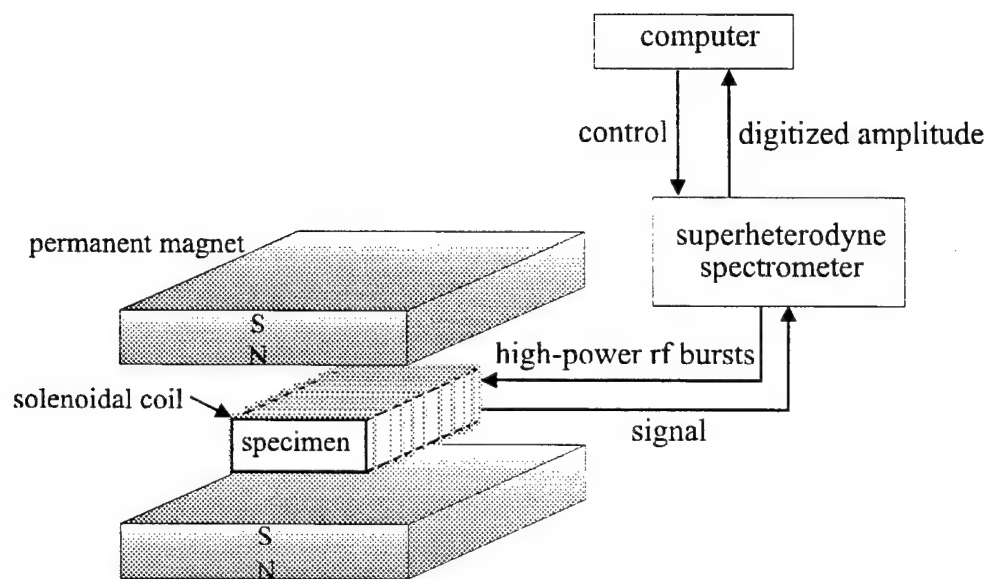


Fig.1 Typical EMAR-measurement setup

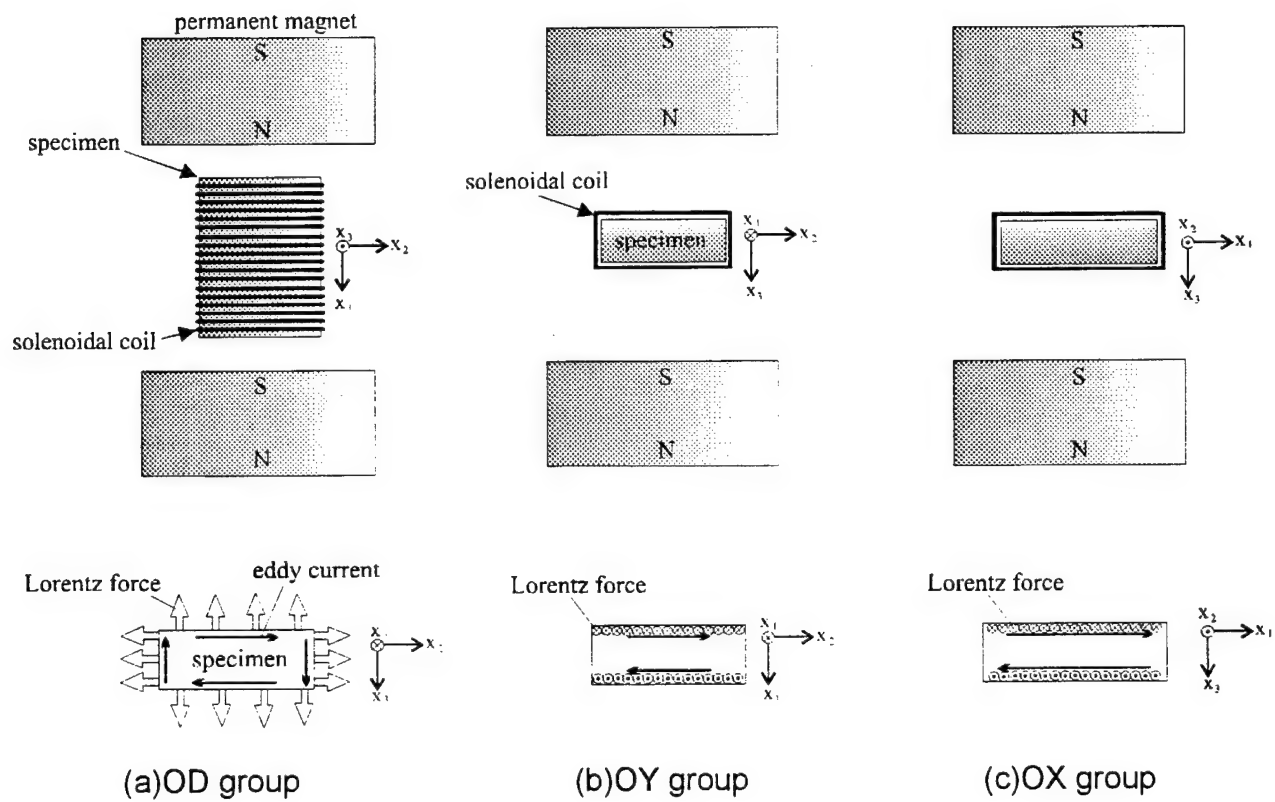
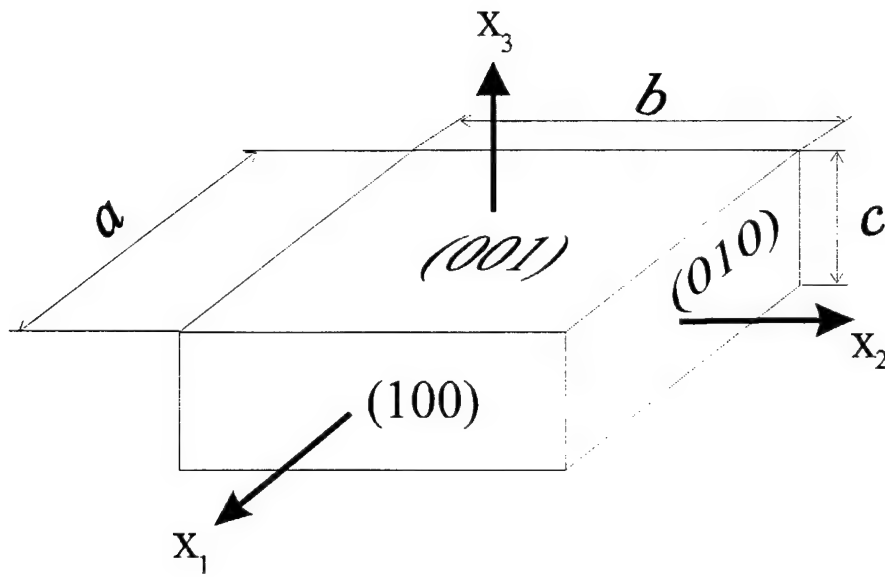
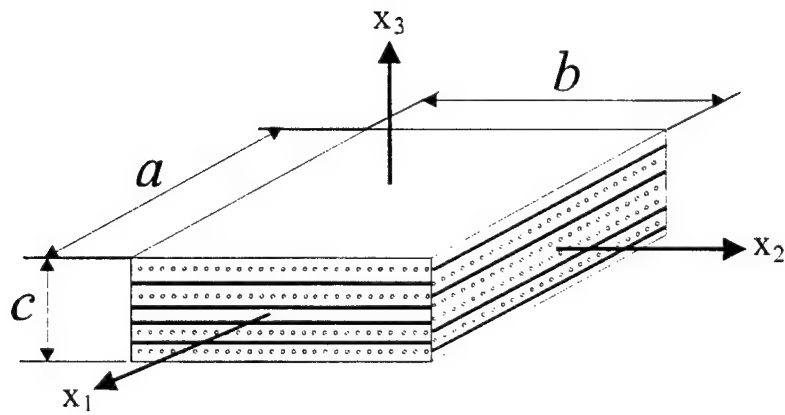


Fig.2 Selection of vibration group with EMAR.



$a = 6.195 \text{ mm}$ ,  $b = 6.167 \text{ mm}$ , and  $c = 3.404 \text{ mm}$



$a = 4.977 \text{ mm}$ ,  $b = 4.015 \text{ mm}$ , and  $c = 1.905 \text{ mm}$

Fig. 3 Copper monocrystal (upper) and  $\text{SiC}_f/\text{Ti-6Al-4V}$  crossply composite (bottom).



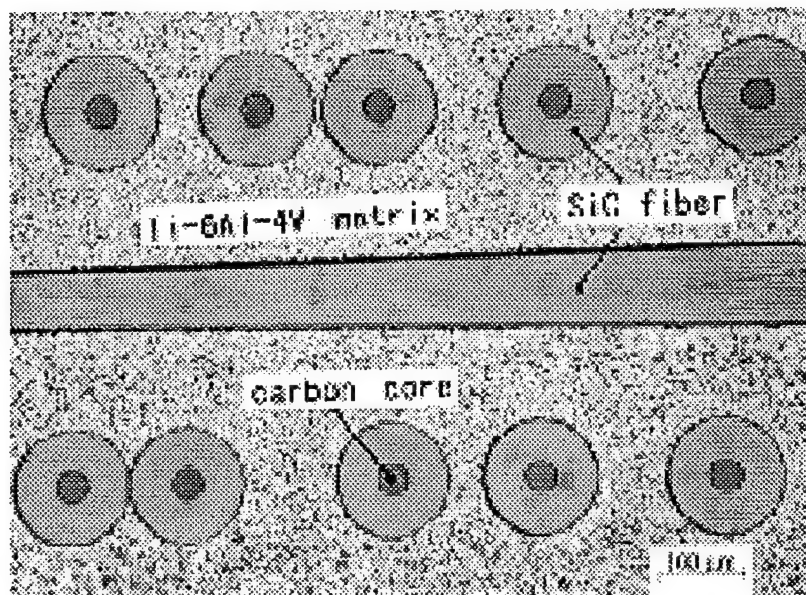


Fig. 4 Microstructure of SiC/Ti-6Al-4V crossply composite.

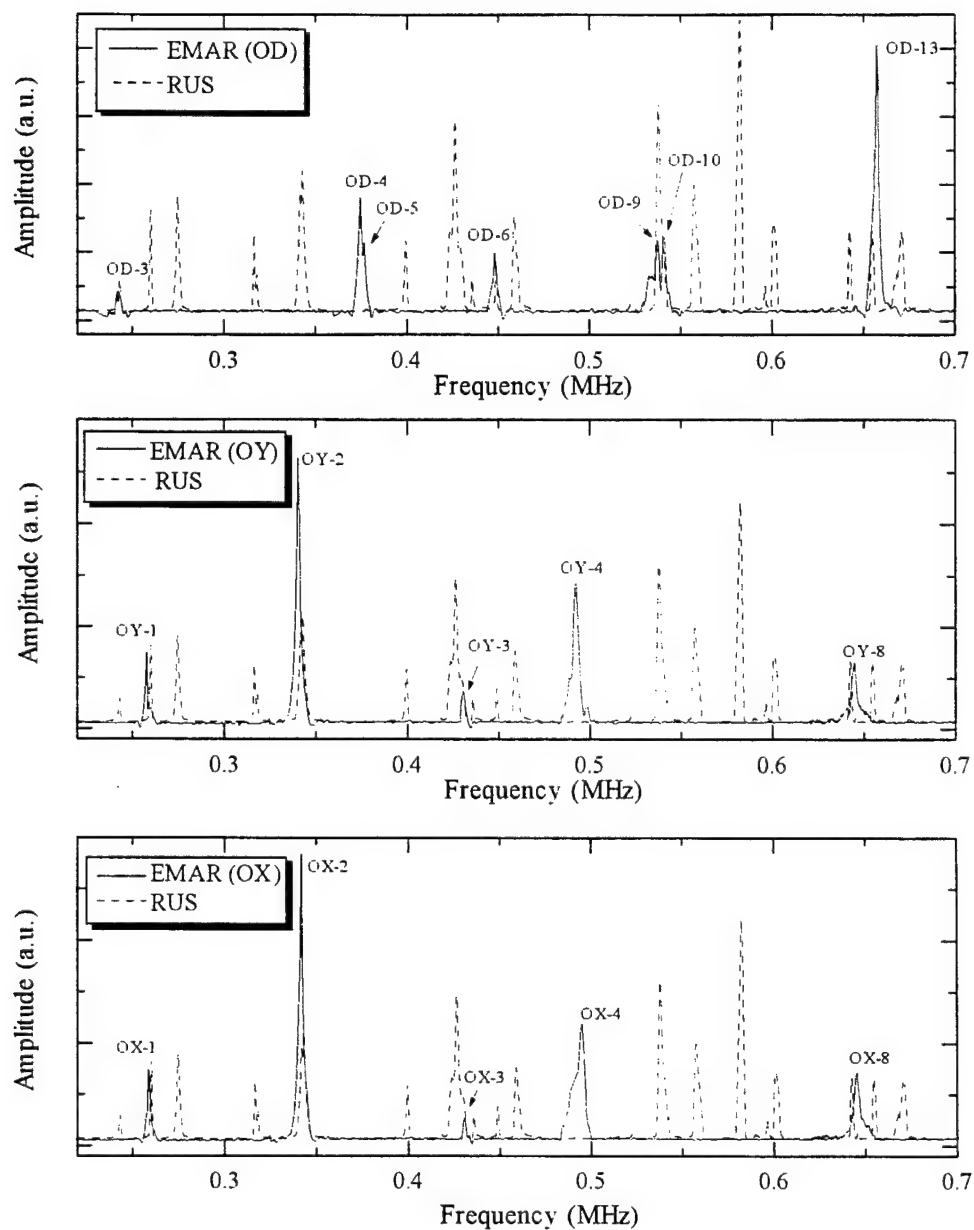


Fig. 5 Resonance spectra for the copper monocrystal measured by RUS and EMAR.

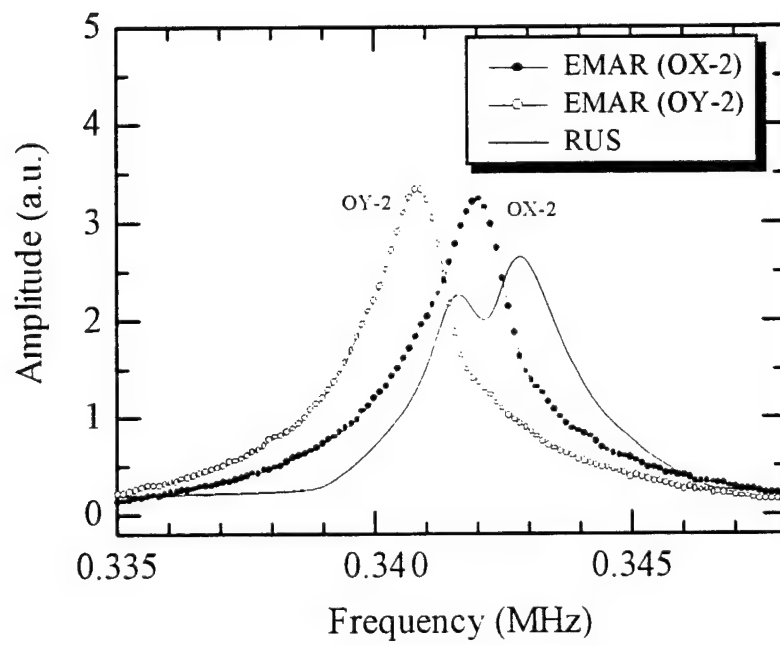


Fig. 6 EMAR and RUS spectra for the copper monocrystal.

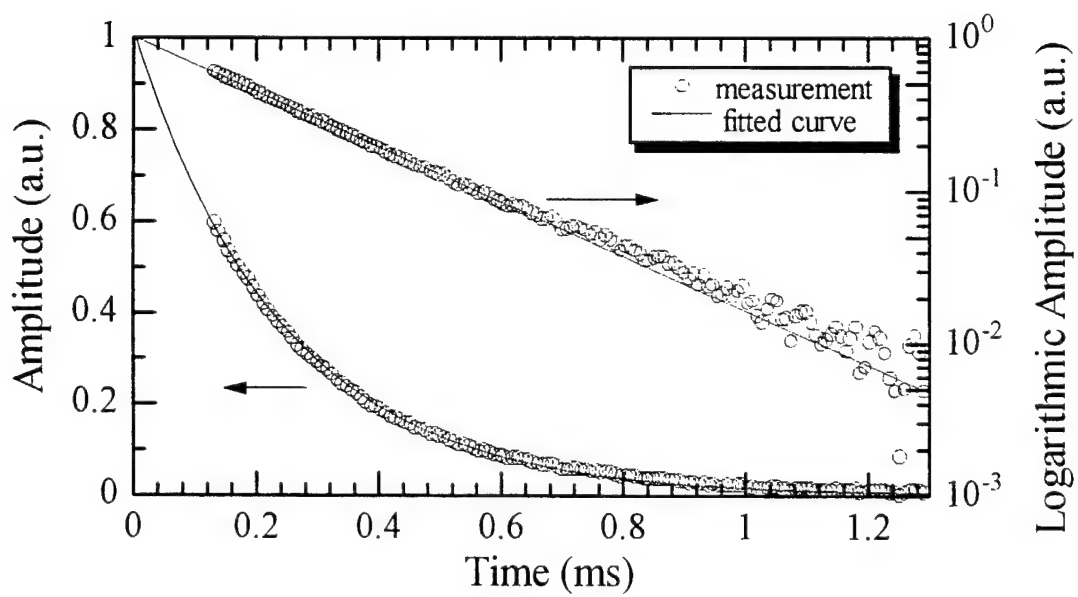


Fig. 7 Amplitude decay and fitted exponential curve for internal friction (copper monocrystal).

Table I Elastic constants and internal friction of a copper monocrystal determined by EMAR and RUS.

	$C_{ij}(\text{GPa})$				$Q_{ij}^{-1} (10^{-3})$	
	EMAR	RUS	Ref. 1	Ref. 2	EMAR	RUS
$C_{11}$	167	168.7	169.7	168.4	1.14	1.74
$C_{12}$	120.9	121.6	122.6	121.4	0.3	1.34
$C_{44}$	74.64	75.46	74.49	75.4	2.59	2.46
$B=(C_{11}+2C_{12})/3$	136.2	137.3	137.3	137.1	0.64	1.5
$C'=(C_{11}-C_{12})/2$	23.05	23.52	23.35	23.5	3.34	2.8
$C_{110,110}=(C_{11}+C_{12}+2C_{44})/2$	218.6	220.6	220.6	220.3	1.4	1.87
$C_{111,111}=(C_{11}+2C_{12}+4C_{44})/3$	235.7	237.9	237.6	237.7	1.47	1.9
$C_{111,arb}=(C_{11}-C_{12}+C_{44})/3$	40.25	40.84	40.54	40.8	2.88	2.59
$C_L$	199.6	201.6	-	-	1.34	1.84
$G$	47.54	48.22	-	-	2.83	2.57
$E$	127.8	129.5	-	-	2.6	2.46
$\nu$	0.3437	0.3428	-	-	-	-

1. H. Ledbetter and S. Kim, Personal communication, unpublished.
2. R. Hearmon, Adv. Phys., 5, 323 (1956).

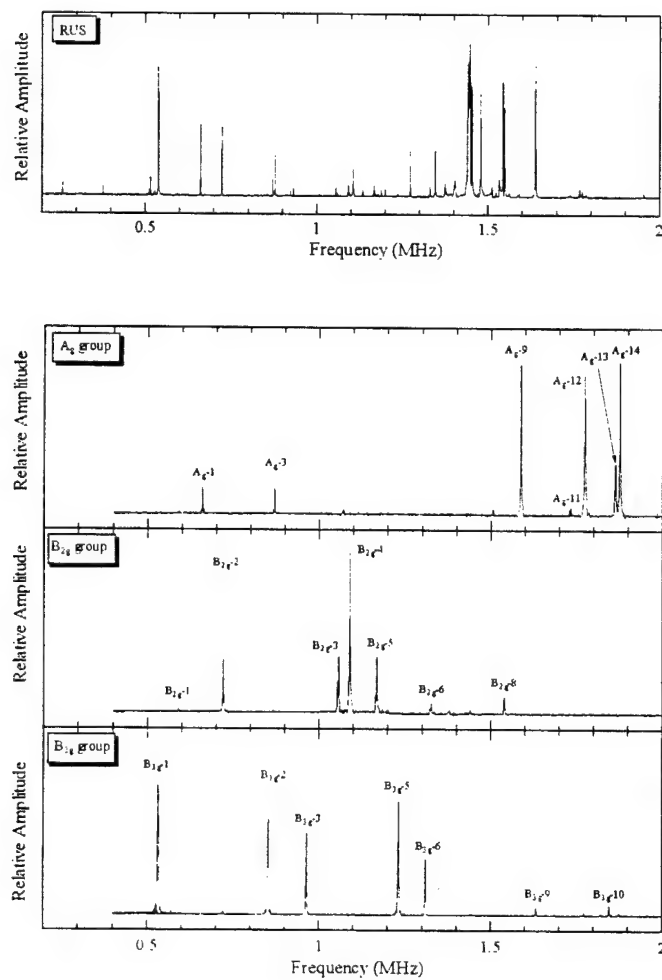


Fig. 8 Resonance spectra of  $\text{SiC}/\text{Ti-6Al-4V}$  crossply composite measured by EMAR and RUS methods.

Table II Elastic constants (GPa) and internal friction of SiC/Ti-6Al-4V crossply composite measured by EMAR, RUS, and pulse-echo methods.

	EMAR	RUS	pulse echo	EMAR	EMAR*	$Q^{-1}_{ij} (10^{-4})$
$C_{11}$	225.4	230.4	—	221.7	222.1	7.79
$C_{33}$	191.6	185.8	188	182.6	183.3	15.6
$C_{44}$	54.58	54.98	53.7	54.6	54.63	9.33
$C_{66}$	51.00	51.73	—	53.02	52.08	6.33
$C_{12}$	80.82	85.14	—	74.2	74.36	16.1
$C_{13}$	79.16	80.9	—	71.29	71.65	23.7
$E_{11}$	180.6	182.3	—	182.7	183.1	3.54
$E_{33}$	149.7	142.1	—	146.8	147.3	9.59
B	124.5	126.7	—	117.7	118.1	15.2

\* Determined from OD and OY vibration groups.

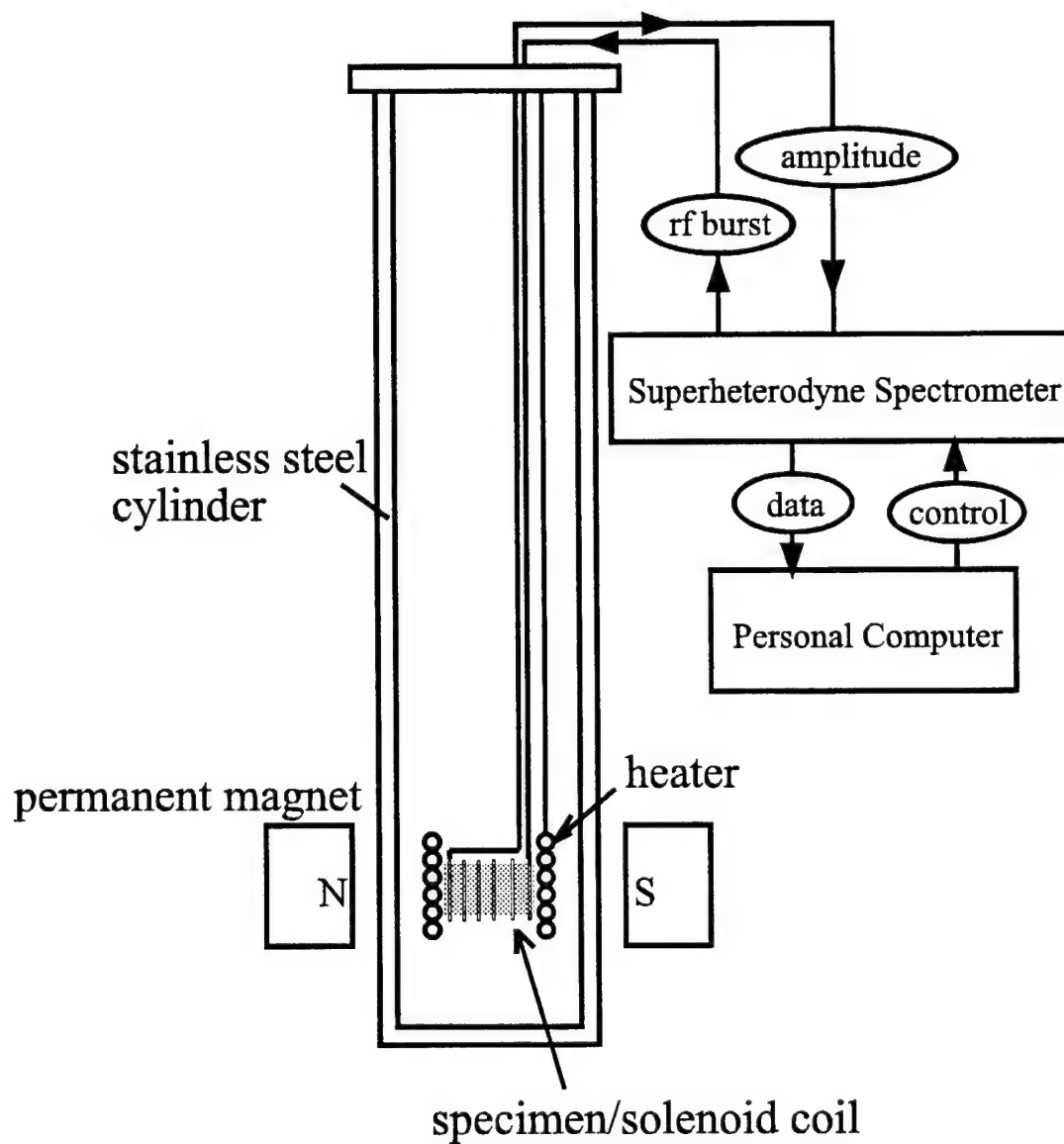


Fig. 9 EMAR measurement setup at elevated temperatures.



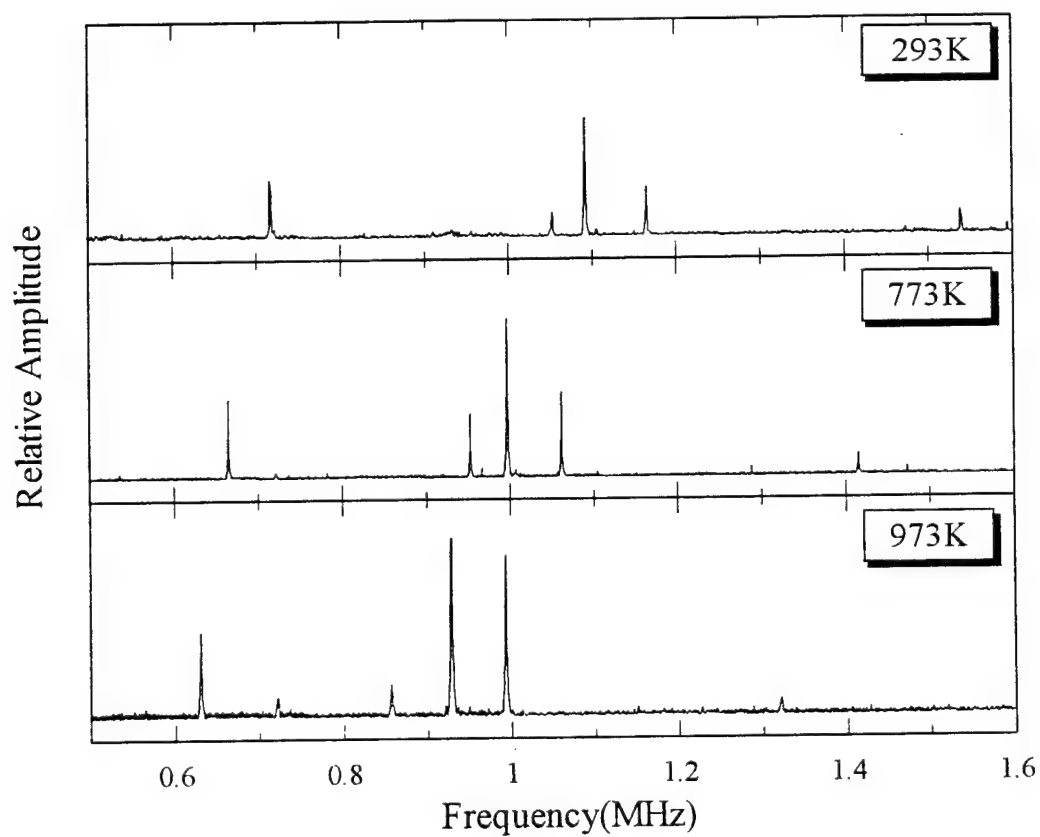


Fig. 10 EMAR spectra of OY group at various temperatures for SiC<sub>f</sub>/Ti-6Al-4V crossply composite.

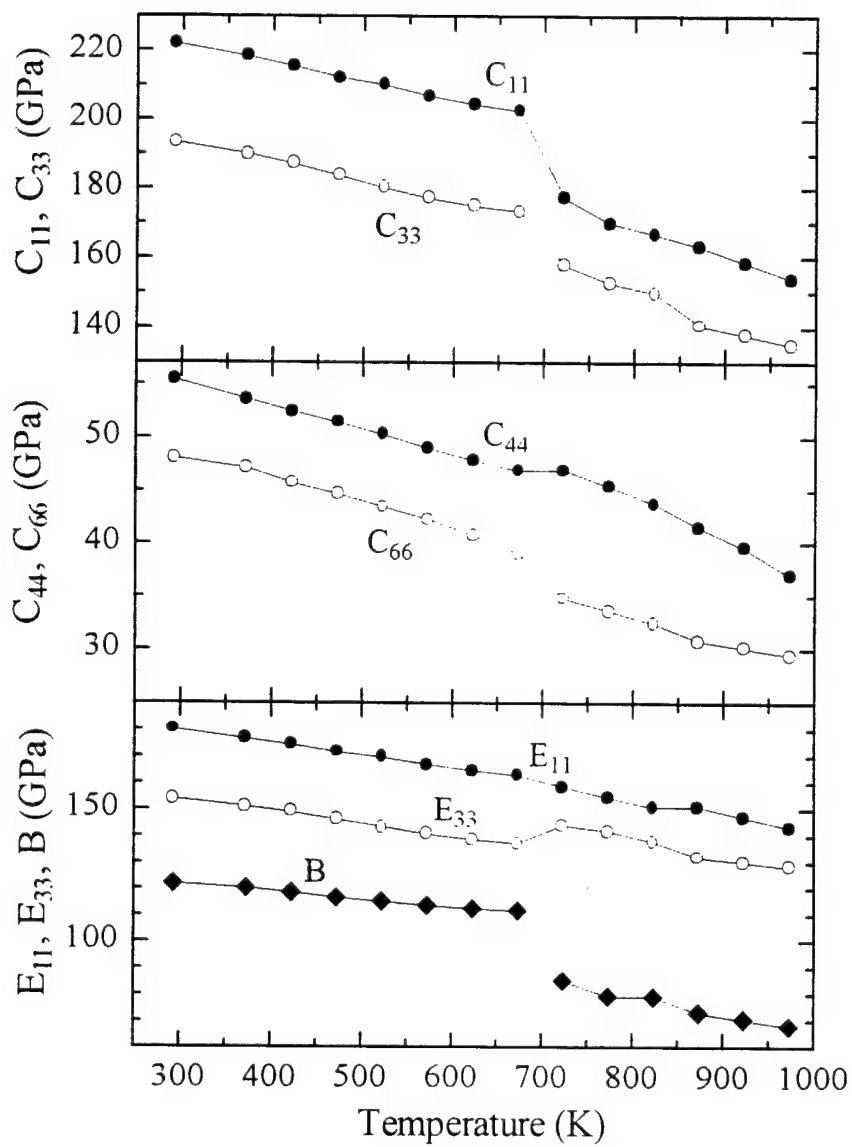


Fig. 11 Temperature dependence of elastic constants of  $\text{SiC}_f/\text{Ti-6Al-4V}$  crossply composite.

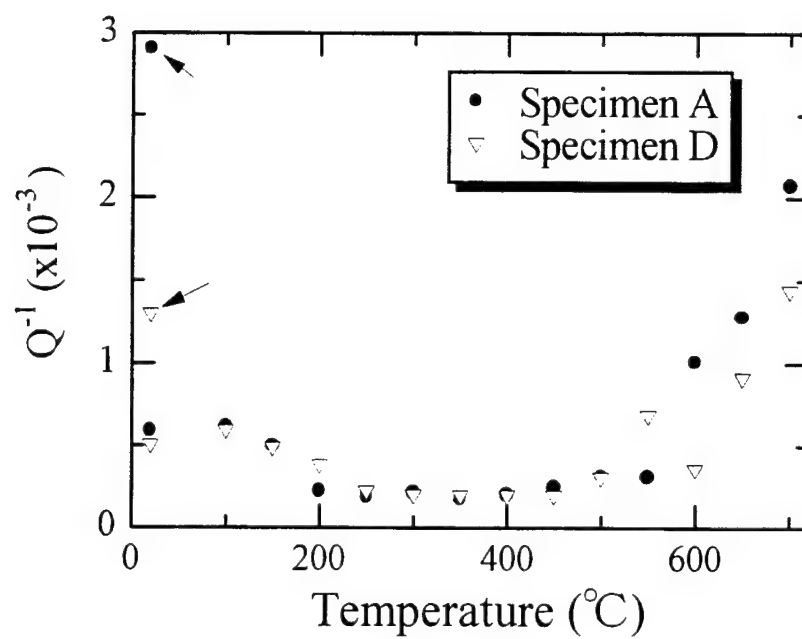


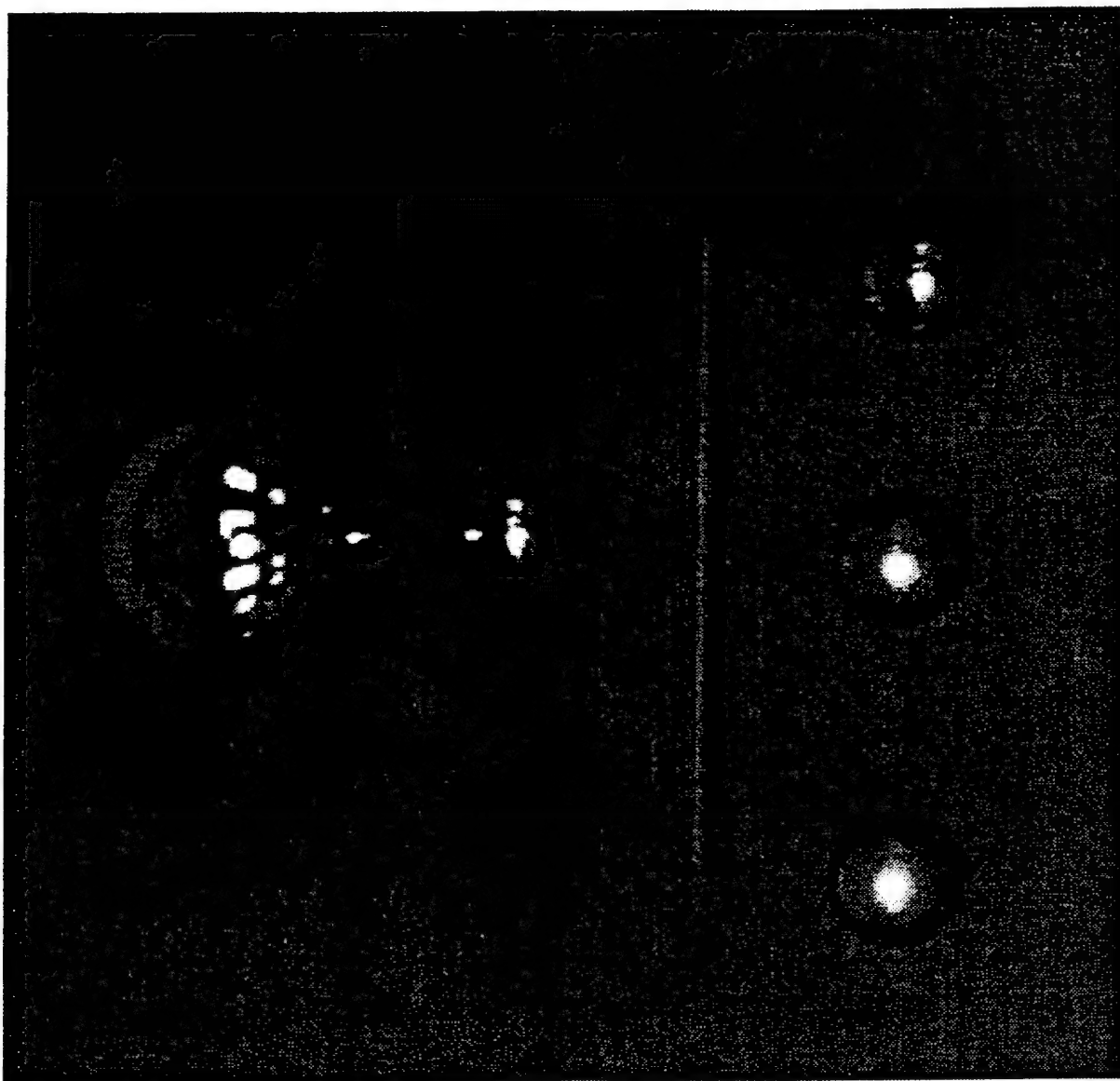
Fig. 12 Temperature dependence of internal friction of a OY mode of SiC<sub>r</sub>/Ti-6Al-4V crossply composite. Arrows indicate internal friction after cooling down to room temperature.

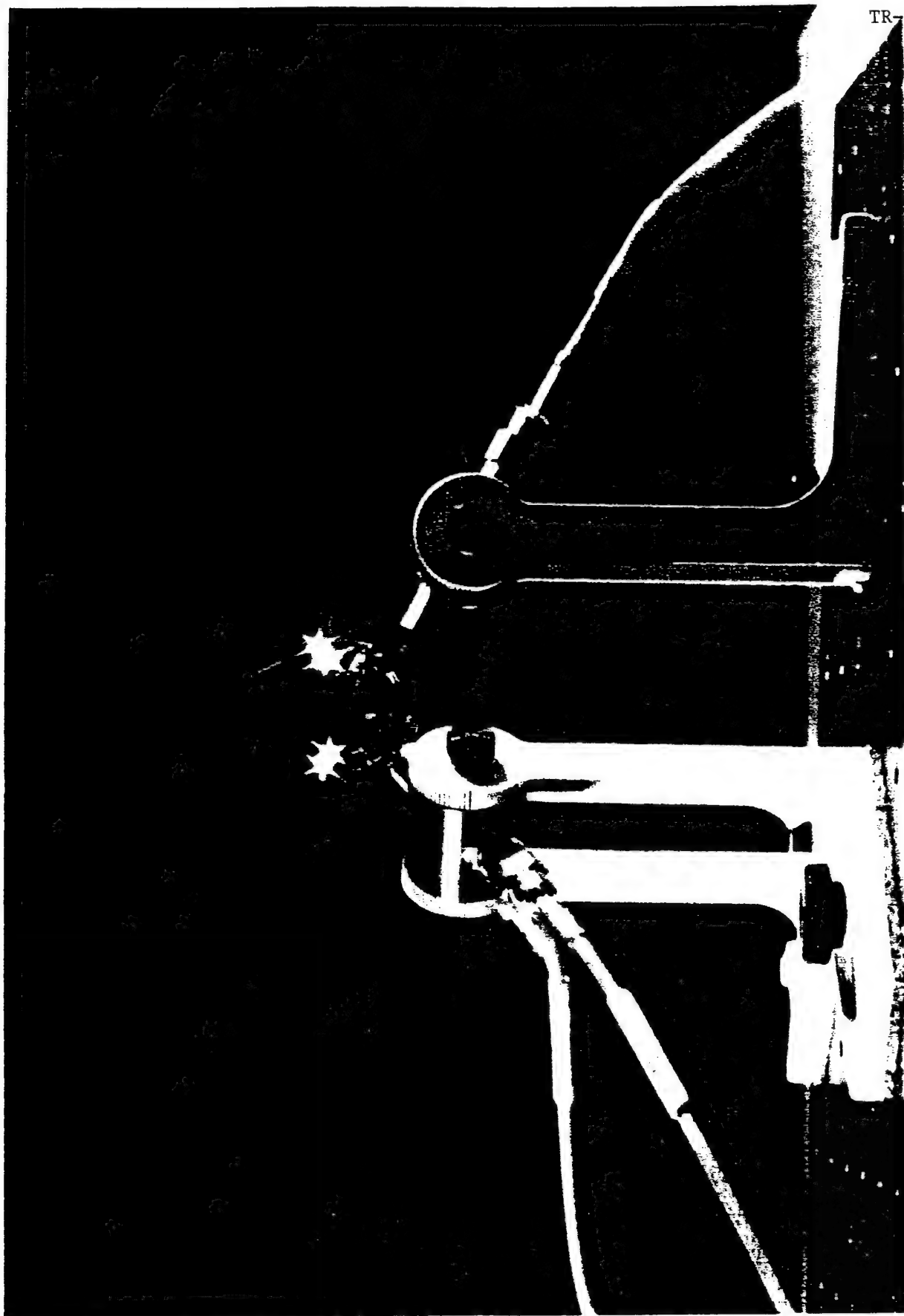
# **Elastic Coefficients and Internal Friction Of Silicon Monocrystal Spheres**

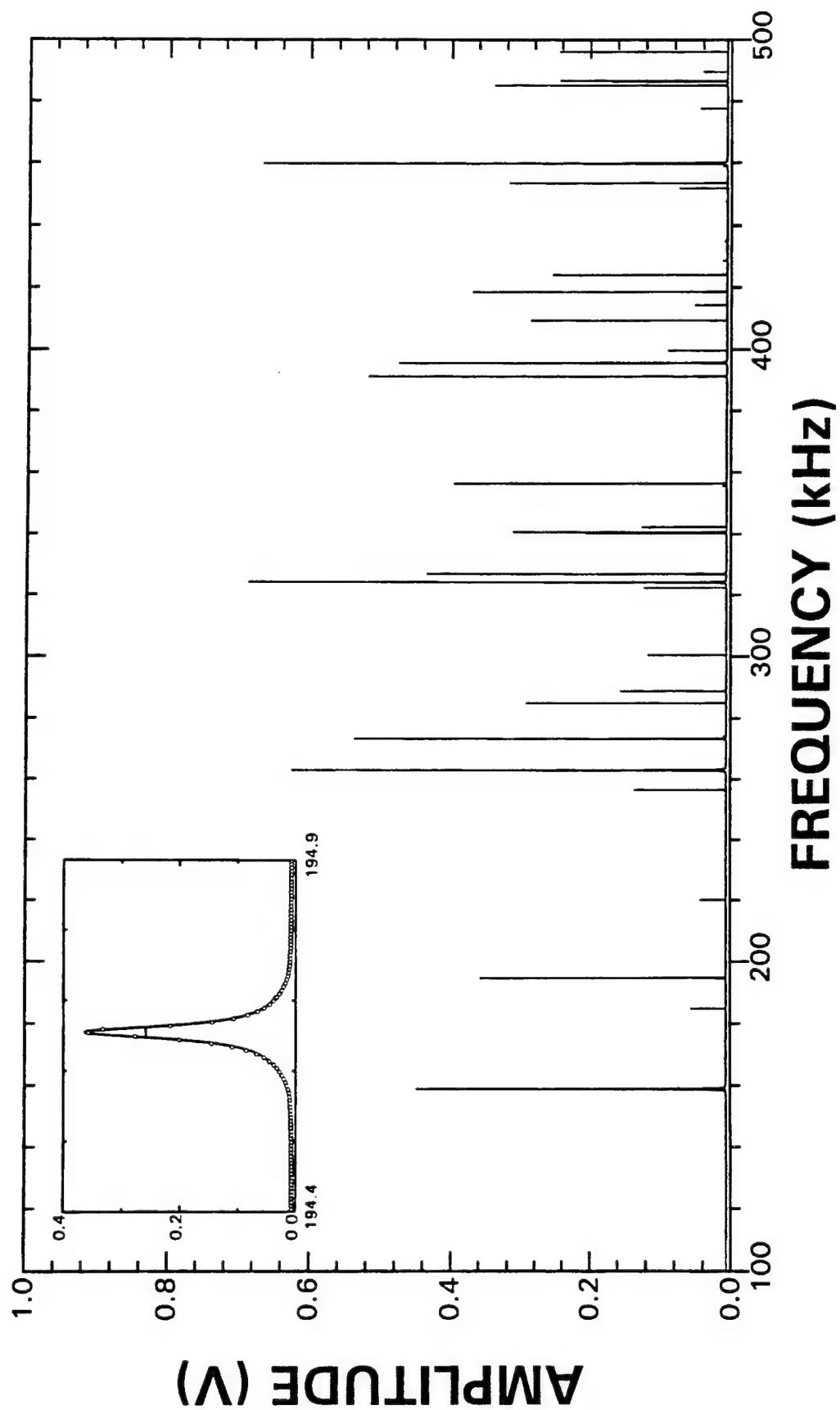
**Hassel Ledbetter  
NIST, Boulder, Colorado**

## **Monocrystal silicon**

- .low impurity content**
- .low dislocation density**
- .used in international standards efforts**
  - .silicon mole**
  - .kilogram**
  - .Avogadro's number**







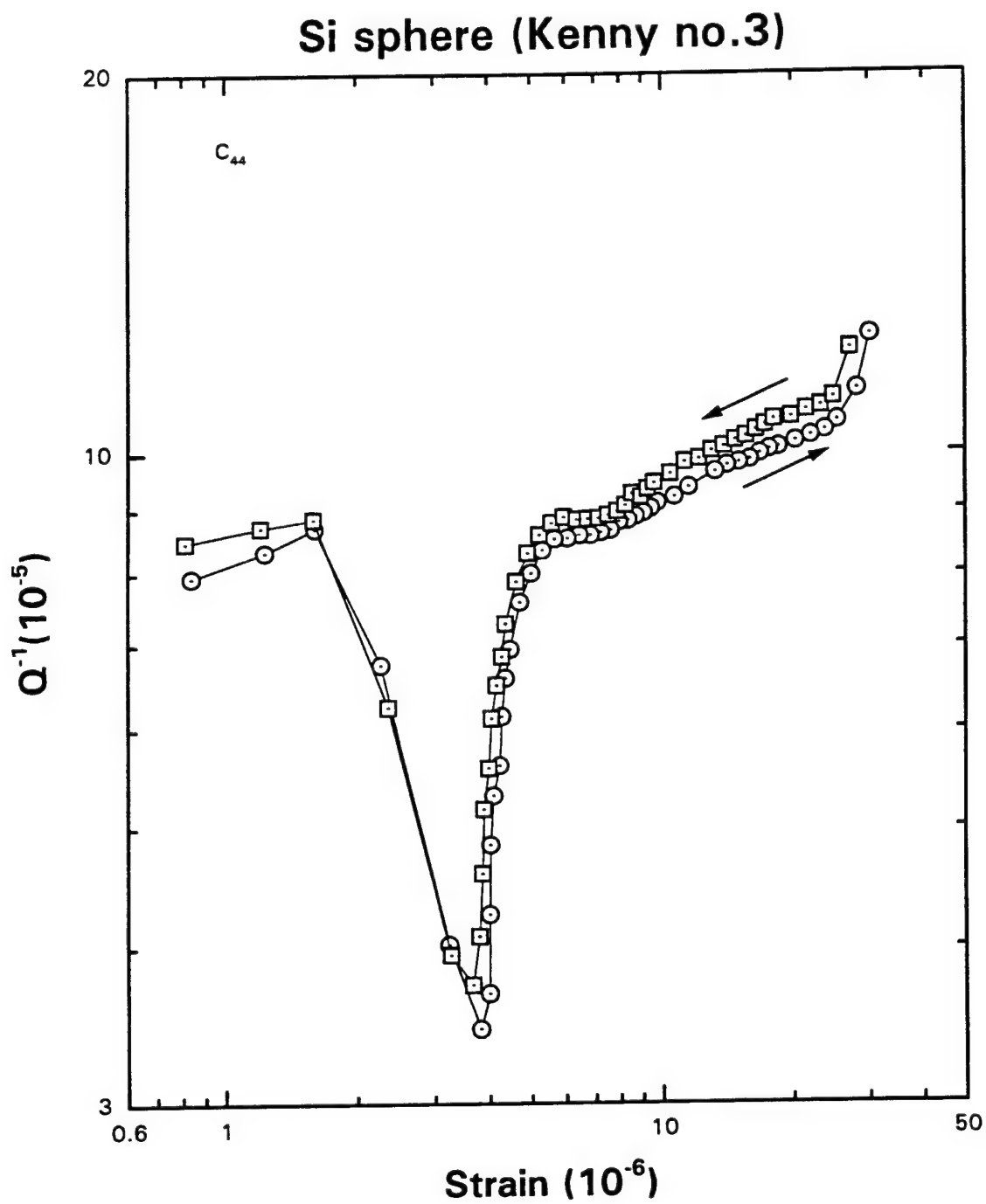


## Si elastic-stiffness coefficients

	K1	K2	K3	T1	T2
T (K)	294.8	294.8	294.8	298.4	299.3
$\rho$ (g/cm <sup>3</sup> )	2.33275	2.33292	2.33219	2.32903	2.32999
$C_{11}$ (GPa)	165.73 $\pm$ 0.03	165.73 $\pm$ 0.03	165.69 $\pm$ 0.03	165.58 $\pm$ 0.05	165.67 $\pm$ 0.02
$C_{12}$ (GPa)	63.917 $\pm$ 0.03	63.919 $\pm$ 0.04	63.914 $\pm$ 0.03	63.840 $\pm$ 0.04	63.924 $\pm$ 0.03
$C_{44}$ (GPa)	79.596 $\pm$ 0.008	79.579 $\pm$ 0.008	79.584 $\pm$ 0.008	79.550 $\pm$ 0.02	79.577 $\pm$ 0.008
$C'$ (GPa)	50.906	50.905	50.890	50.868	50.875
$C_{110,110}$ (GPa) = $(C_{11} + C_{12} + 2C_{44})/2$	194.42	194.40	194.39	194.26	194.33
$C_{111,111}$ (GPa) = $(C_{11} + 2C_{12} + 4C_{44})/3$	203.98	203.96	203.95	203.82	203.94
$C_{111,110}$ (GPa) = $(C_{11} - C_{12} + C_{44})/3$	60.469	60.463	60.455	60.429	60.442
B (GPa)	97.854	97.856	97.841	97.752	97.840
G (GPa)	66.604	66.595	66.589	66.561	66.578
E (GPa)	162.86	162.84	162.83	162.74	162.81
$C_L$ (GPa)	186.66	186.65	186.63	186.50	186.61
$\nu$	0.22260	0.22261	0.22265	0.22249	0.22270
rms (%)	0.021	0.023	0.019	0.026	0.016

Source: S. Kim, H. Ledbetter, NIST—Boulder.

26 MAY 1999

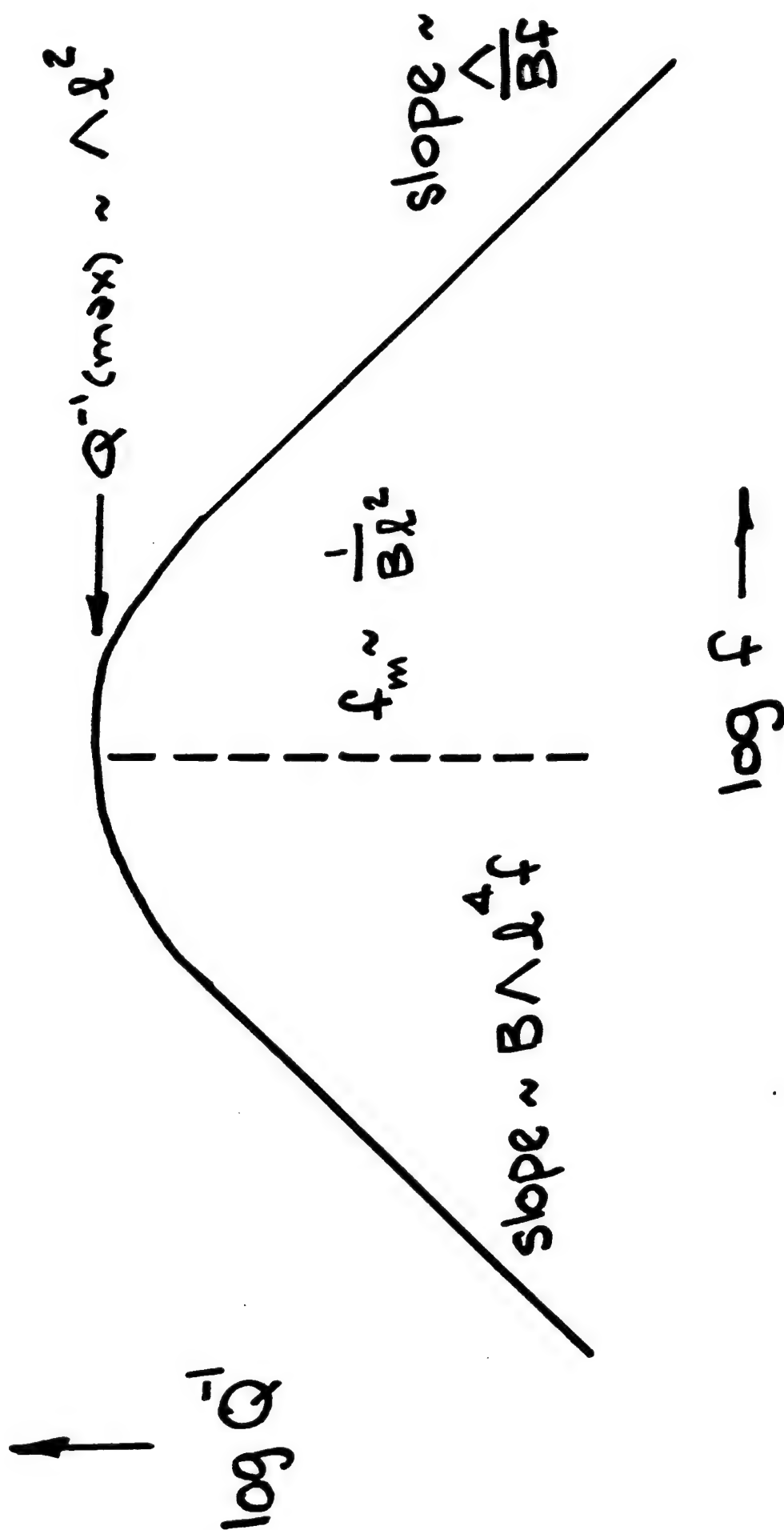


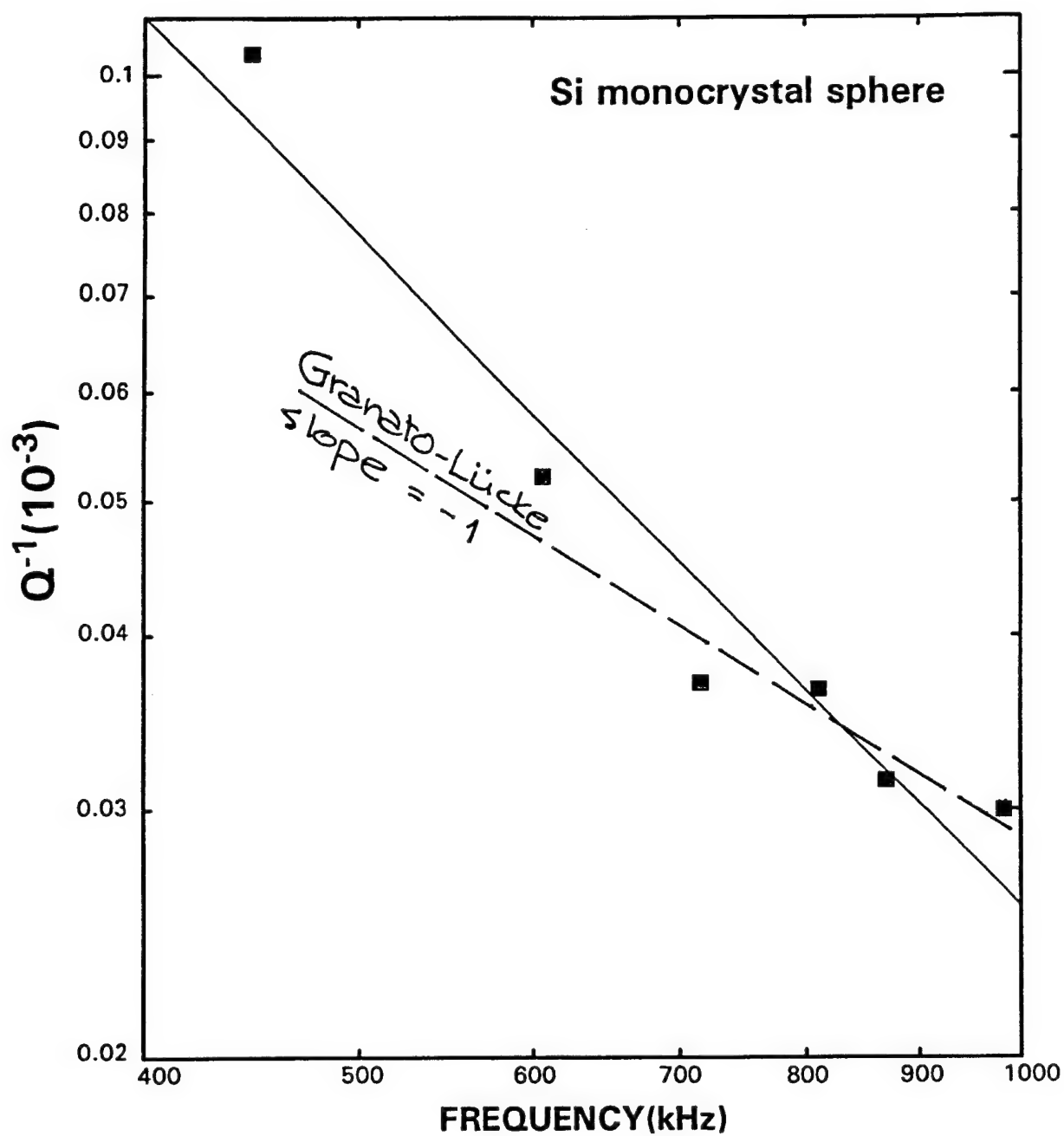
From the Koehler-Granato-Lücke stretched-vibrating-string model, we have for low frequencies (in modified form) :

$$Q^{-1} = \frac{576\Omega(1 - \nu)^2 \Lambda L^4 kT \omega}{\pi^3 \rho \nu_t^3 a^4}.$$

Here  $\Omega$  denotes an orientation factor,  $\nu$  is Poisson's ratio,  $\Lambda$  dislocation density,  $L$  dislocation-loop length,  $k$  Boltzmann's constant,  $T$  temperature,  $\omega$  circular frequency,  $\rho$  mass density,  $\nu_t$  shear-wave sound velocity, and  $a$  unit-cell dimension.

# Granato-Lücke model





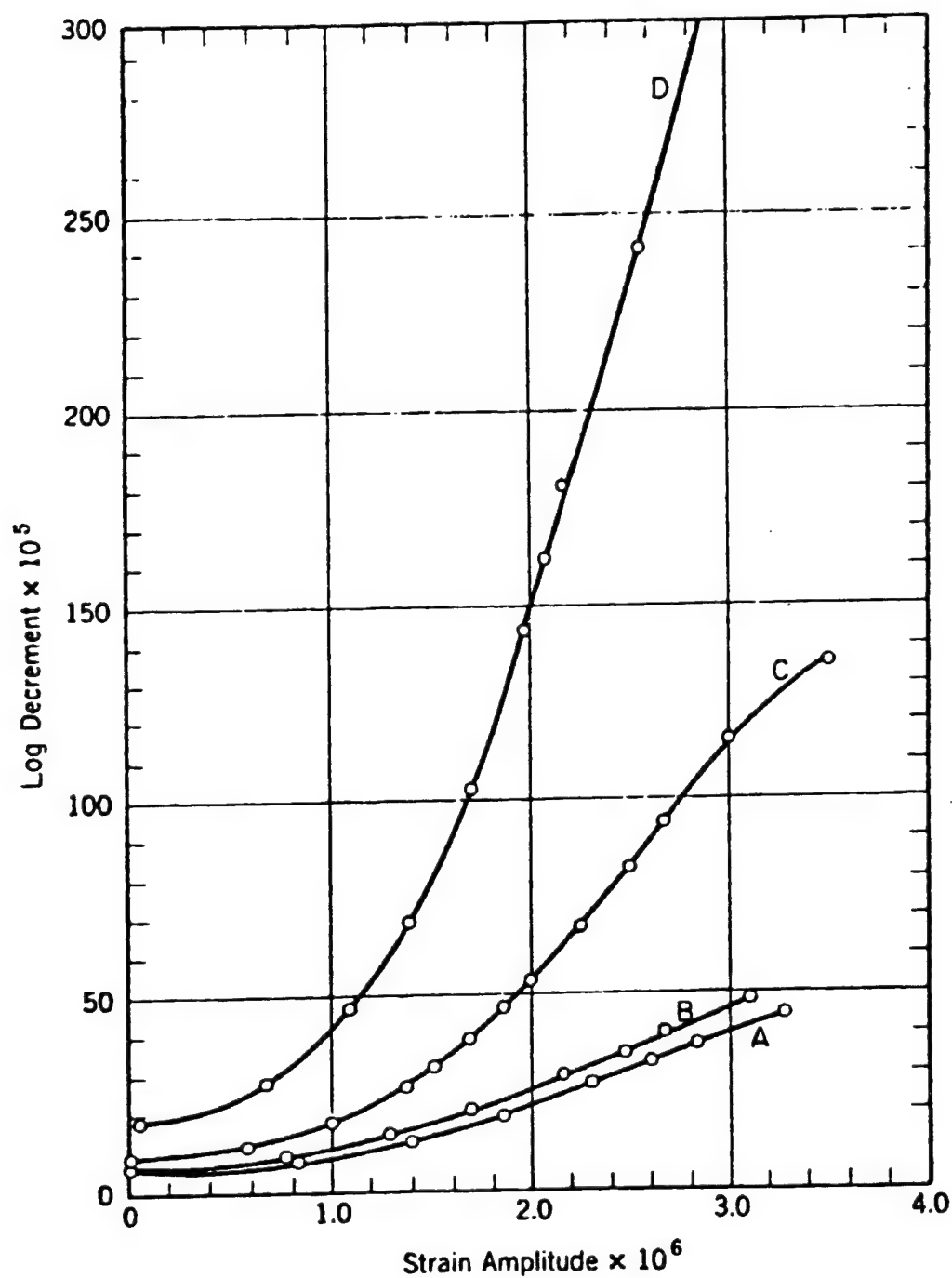


FIG. 14-1. Internal friction as a function of strain amplitude for an undeformed copper crystal (curve A) and after application of stresses of 60, 120, and 150 psi, respectively (curves B, C, and D). Measurements in longitudinal vibration at about 30 kHz. (From Read, 1941.)

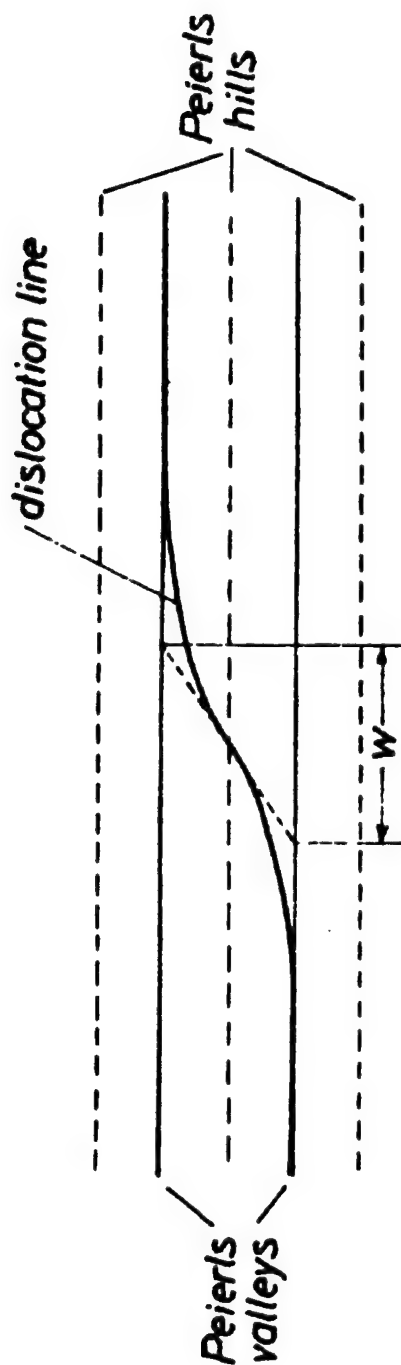


FIG. 3. Kink in a dislocation line at the transition from one Peierls valley to neighboring valley.  $w$  = kink width.

Seeger, Schiller (1966)

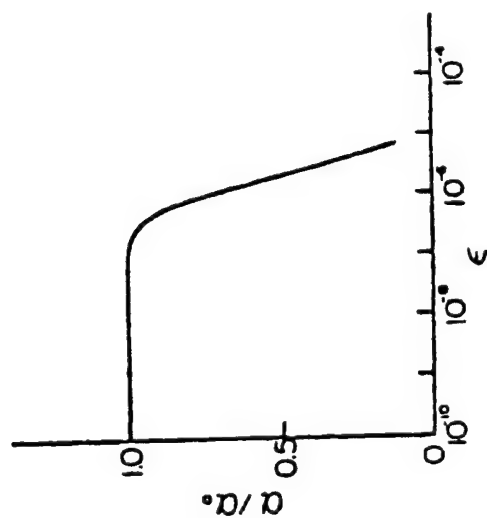
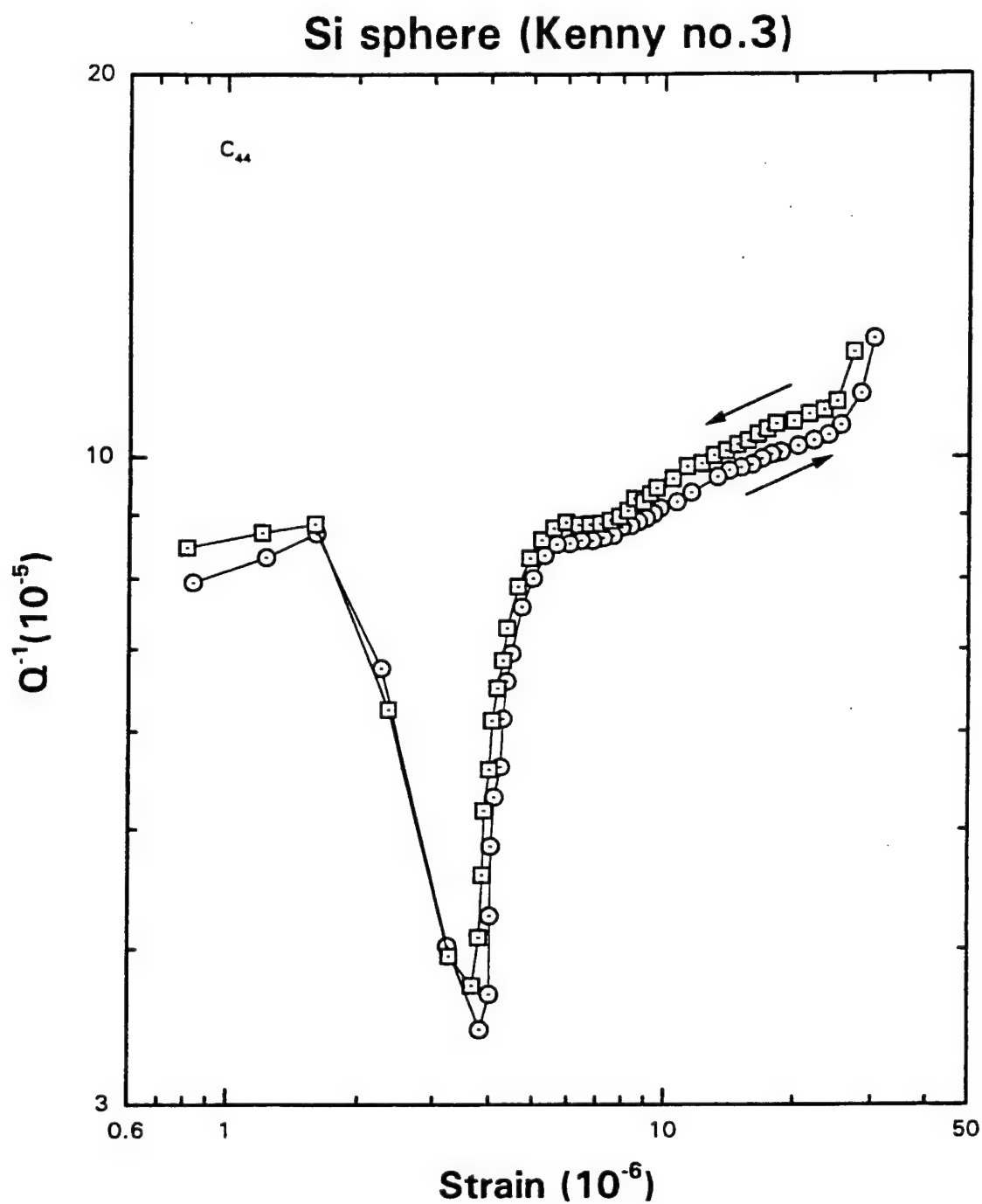


FIG. 4. The ratio of the ultrasonic attenuation  $\alpha$  at finite strain amplitude to the attenuation  $\alpha_0$  at infinitely small strain amplitude, when  $l = 2d_0 = 2 \times 10^{-5}$  cm and  $\omega = 2\pi 10^5$  sec $^{-1}$ .

SUZUKI, ELBAUM (1964)





## CONCLUSIONS

1. Within 1 in 1000, new-silicon Cij agree with old-silicon Cij.
2. At low strains, dislocation kinks contribute most of  $Q^{-1}$ .
3. At high strains, Koehler-Granato-Lücke defined string-model dislocations contribute most to  $Q^{-1}$ .

# **Elastic Constants and Internal Friction of Quartz and Langasite**

**Ming Lei\***

**Dynamic Resonance Systems, Powell, Wyoming**

**Hassel Ledbetter**

**National Institute of Standards and Technology, Boulder, Colorado**

**Resonance Workshop, Oxford, Mississippi, June 1999**

**\*Now at Lucent, Denver**

	<u>Langasite</u>	<u>Quartz</u>
Phase transform.	-----	573°C
$Q^{-1} (10^{-5})^{**}$	17.3	12.6
$\epsilon_{11}$	168	39
$\epsilon_{33}$	460	41
$e_{11} (C/m^2)$	-0.43	-0.173
$e_{14} (C/m^2)$	0.11	0.04
$k^2$	4.2	2.5

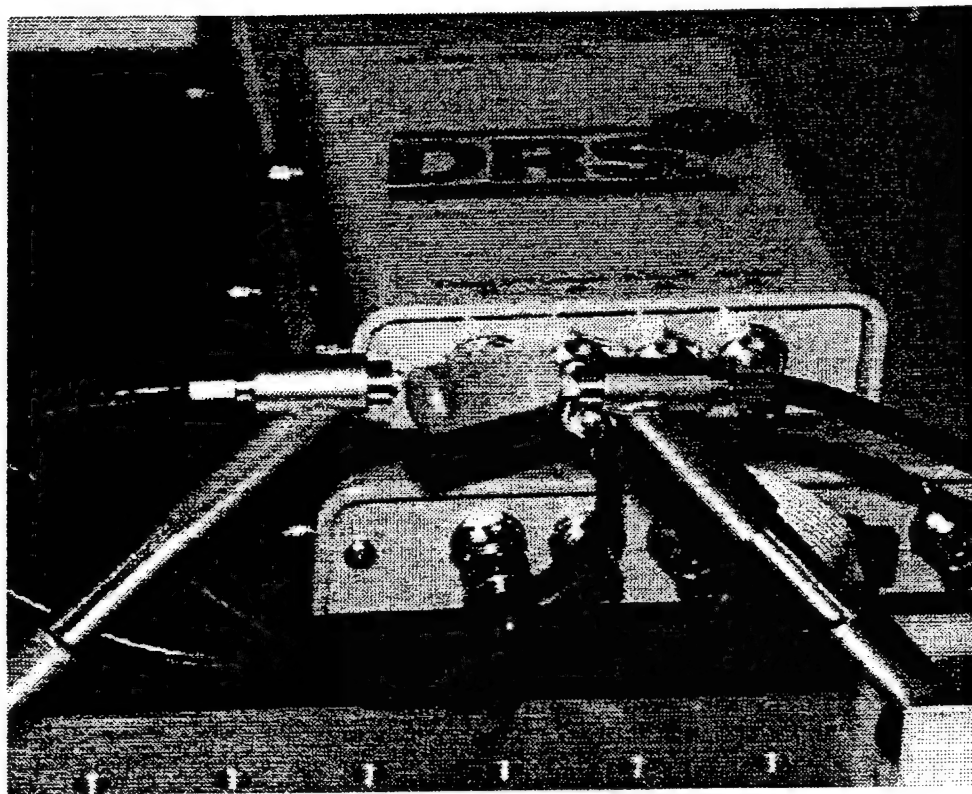
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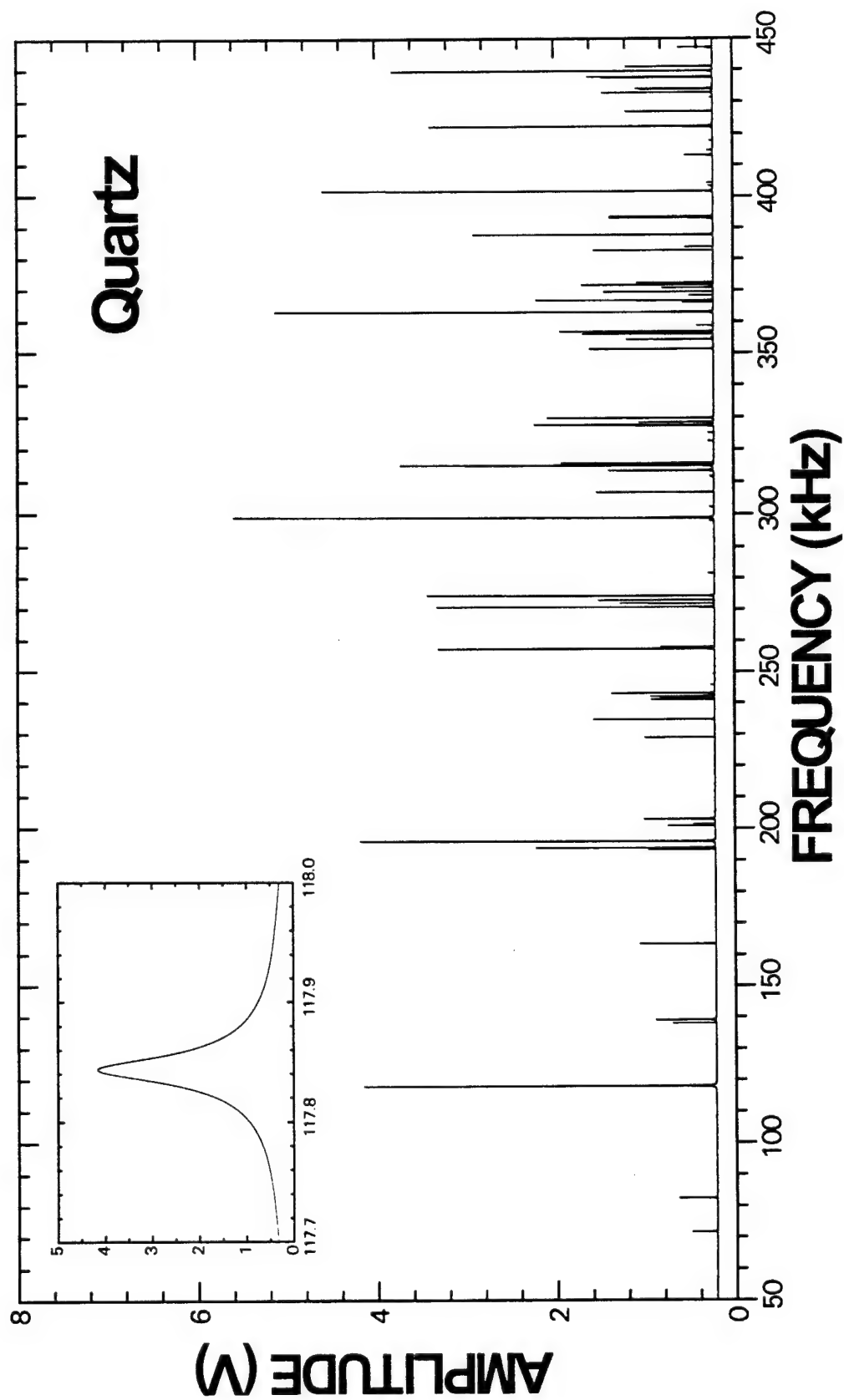
\*Equivalent designations: P321,  $D_3^2$ , hP9.

\*\*First torsional mode of a cylinder.

**Table 1. Langasite and Quartz: basic properties**

	<u>Langasite</u>	<u>Quartz</u>
<b>Formula</b>	<b>La<sub>3</sub>Ga<sub>5</sub>SiO<sub>14</sub></b>	<b>SiO<sub>2</sub></b>
<b>Mass density (g/cm<sup>3</sup>)</b>	<b>5.744</b>	<b>2.6500</b>
<b>Point (space) group</b>	<b>Trigonal 32 (150)*</b>	<b>Trigonal 32 (150)</b>
<b>Atoms/cell</b>	<b>23</b>	<b>9</b>
<b>C<sub>11</sub> (GPa)</b>	<b>186.1</b>	<b>87.05</b>
<b>C<sub>33</sub> (GPa)</b>	<b>259.8</b>	<b>106.9</b>
<b>V<sub>11</sub> (cm/μs)</b>	<b>0.5692</b>	<b>0.5731</b>
<b>V<sub>33</sub> (cm/μs)</b>	<b>0.6725</b>	<b>0.6328</b>





# Quartz elastic-stiffness coefficients

	Present	Ohno	McSkimin
$\rho$ (g/cm <sup>3</sup> )	2.6496	2.648	2.6485
$C_{11}$ (GPa)	87.10	88.1	86.8
$C_{33}$ (GPa)	106.1	104.7	105.75
$C_{44}$ (GPa)	58.26	58.6	58.20
$C_{66}$ (GPa)	40.17	40.25	39.88
$C_{12}$ (GPa)	6.76	8.1	7.04
$C_{13}$ (GPa)	12.17	12.0	11.91
$C_{14}$ (GPa)	-18.07	-18.1	-18.04

26 May 1999



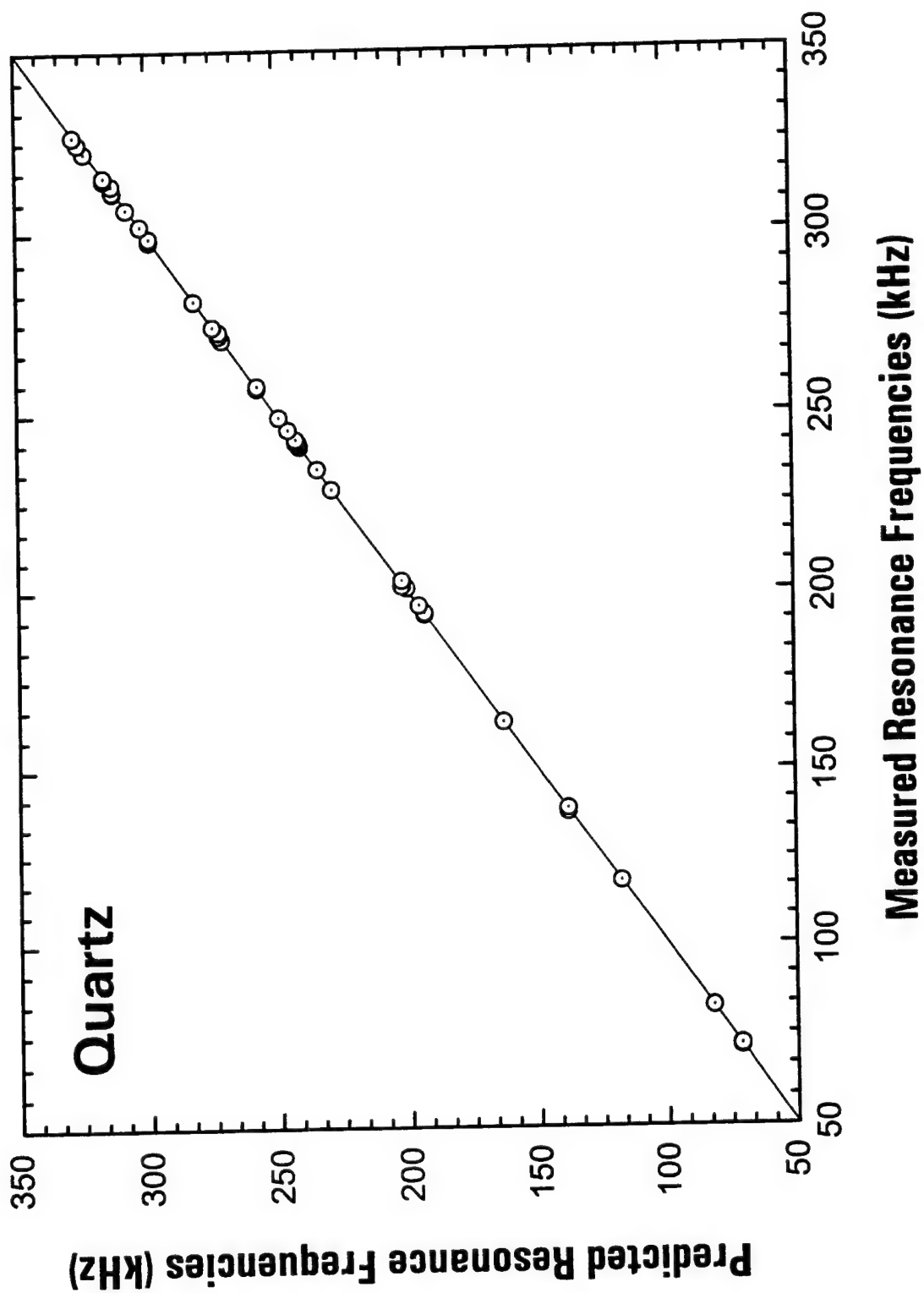
# Quartz and Langasite elastic-stiffness coefficients

	Quartz		Langasite	
	$C_{ij}$ (GPa)	$Q_{ij}^{-1}(10^{-3})$	$C_{ij}$ (GPa)	$Q_{ij}^{-1}(10^{-3})$
$\rho$ (g/cm <sup>3</sup> )	2.650		5.744	
$C_{11}$	87.10	-0.09	186.1	0.18
$C_{33}$	106.1	0.66	259.8	0.27
$C_{44}$	58.26	-0.07	53.87	0.07
$C_{66}$	40.17	-0.17	42.68	0.03
$C_{12}$	6.76		100.7	
$C_{13}$	12.17		93.50	
$C_{14}$	-18.07		14.08	
rms (%)	0.19		0.26	
.....				
$C_L$ (GPa)	100.7	0.11	202.4	0.22
$B$ (GPa)	37.80	0.41	133.0	0.31
$G$ (GPa)	47.18	-0.07	52.00	0.05
$E$ (GPa)	99.94	0.07	138.0	0.08
$\nu$	0.0593		0.3271	

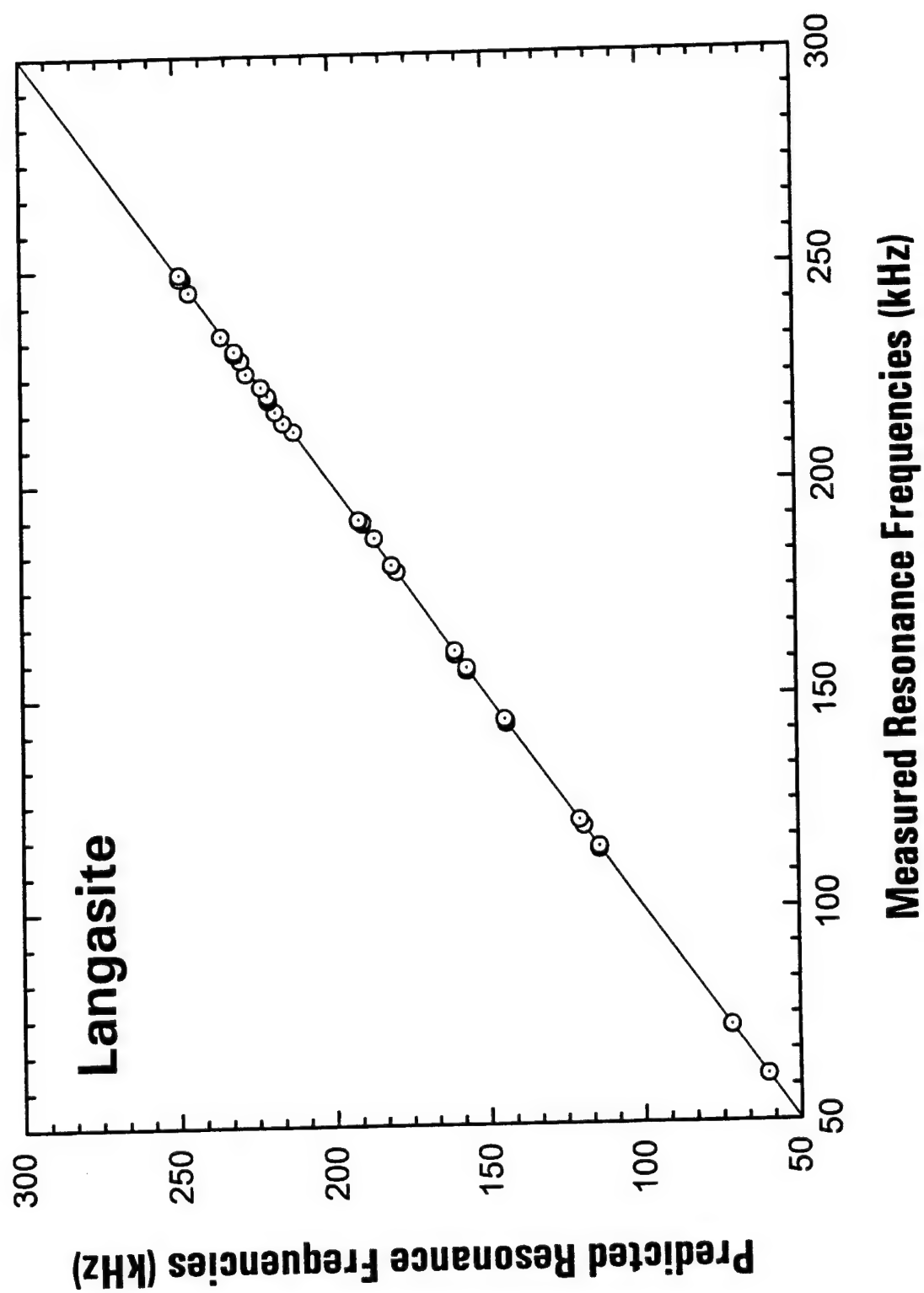
21 June 1999

$$\left| C_{ijkl} + \left[ \frac{e_{rij} q_r e_{skl} q_s}{K_{mn} q_m q_n} \right] - \rho v^2 \delta_{ijkl} \right| = 0$$

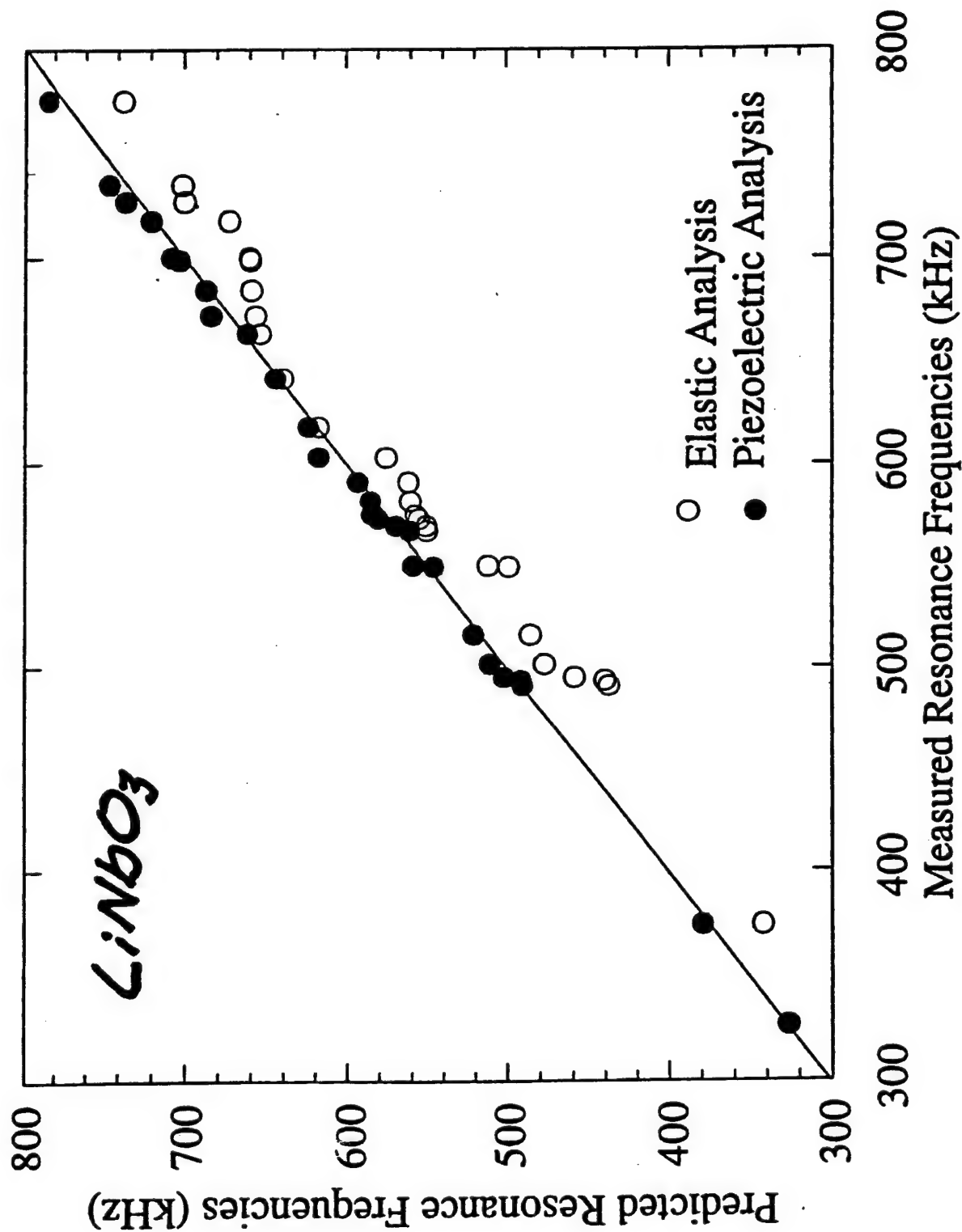
24 May 1999  
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24 May 1999  
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# Elastic vs. Piezoelectric Analyses



## Plans

1.  $Q_{ij}^{-1}$  both quartz and langasite.
2. Relationships between  $Q_{\text{mech}}$  and  $Q_{\text{elec}}$ .
3. From mechanical spectrum, determine  $e_{ijk}$ , especially for  $\text{LiNbO}_3$ .

**RUS STUDIES OF CRYPTO-CLATHRATES:  
PERFECT CRYSTALS WITH THE ELASTIC  
PROPERTIES OF GLASSES**

*Veerle Keppens*

*K. U. Leuven*

*and*

*NCPA, University of Mississippi*

## Collaborators

### ORNL:

B.C. Sales

D. Mandrus

B. C. Chakoumakos

### KUL:

C. Laermans

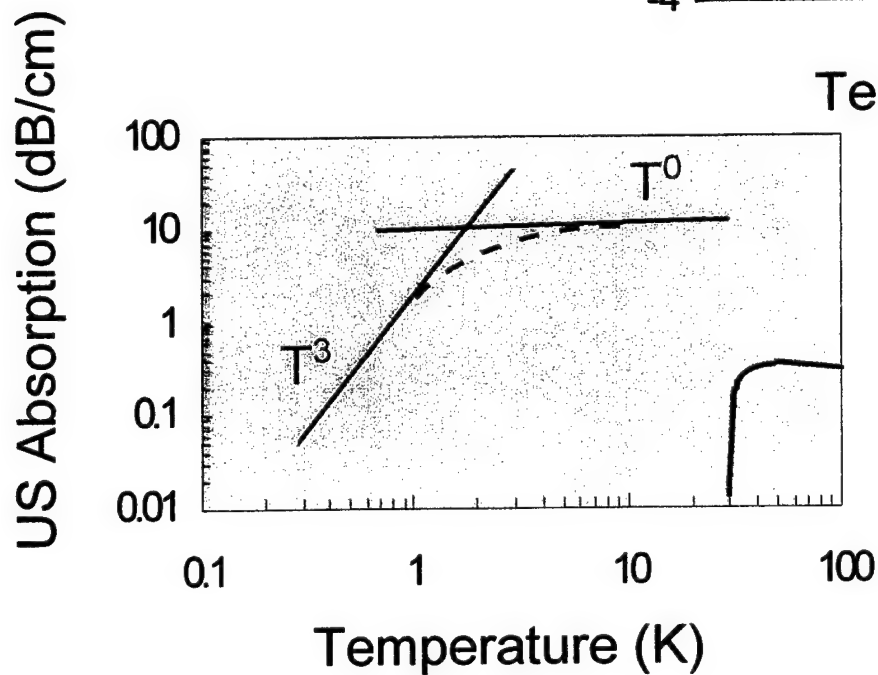
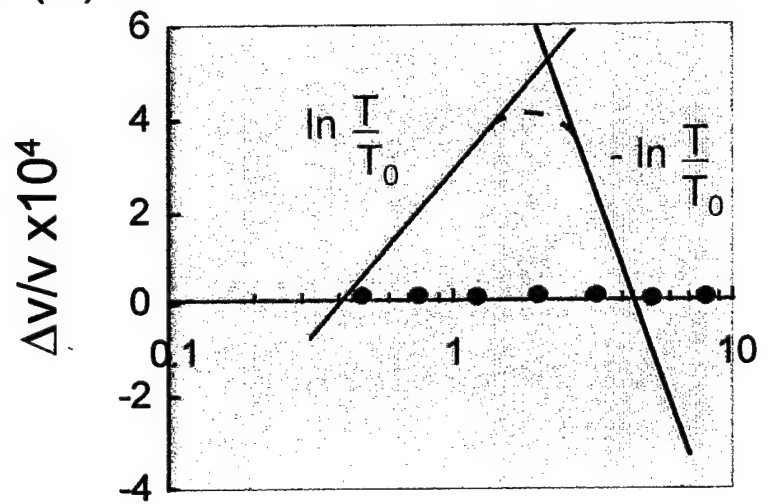
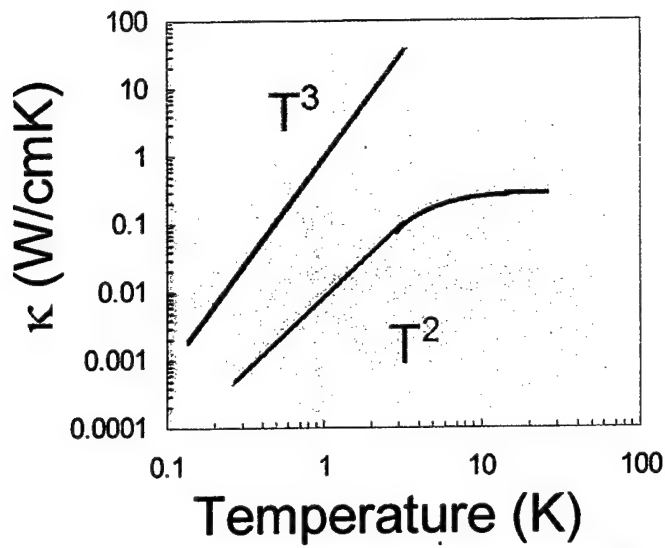
### LANL:

A. Migliori

T. Darling



# glasses vs. crystals



Filled skutterudites:  $\text{RM}_4\text{X}_{12}$

R=Sr, Ba, La, Ce, Pr, Nd,  
Eu, Gd, U, Th

M=Ru, Fe, Os

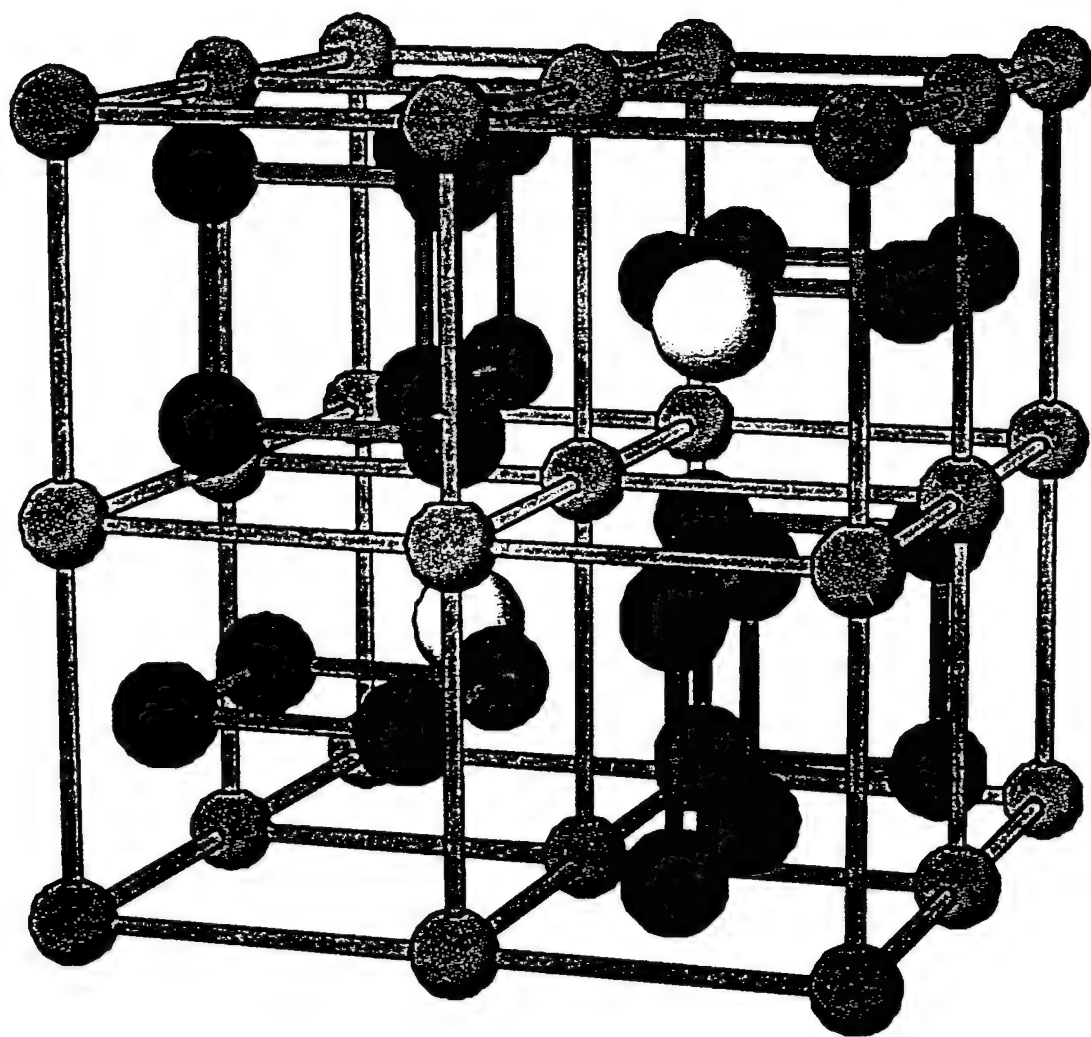
X=P, As, Sb

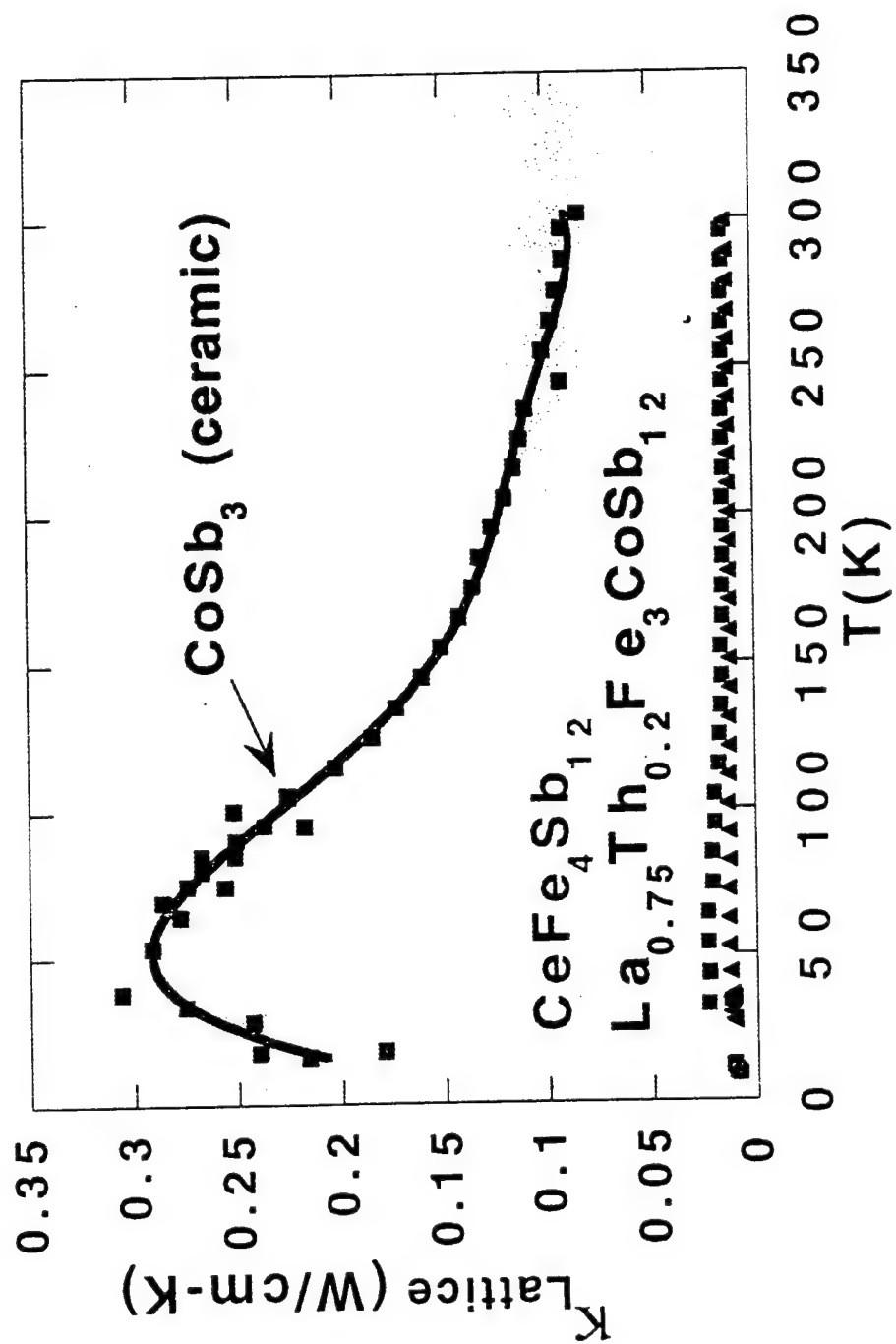
R-atoms “rattle” in atomic cage

→ reduce  $\kappa_{\text{lattice}}$

excellent example of “electron-crystal  
phonon glass”

New class of thermoelectric  
materials

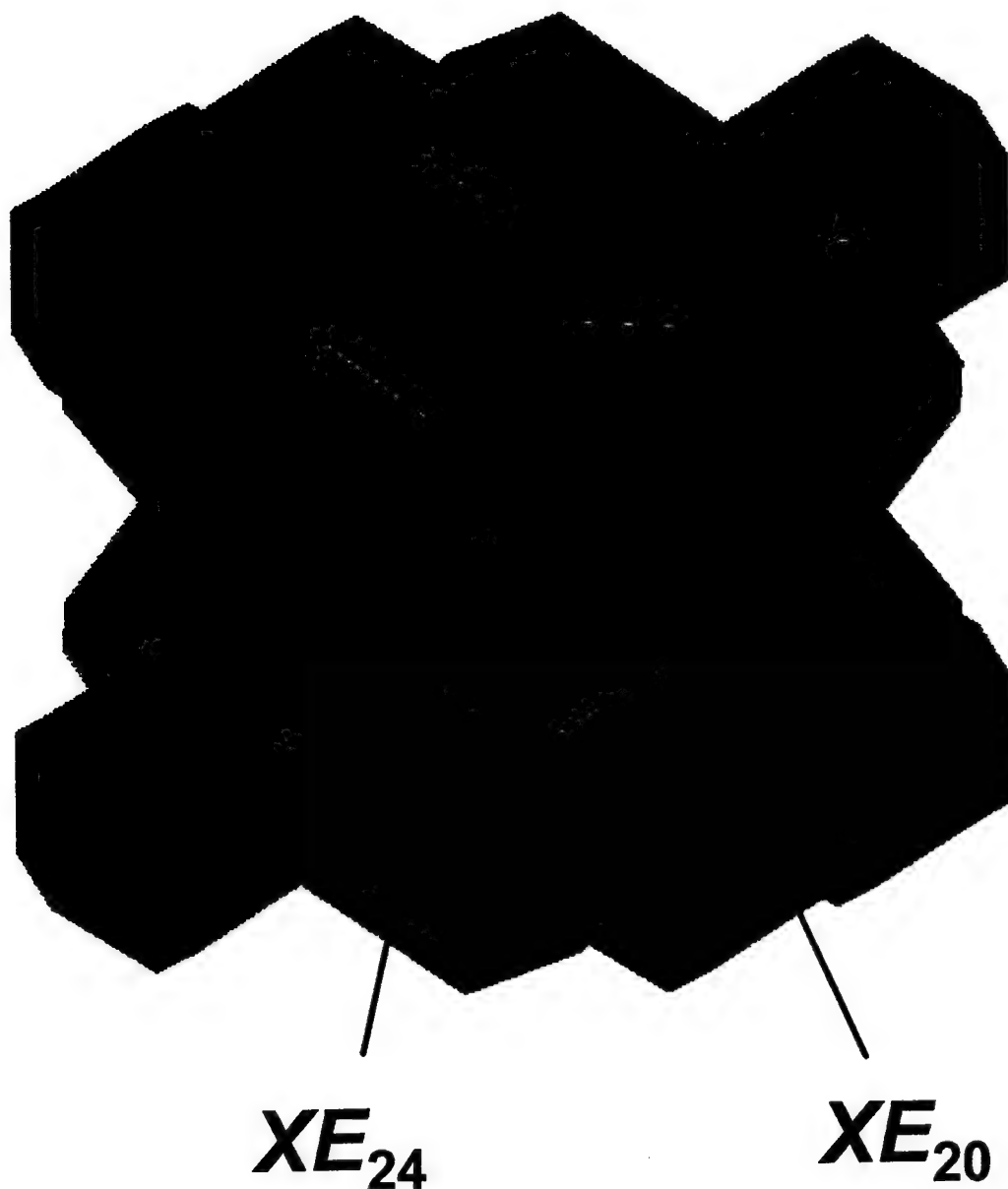




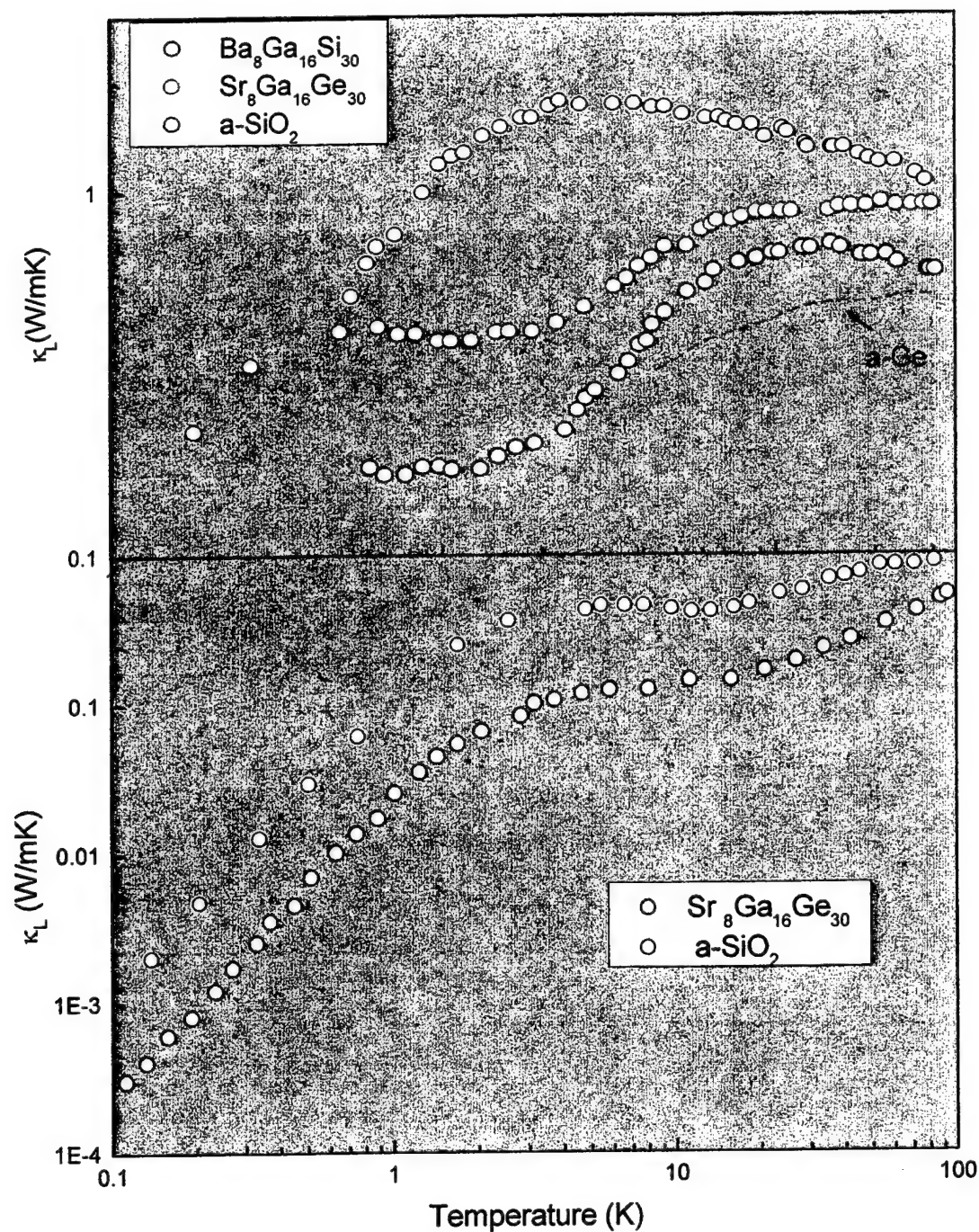
*Clathrates:  $X_8E_{46}$*

**$X = \text{Na, K, Rb, Cs, Sr, Ba}$**

**$E = \text{Si, Ge, Sn, Al, Ga}$**



J. L. Cohn, G. S. Nolas, V. Fessatidis,  
T. H. Metcalf, and G.S. Slack,  
PRL 82, 779 (1999)



# ELASTIC PROPERTIES OF CRYPTO-CLATHRATES:

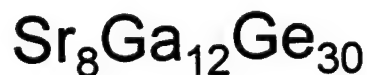
RUS on filled and unfilled skutterudite  
antimonides:



polycrystalline  $\longrightarrow$   $c_{11}$  and  $c_{44}$

T: 5 - 300 K

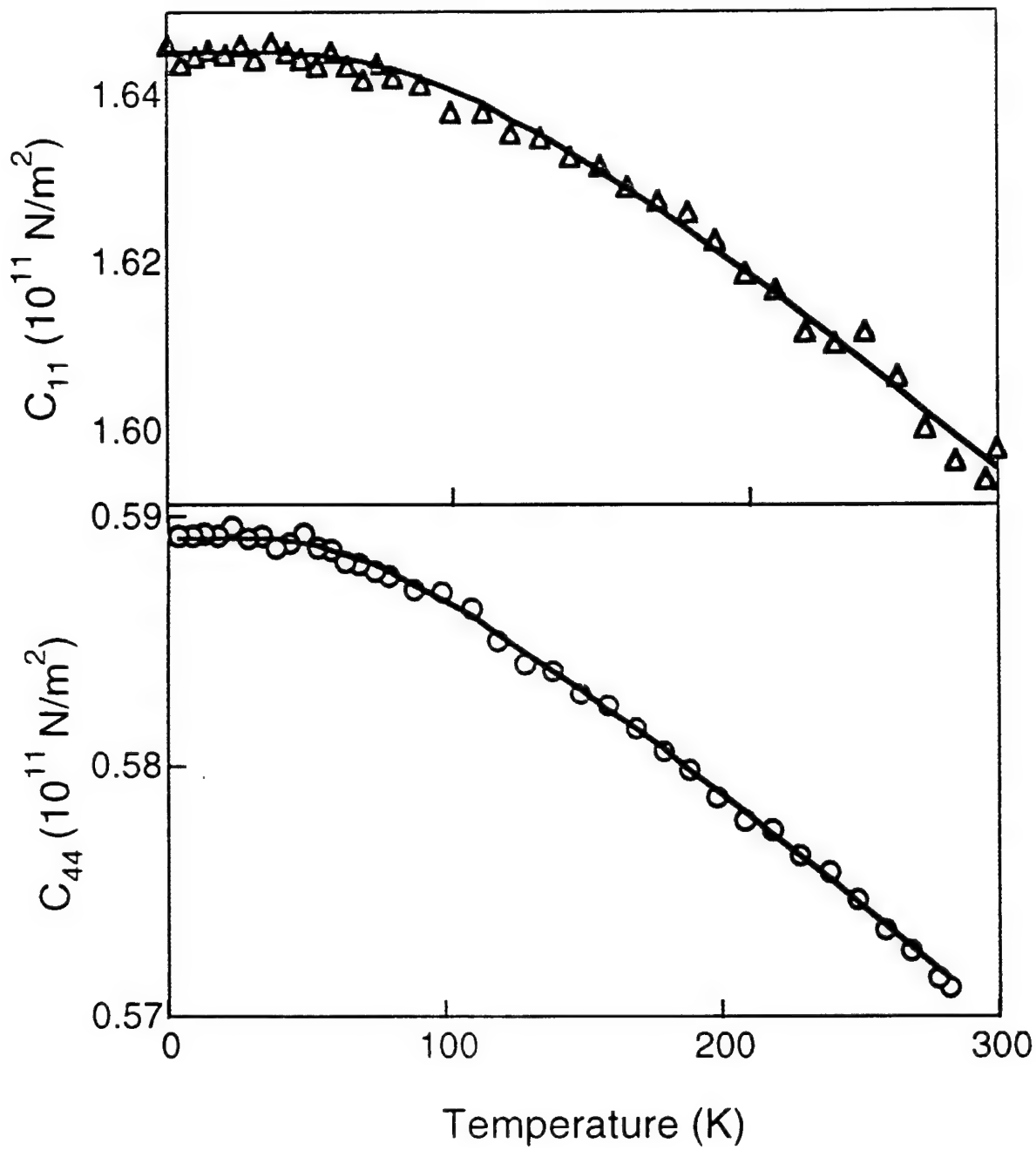
US absorption (pulse-echo) on Ge-clathrates:



single crystal

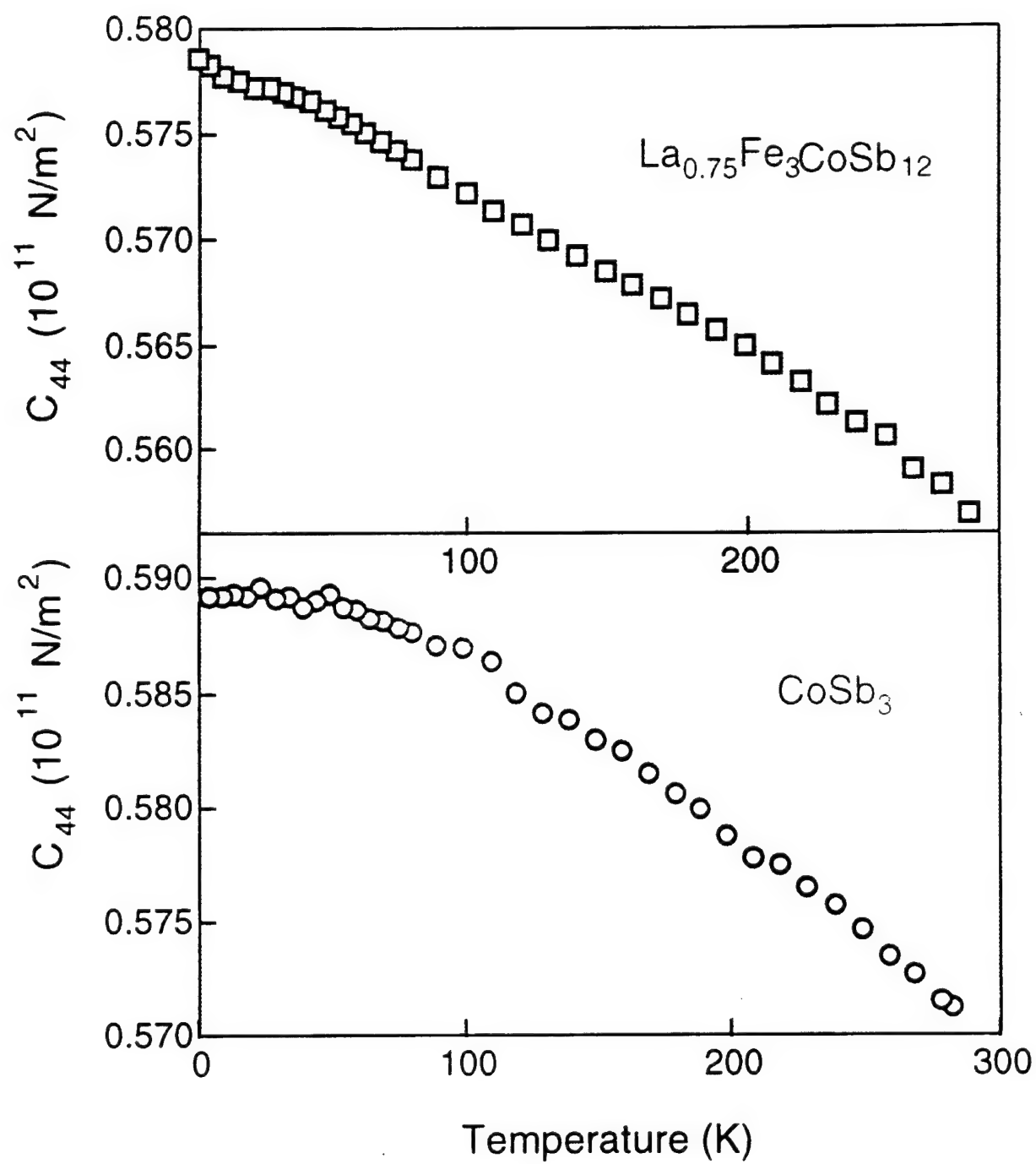
T: 0.4 - 30 K

# CoSb<sub>3</sub>



Varshni-behavior:  $c_{ij} = c_{ij}^0 - s / (e^{(t/T)} - 1)$





elastic constants for two-level system

$$c = \partial^2 F / d\epsilon^2$$

$$F = -N_A k_B T \ln(1 + e^{-\Delta/T})$$

$$\Delta = \Delta_0 + A\epsilon$$

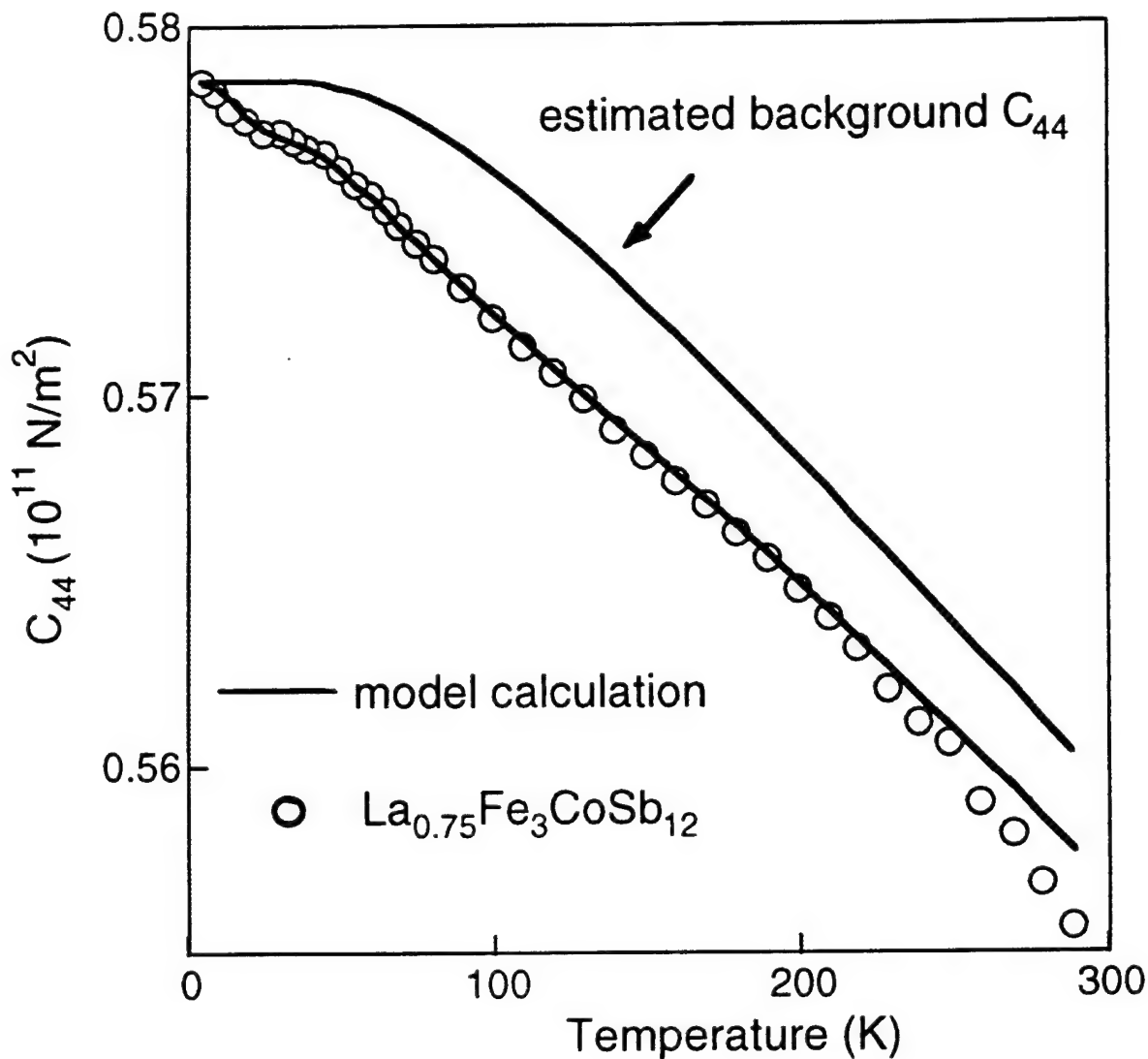
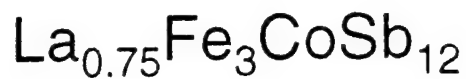
$\text{La}_{0.75}\text{Fe}_3\text{CoSb}_{12}$ :

$$c = c(\text{background})$$

$$+ c(\text{TLS \#1}, \Delta_{01} = 50 \text{ K})$$

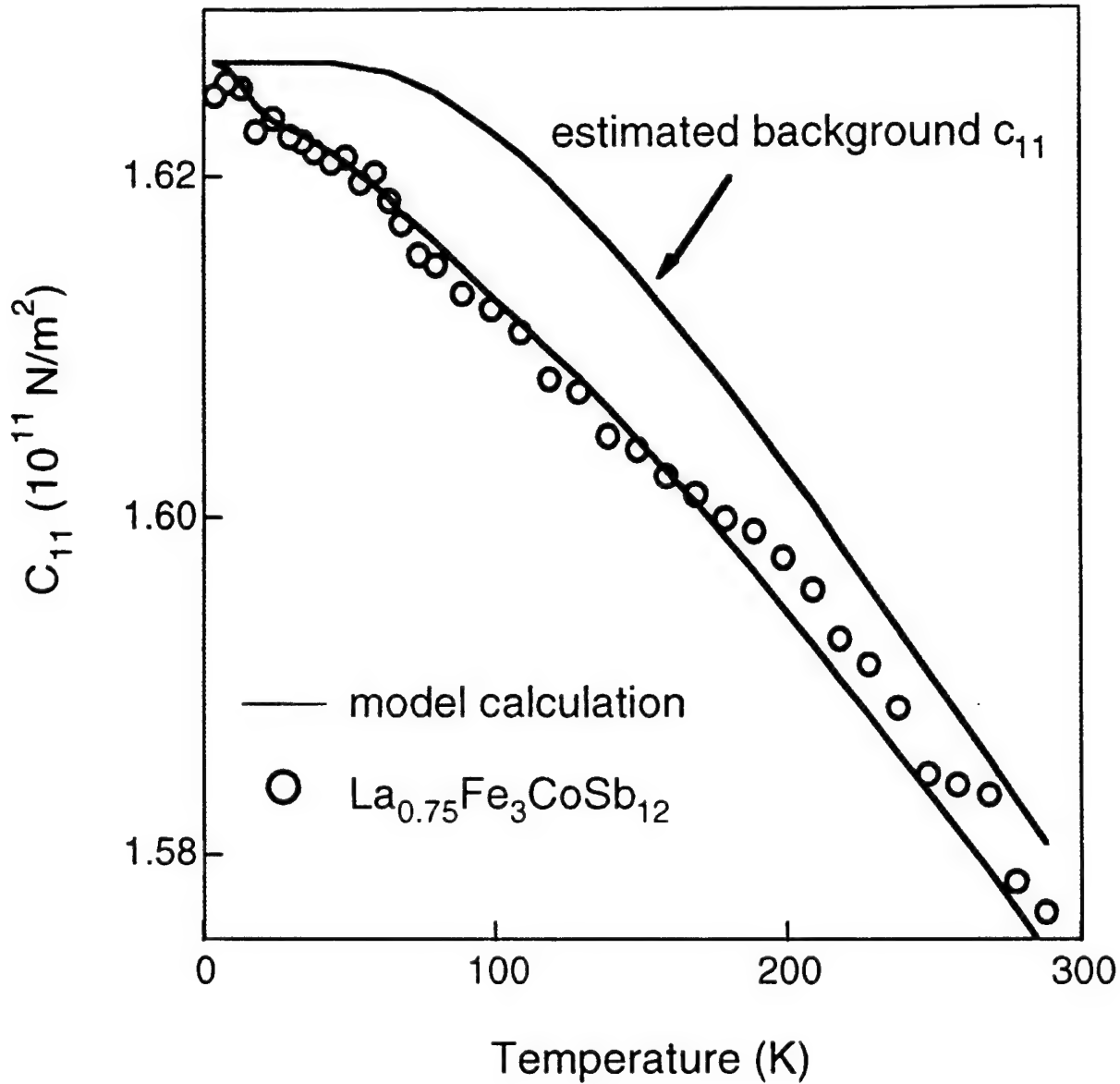
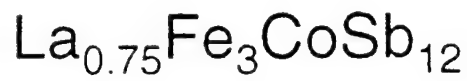
$$+ c(\text{TLS \#2}, \Delta_{02} = 200 \text{ K})$$

$c(\text{background}) = \text{Varshni-fit for unfilled sample}$

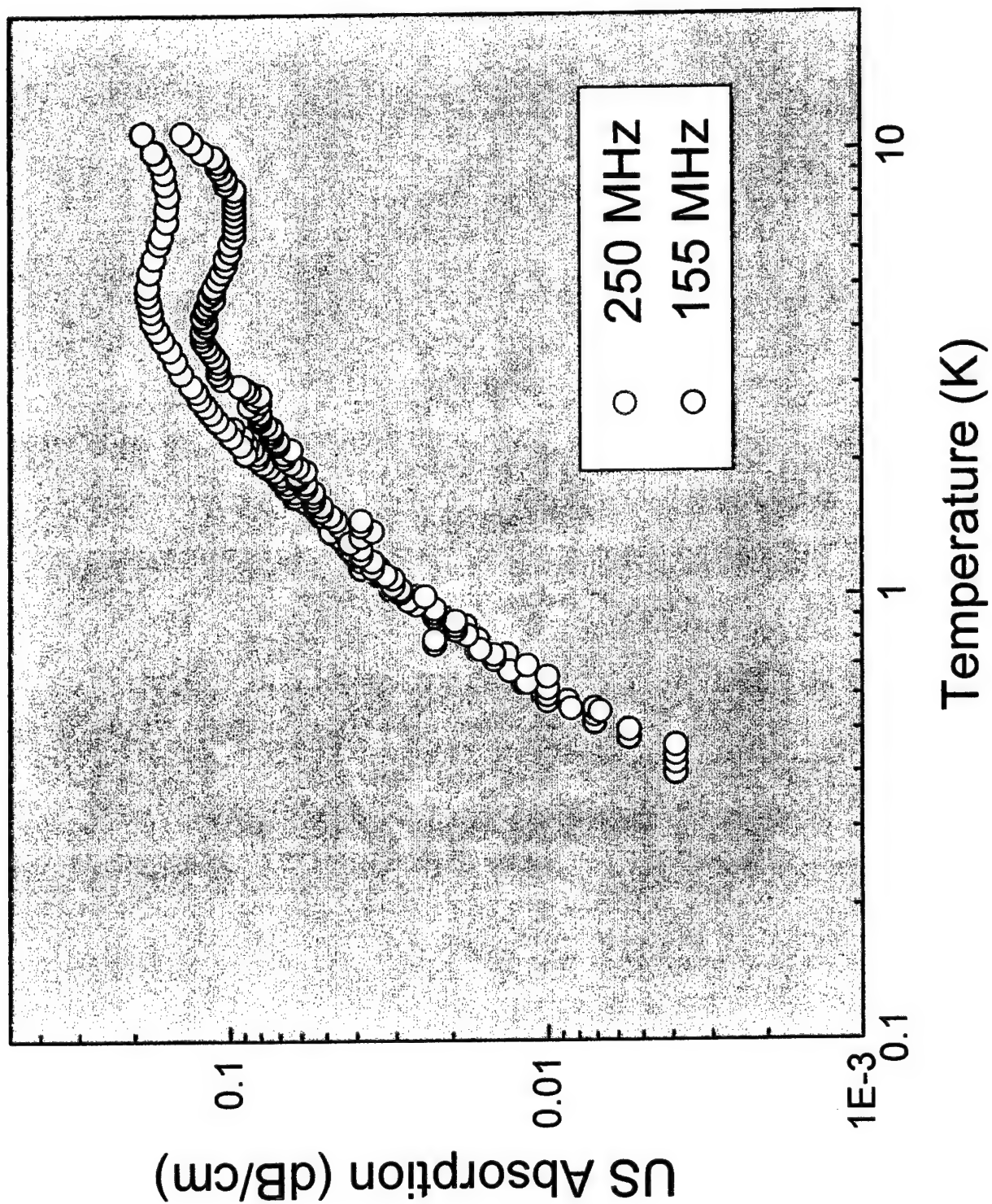
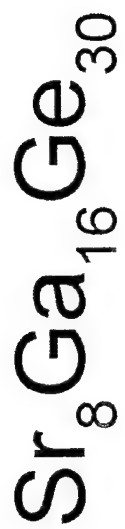


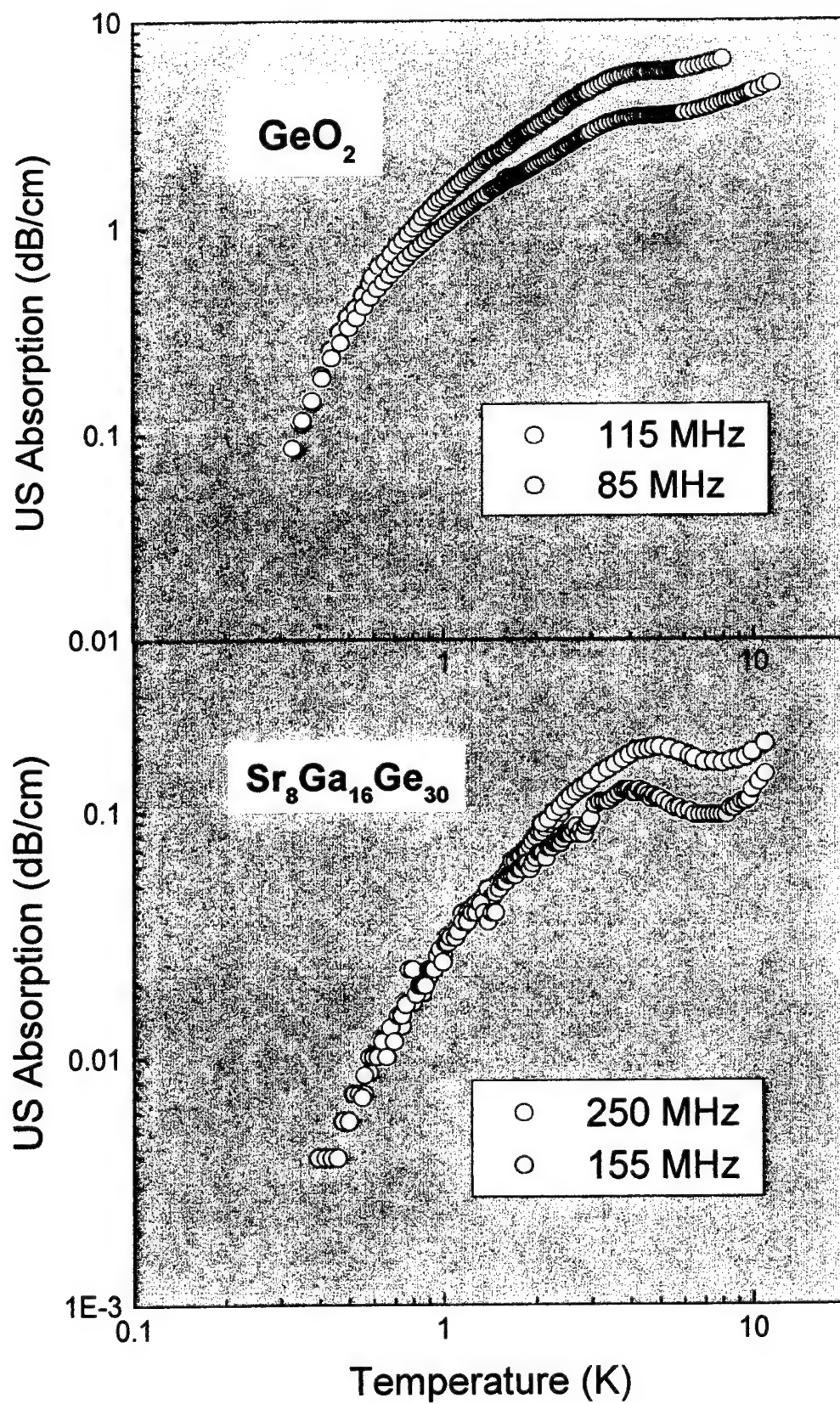
$$c_{44}(\text{model}) = c_{44}(\text{background}) + c_{44}(\text{TLS \#1}, \Delta_{01} = 50 \text{ K}) + c_{44}(\text{TLS \#2}, \Delta_{02} = 200 \text{ K})$$

Keppens et al., Nature 395, 876 (1998)

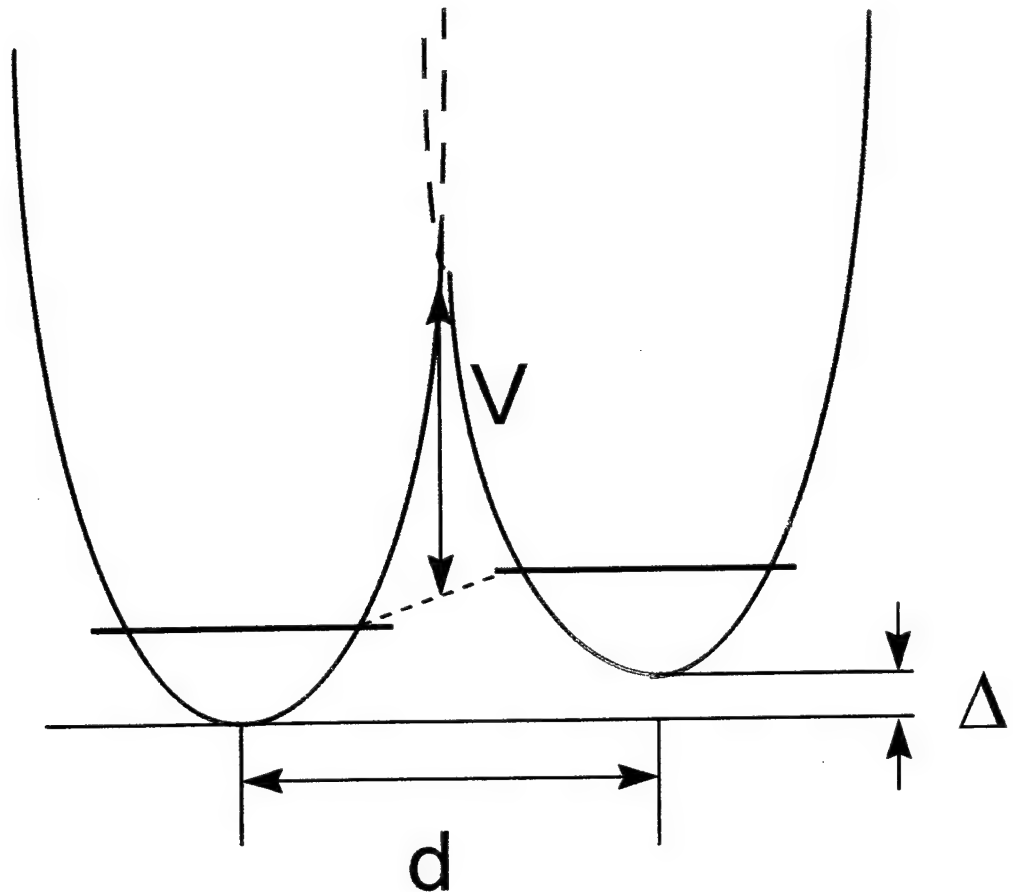


$$\begin{aligned}
 c_{11}(\text{model}) = & c_{11}(\text{background}) \\
 & + c_{11}(\text{TLS \#1}, \Delta_{01} = 50 \text{ K}) \\
 & + c_{11}(\text{TLS \#2}, \Delta_{02} = 200 \text{ K})
 \end{aligned}$$





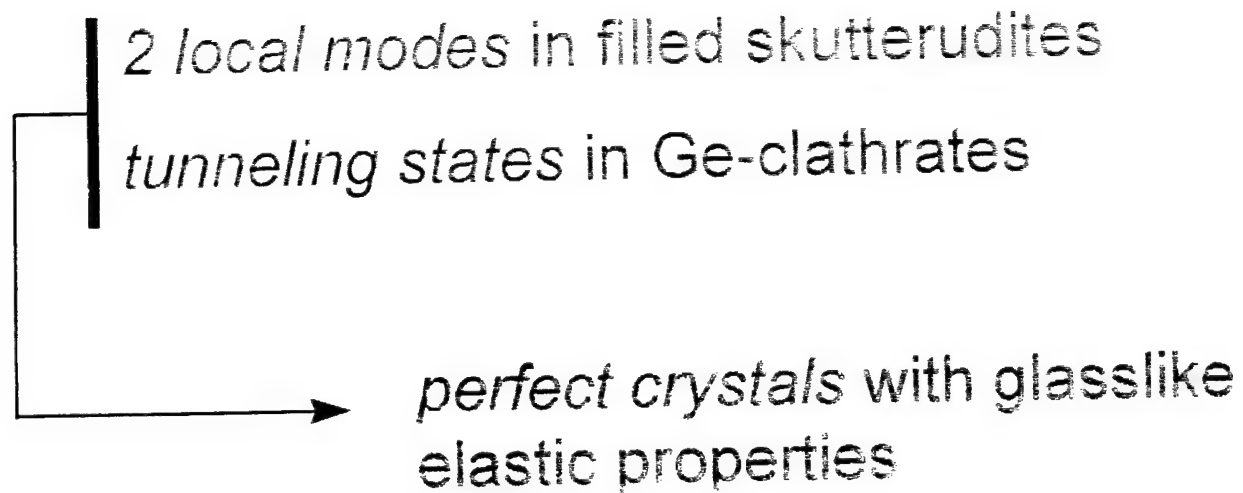
## Tunneling states



$\Delta$ : asymmetry  
 $\Delta_0$ : energy-overlap

→ broad and uniformly distributed

## CONCLUSIONS - FUTURE WORK



### future work:

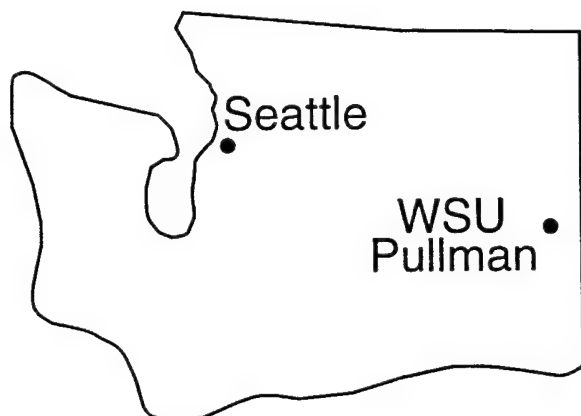
RUS study of Ge-clathrate

US attenuation of skutterudites



Comparison of radiation and scattering mechanisms for objects having Rayleigh wave velocities greater than or smaller than the speed of sound in the surrounding water

Philip L. Marston  
Florian J. Blonigen  
Brian T. Hefner  
Karen Gipson\*  
Scot F. Morse\*\*  
Dept. of Physics  
Washington State University  
Pullman, WA 99164-2814

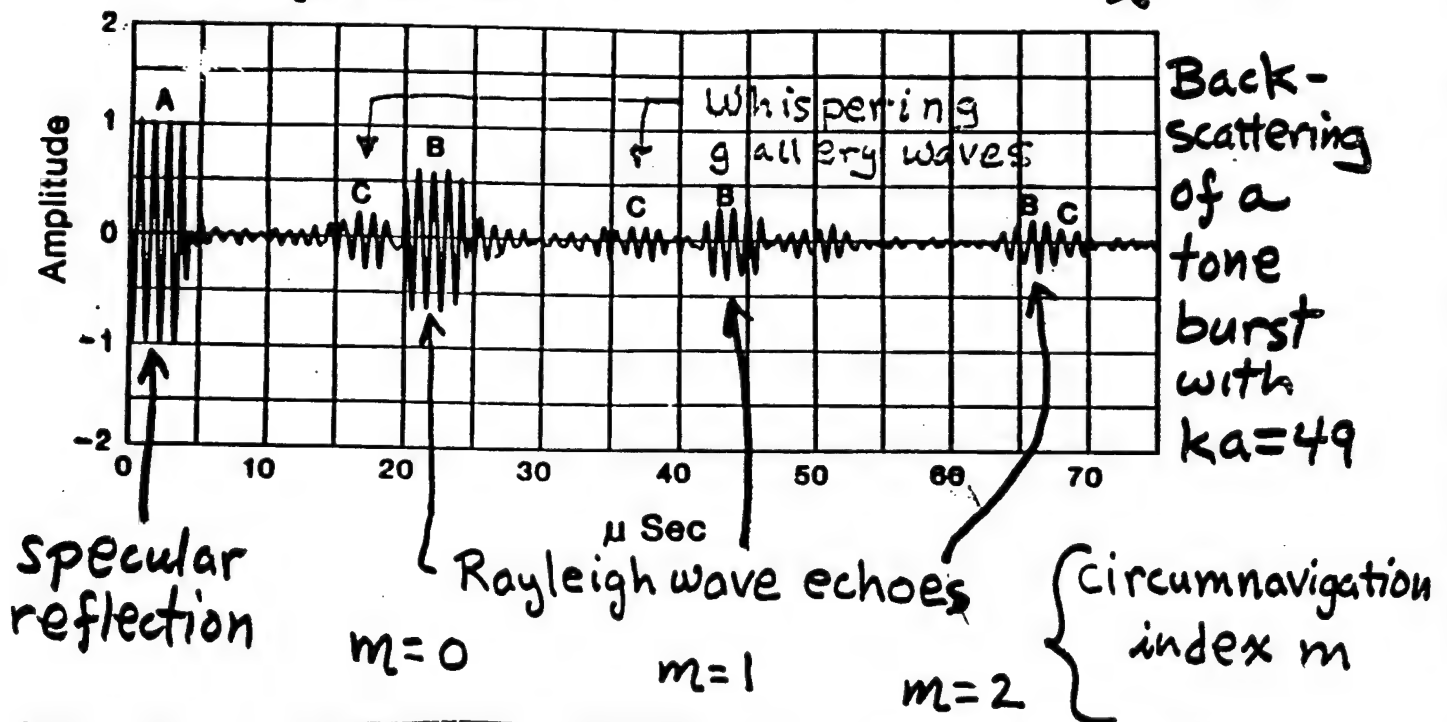
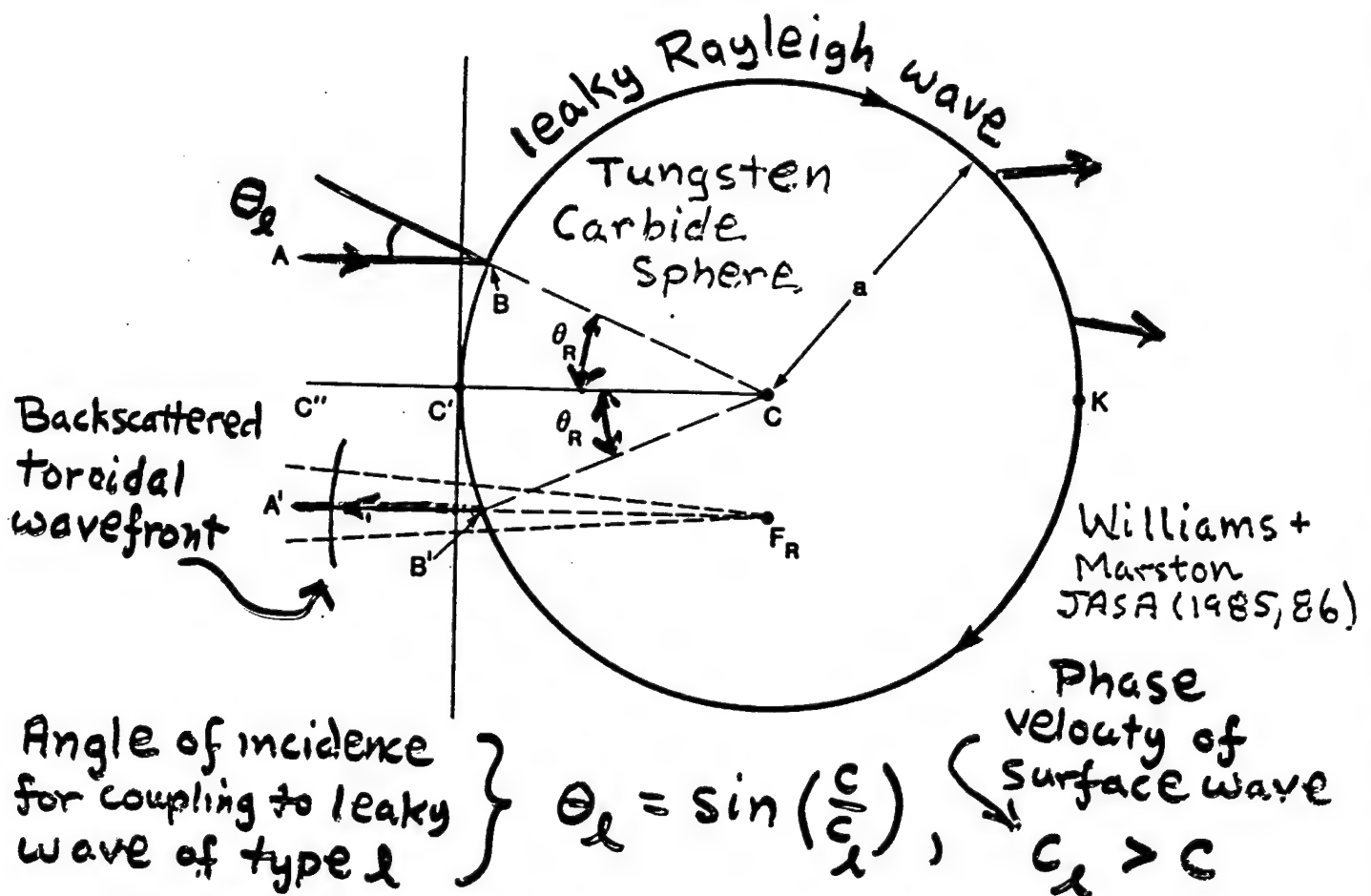


Supported by the Office of Naval Research.

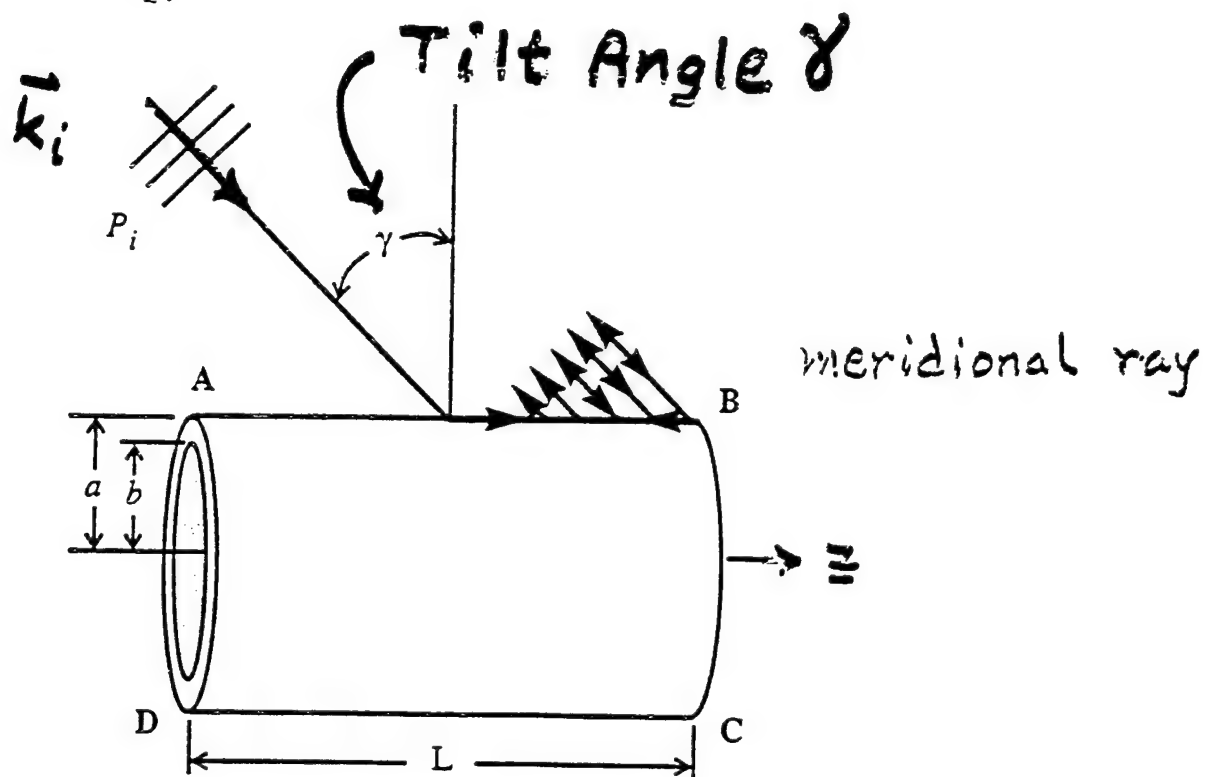
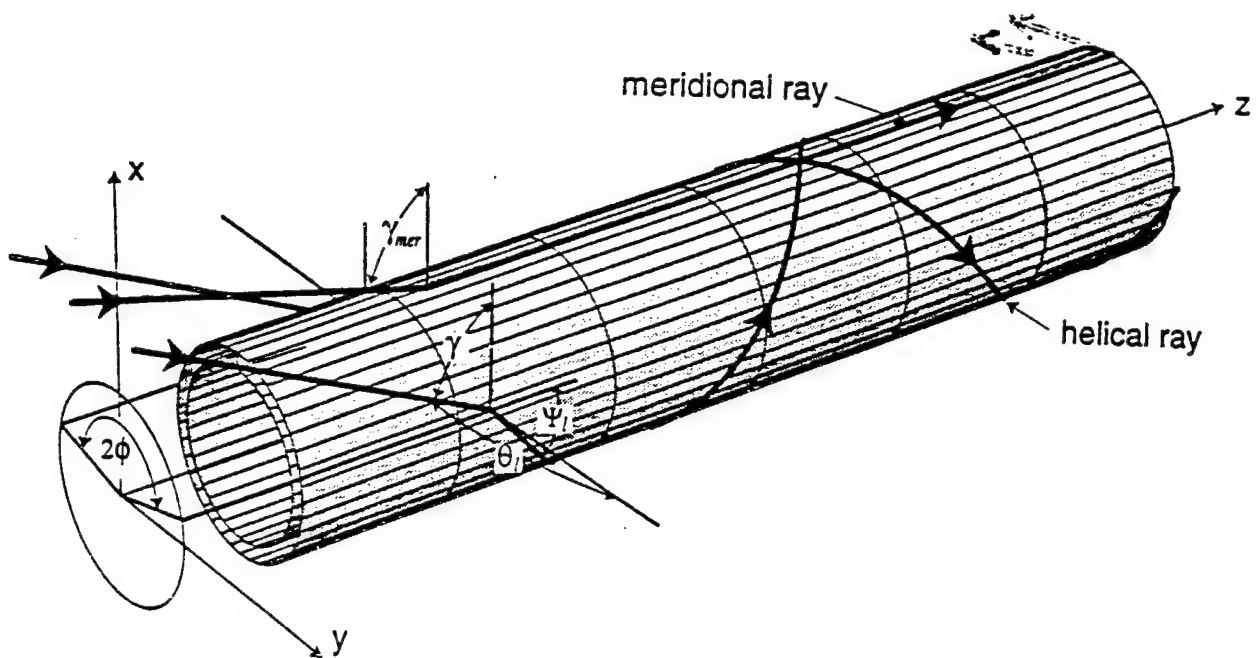
\*Physics Department, University of Puget Sound, 1500 North Warner, Tacoma, WA 98416

\*\*Physical Acoustics Branch, Code 7136, Naval Research Laboratory, 4555 Overlook Ave., SW, Washington, DC 20375

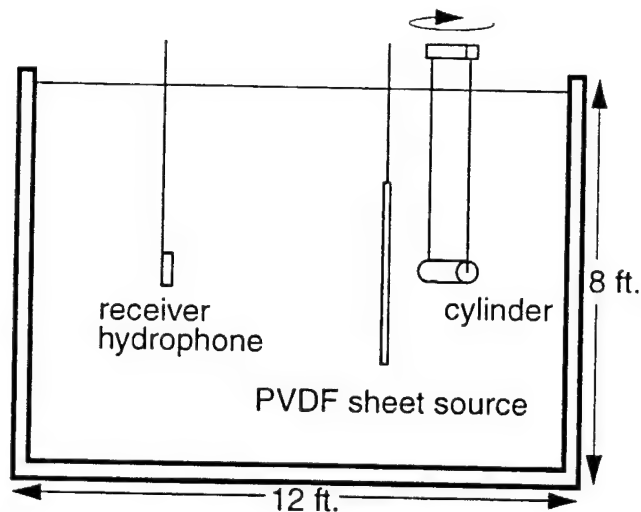
- Fluid loading mechanisms for metallic objects in water: review and recent progress (Morse, Gipson, Blonigen).
- Reflection from plastics, fluid loading, and the subsonic Rayleigh wave.
- Novel backscattering enhancements for plastic (and rubber) objects in water (Blonigen, Hefner).



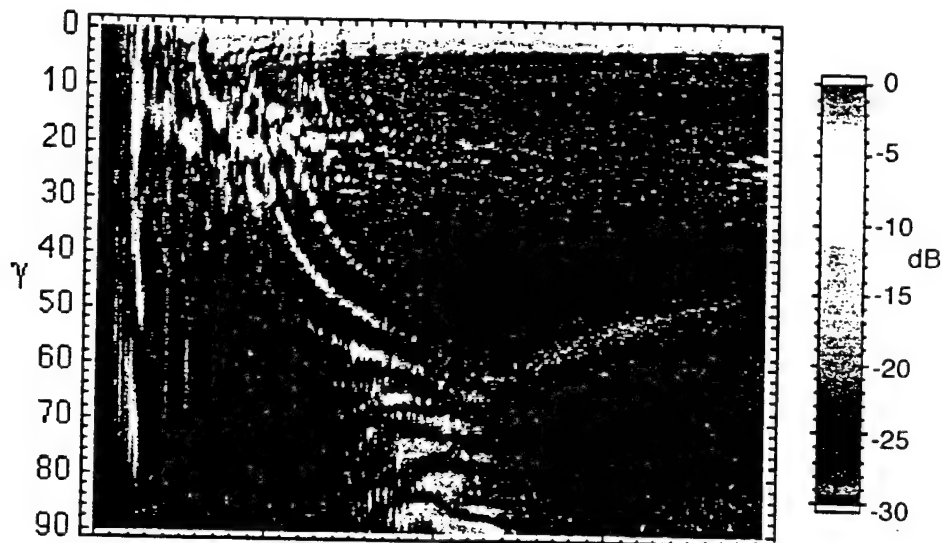
$$ka = \left( \frac{2\pi}{\lambda_{\text{in water}}} \right) (\text{sphere radius}) = \left( \frac{\omega_{\text{burst}}}{c} \right) a$$



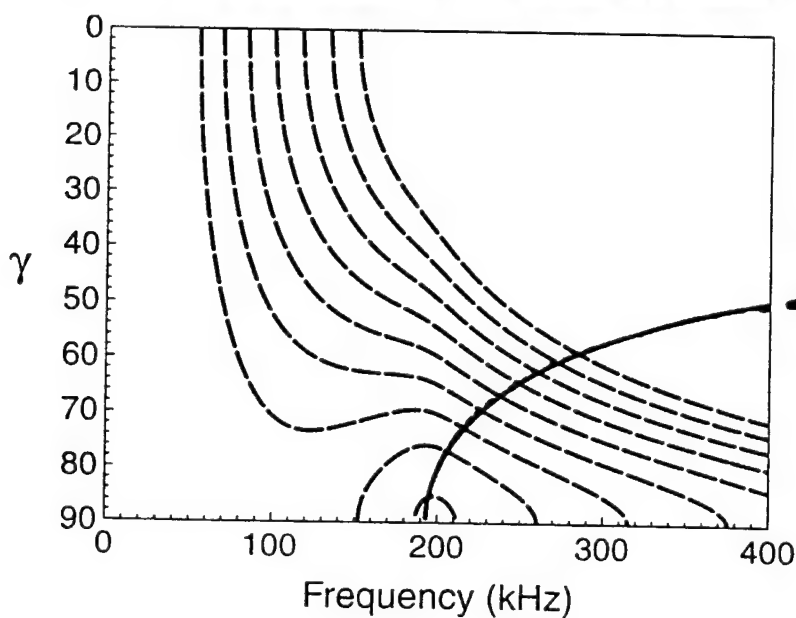
Mar. E,  
Marston  
& Kaduchuk  
JASA (1996)



(a)



(b)



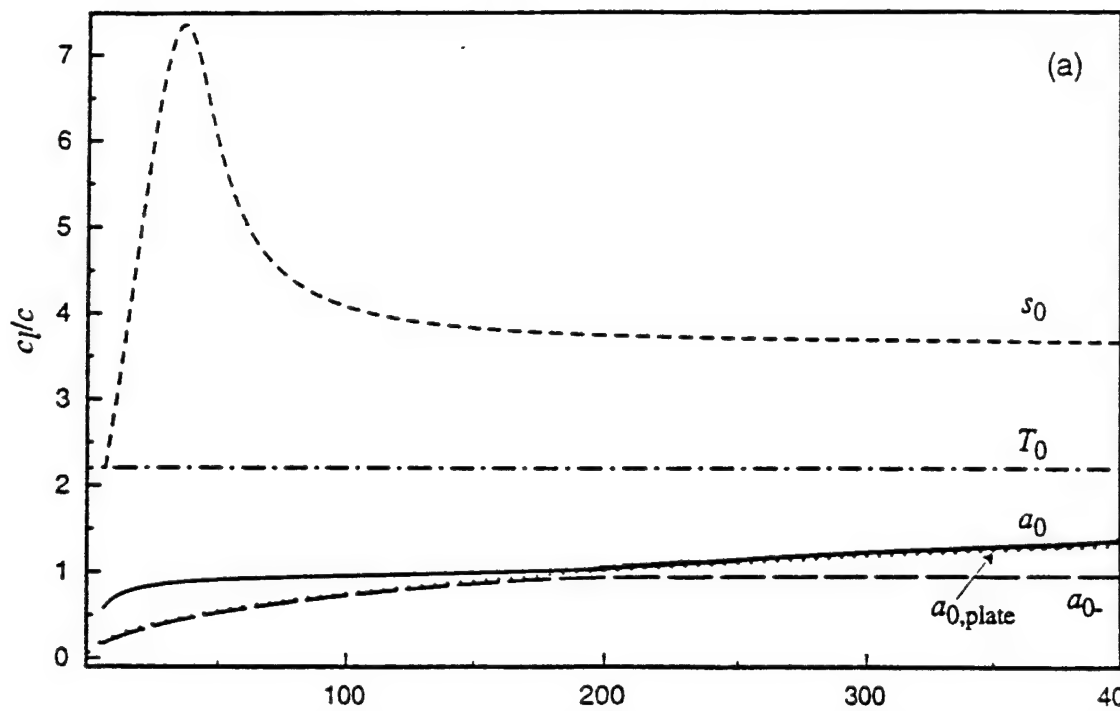
Coupling  
Condition for  
meridional leaky Lamb wave  
(c)

TR-4

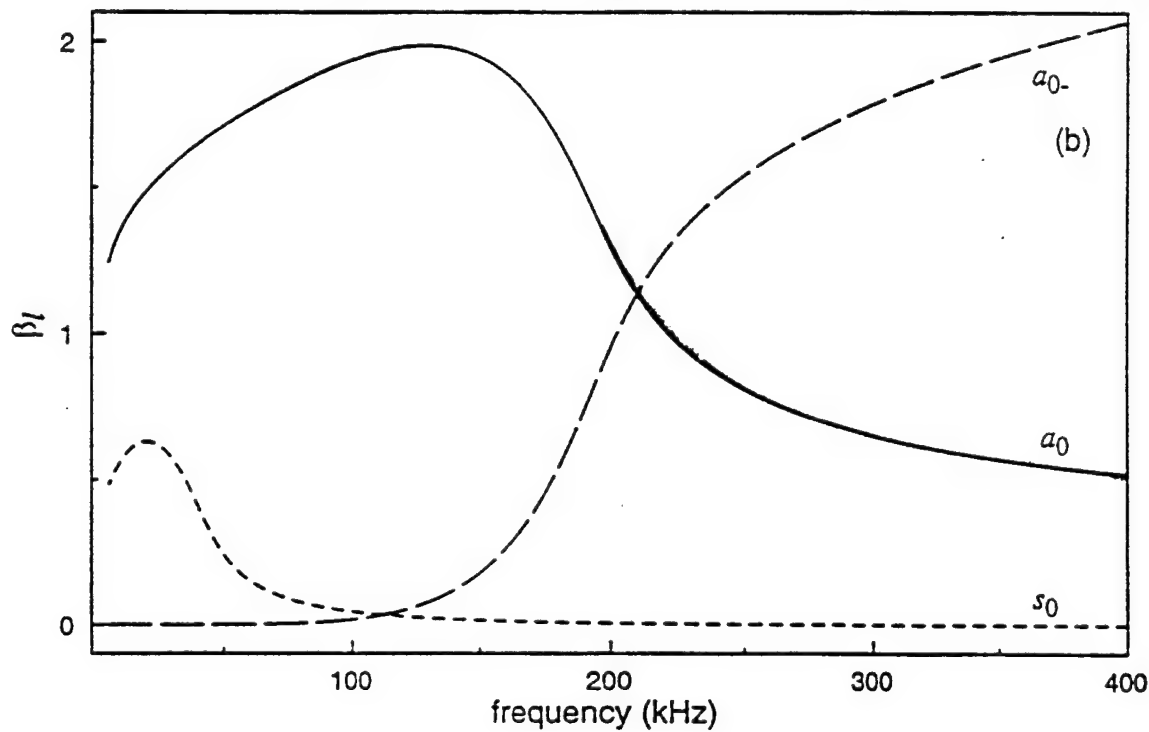
$h$  = shell thickness

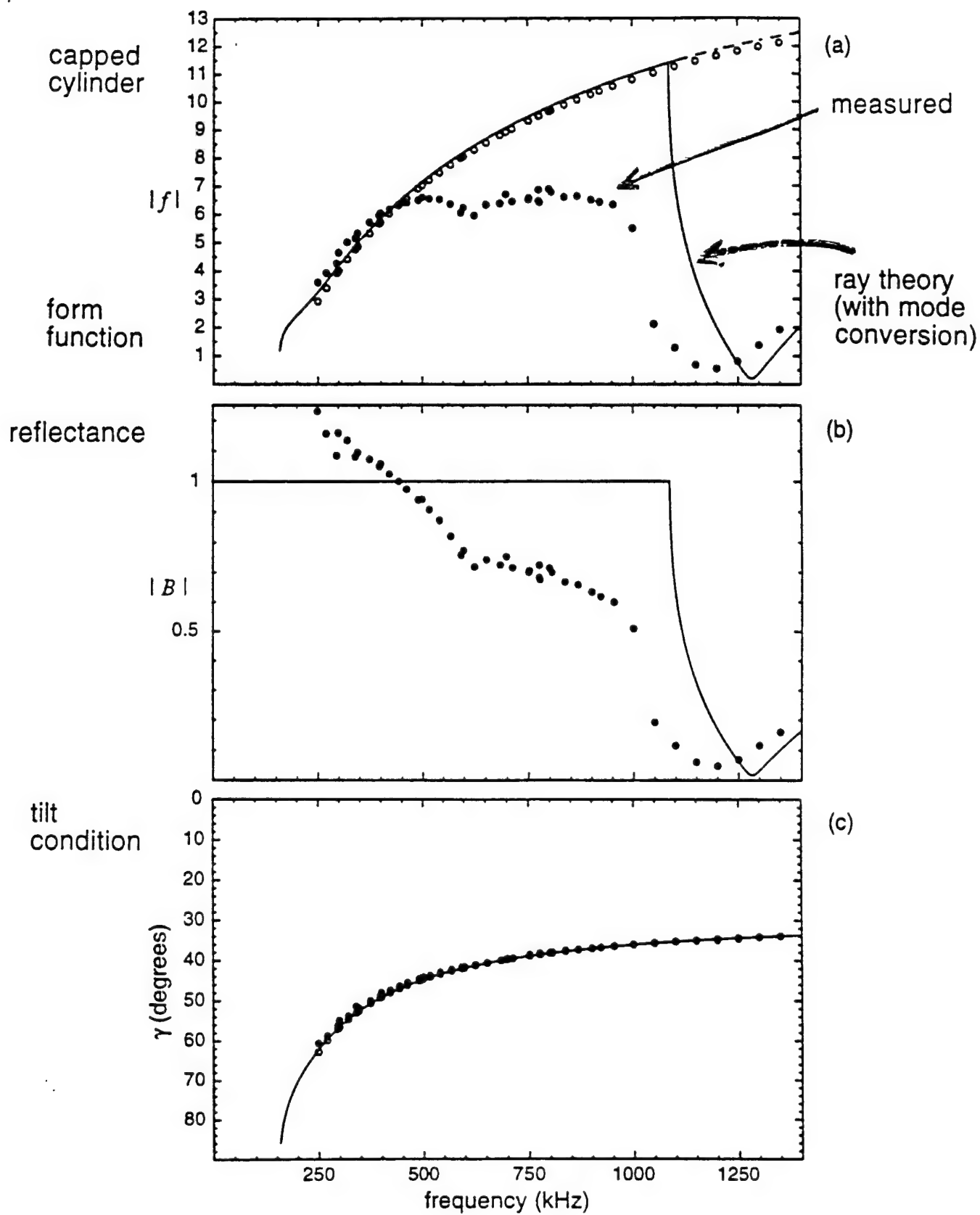
$a$  = radius

$h/a = 0.076$



attenuation  $\uparrow$



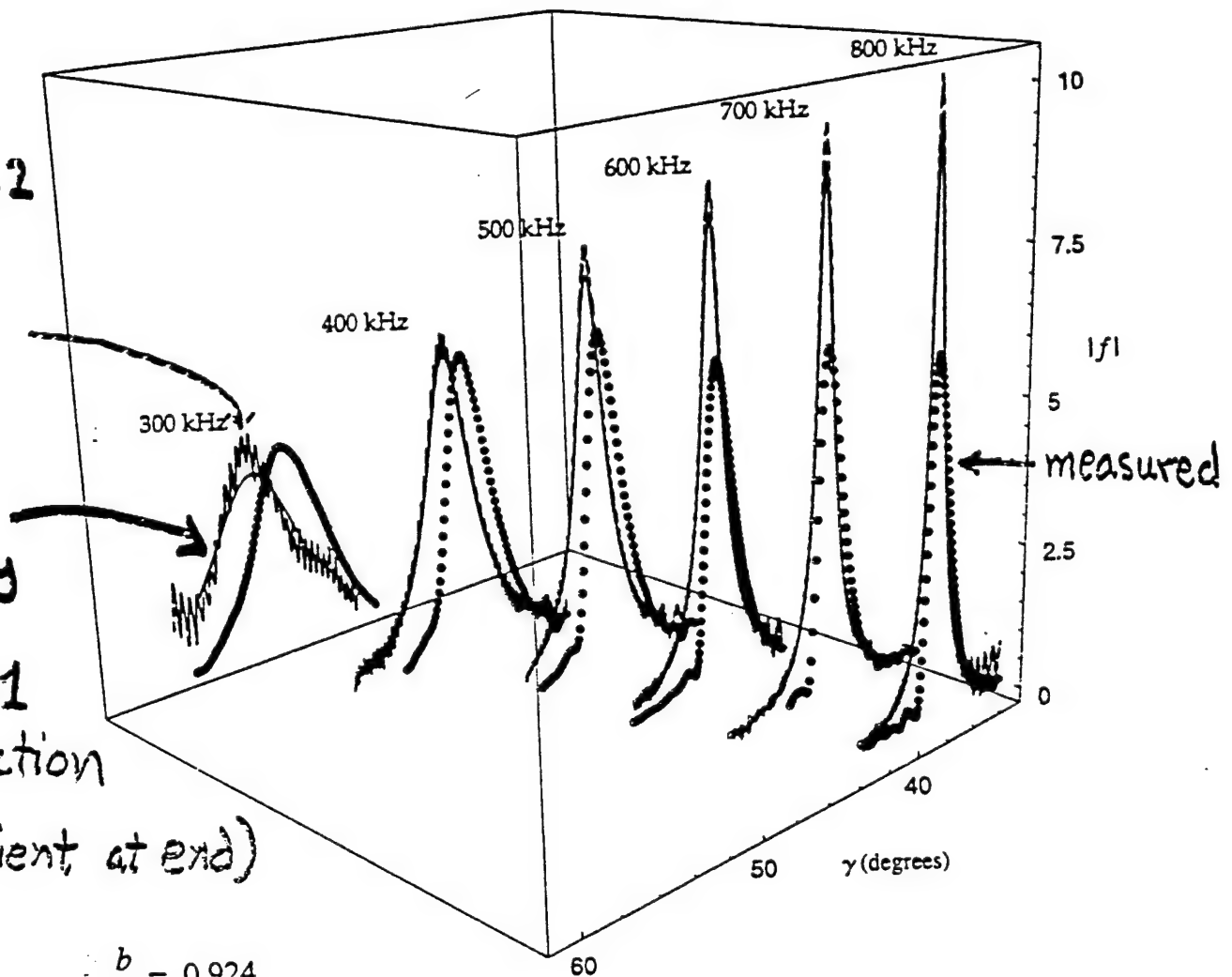


TR-6

$ka = 24.2$   
partial  
wave  
series

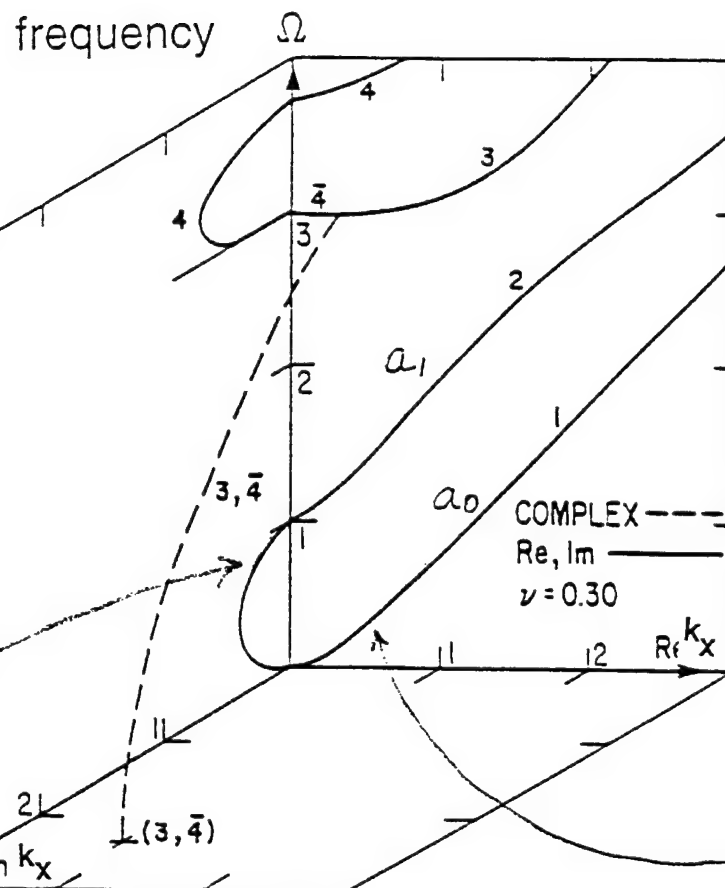
ray  
theory  
with  
 $|B|=1$   
(reflection  
coefficient at end)

$$\frac{b}{a} = 0.924$$



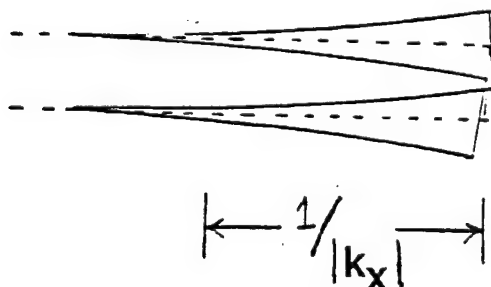
For a rigid cylinder, edge diffraction  
at the end gives  $|f| = O(ka)^{-1/2}$

# Mindlin (1959) Analysis of Plate Modes



Real, imaginary, and complex segments of branches of an isotropic plate for antisymmetric family of modes. (Source: Courtesy of Dr. P. C. Y. Lee.)

Subthreshold  $a_1$  "wave"

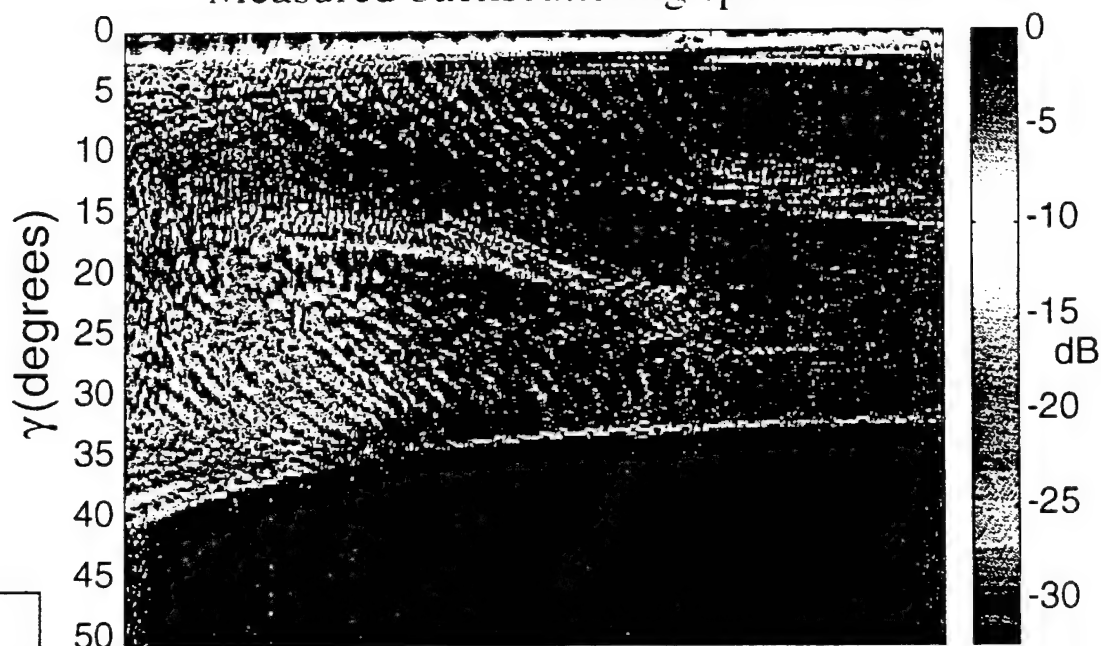


$a_0$  Lamb wave



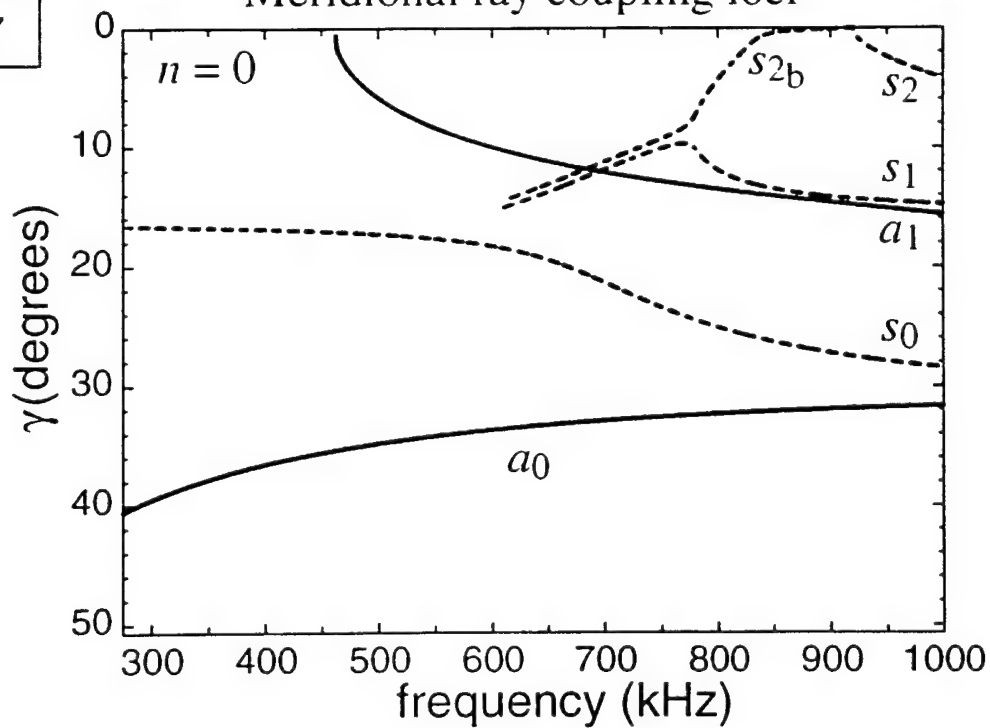


Measured backscattering spectrum



SS304  
 $b/a = 0.838$   
 $L/a = 11.7$

Meridional ray coupling loci



# Space-Time Approach to Non-Relativistic Quantum Mechanics

R. P. FEYNMAN

Cornell University, Ithaca, New York

(Rev. Mod. Phys. 1948)

$$\psi(x_{k+1}, t + \epsilon)$$

Path Integral  
Formulation  
of Q.M.

$$= \int \exp\left[\frac{i}{\hbar} S(x_{k+1}, x_k)\right] \psi(x_k, t) dx_k / A. \quad (18)$$

## 7. DISCUSSION OF THE WAVE EQUATION

### The Classical Limit

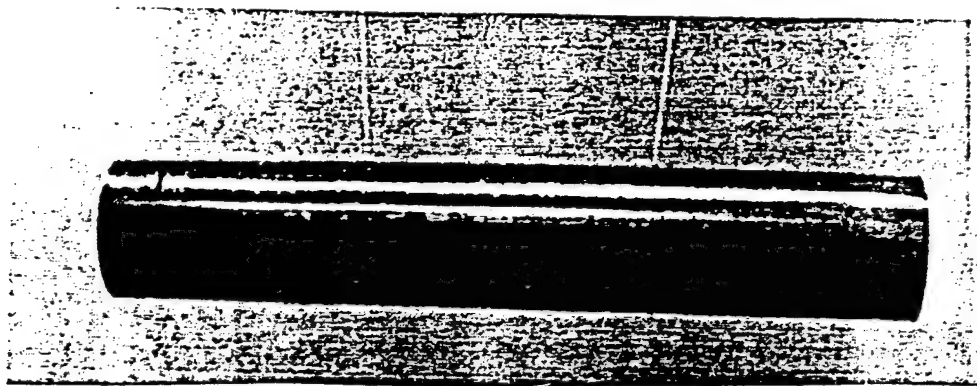
This completes the demonstration of the equivalence of the new and old formulations. We should like to include in this section a few remarks about the important equation (18).

This equation gives the development of the wave function during a small time interval. It is easily interpreted physically as the expression of Huygens' principle for matter waves.\* In geometrical optics the rays in an inhomogeneous medium satisfy Fermat's principle of least time. We may state Huygens' principle in wave optics in this way: If the amplitude of the wave is known on a given surface, the amplitude at a near by point can be considered as a sum of contributions from all points of the surface. Each contribution is delayed in phase by an amount proportional to the *time* it would take the light to get from the surface to the point along the ray of least *time* of geometrical optics. We can consider (22) in an analogous manner starting with Hamilton's first principle of least *action* for classical or "geometrical" mechanics. If the amplitude of the wave  $\psi$  is known on a given "surface," in particular the "surface" consisting of all  $x$  at time  $t$ , its value at a particular nearby point at time  $t + \epsilon$ , is a sum of contributions from all points of the surface at  $t$ . Each contribution is delayed in phase by an amount proportional to the *action* it would require to get from the surface to the point along the path of least *action* of classical mechanics.<sup>16</sup>

Fermat's  
principle

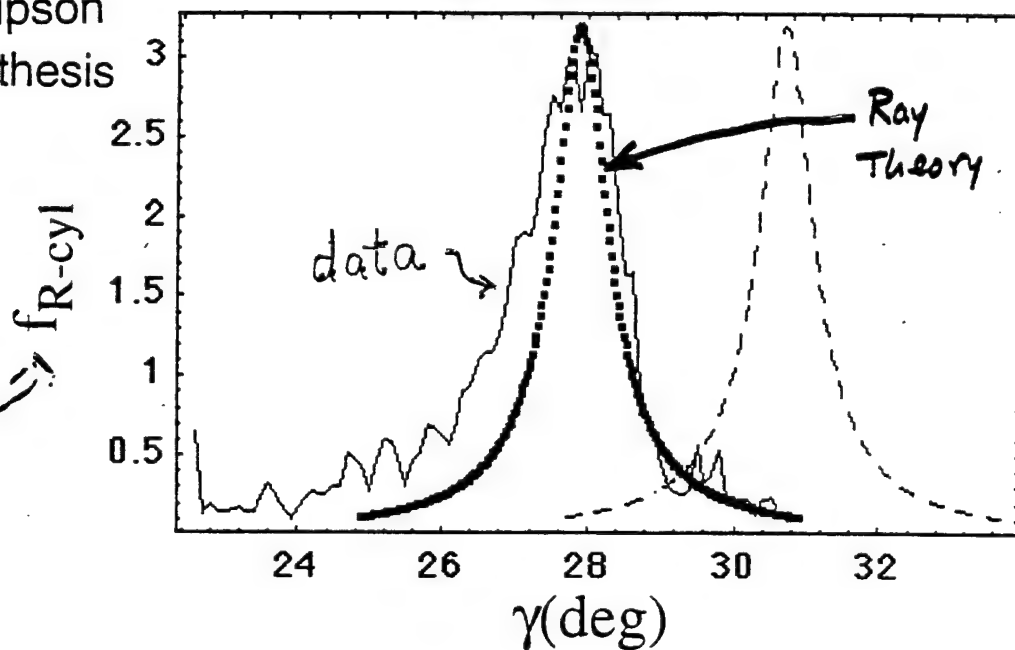
Huygens'  
principle

TR-10



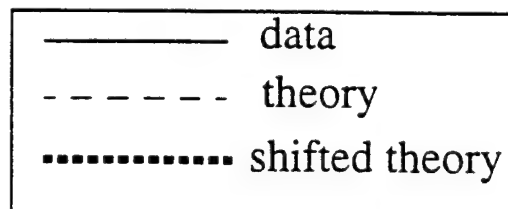
## Observed far-field backscattered form function

Karen Gipson  
1998 Ph.D. thesis



frequency = 1.03 MHz

$$P_{sca} = P_{inc} \frac{f_{cyl} a}{2r} e^{ikr}$$



frequency	data peak	theory peak	% discrepancy
620 kHz	2.60	3.45	25%
1.03 MHz	3.05	3.20	5%

## Fluid Loading Properties Relevant to Scattering:

	c (m/s)	$\rho$ (g/cc)
1. Water	1490	1.0
2. Sandy Sediment (fluid model)	1800	1.8

## Elastic Parameters (Solid or Shell "Target"):

	CR	CS	CL	$\rho$
3. Stainless Steel	2904	3141	5675	7
4. Solid Polymers:				
A. PMMA	1250	1340	2690	1.19
Polystyrene	1075	1150	2400	1.05
B. RTV Rubber	-	-	1080	1.02

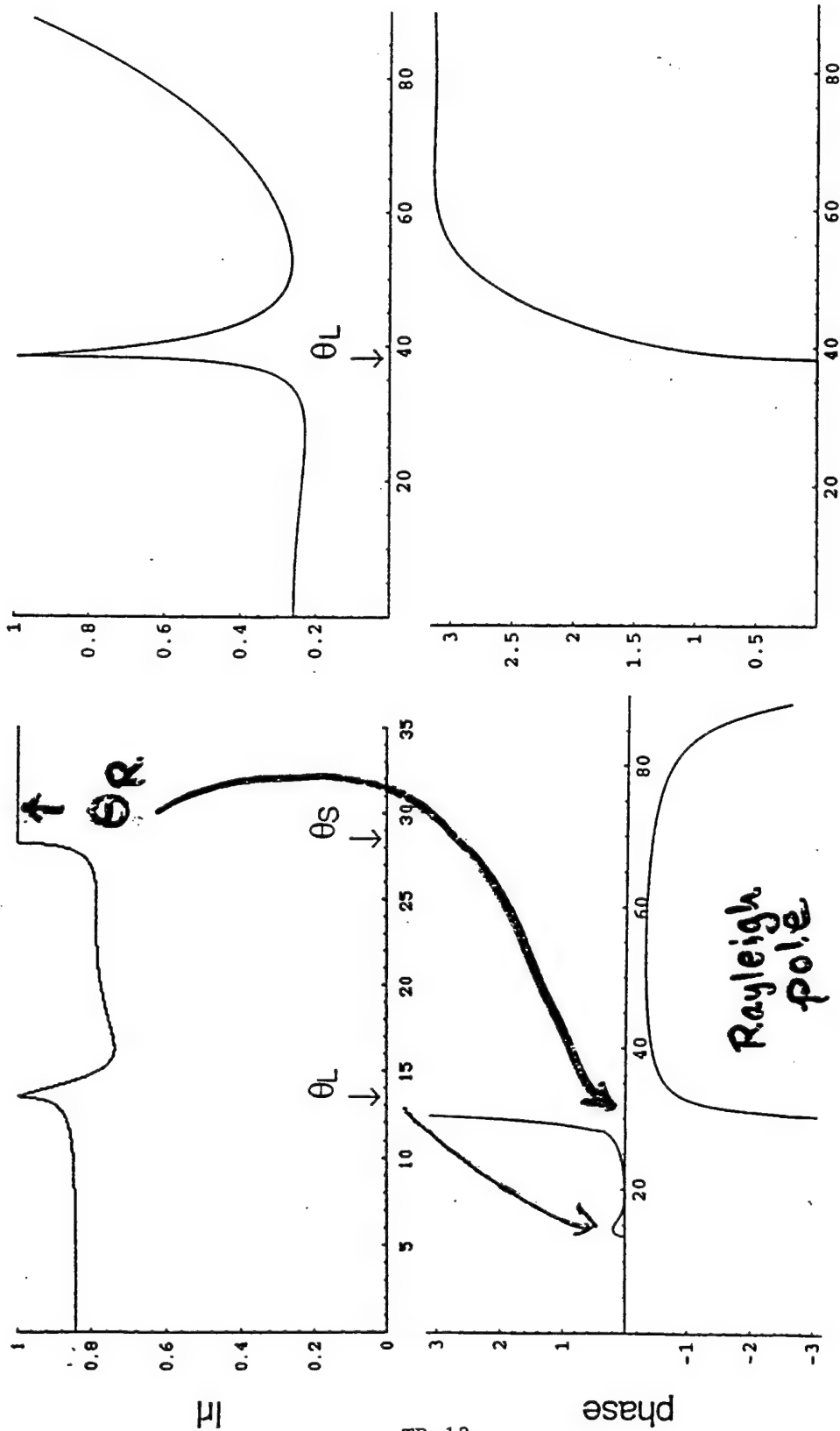
What are some mechanisms for enhanced backscattering from TYPE 4(A) or (B) targets with TYPE 1 or 2 loading?

# Ultrasonic Reflectivities

(lossless)

Aluminum-Water

Polystyrene-Water

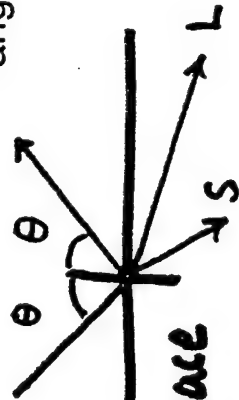


angle of incidence (deg.)

angle of incidence (deg.)

water

elastic half space



## PLASTIC PROPERTIES

**GLASS TRANSITION** - a second order phase transition from the glassy state to the rubbery state having a negligible shear modulus.

**GLASS TRANSITION TEMPERATURE ( $T_g$ )** - the temperature at which the glass transition occurs in the low frequency limit.

	$T_g$ (C)
PMMA	105
Polystyrene	100

- While the shear wave velocity  $c_s$  is relatively dispersive, for  $T \approx 20$  C at low ultrasonic frequencies,

$$\Delta c_s \ll 0.01 c_s$$

for the frequency range in our experiments.

- With  $k_s = (\omega / c_s)(1 + i\delta_s)$

$$k_L = (\omega / c_L)(1 + i\delta_L)$$

$\delta_s \ll 1$  and  $\delta_L \ll 1$  at low ultrasonic frequencies and do not depend on  $\omega$ . Example: PMMA  $\delta_s \approx 0.0053$

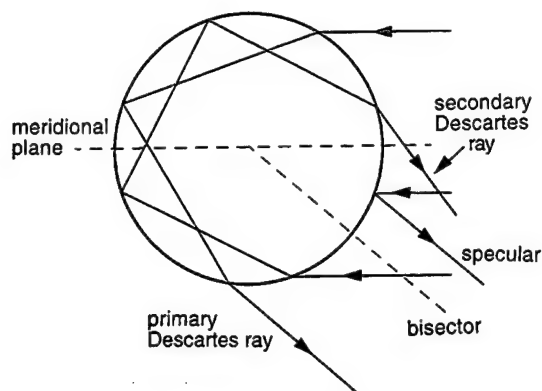


Fig. 4. Primary and secondary Descartes rays projected onto the base plane for a circular ice cylinder with  $n = 1.31$  and illuminated at  $\gamma = 20^\circ$ . As shown for this case in Fig. 3, the Descartes ray scattering angle  $\theta_D$  for  $p = 2$  exceeds  $\theta_D$  for  $p = 3$ . The projected specular reflection and bisector of the cylinder are shown from the perspective of a distant observer at the  $\theta_D$  of the primary rainbow. The meridional plane that contains the cylinder's axis and the incident wave vector is shown. Glare points associated with specular reflection were not as bright as those for the Descartes rays for situations in which the glare points from Descartes rays were visible.

the base plane for the cases of  $p = 2$  and 3, which are analogous to primary and secondary rainbows. The filled circle shown is the location of the corresponding ray-optics discontinuity in Fig. 5 of Ref. 8 for the specific case of  $\gamma = 45^\circ$ . This comparison, as well as laboratory experiments,<sup>15</sup> gives support to this analysis. As the tilt angle increases, the  $p = 2$  caustic

evolves in such a way that  $\theta_D$  reaches  $180^\circ$  at a critical tilt angle  $\gamma_c$  given by taking  $n' = 2$  in Eq. (1). This corresponds to a merging in the meridional scattering plane of the Airy caustics from both sides of the cylinder (see Appendix A).

For small values of tilt, the evolution of the values of  $\theta_D$  and the locations of the associated rays are more closely related to the observations of sunlit icicles. Figure 4 shows the locations of the Descartes rays projected onto the base plane for  $p = 2$  and 3 as well as the specular ray (which may be assigned the value  $p = 0$ ), which has the same scattering angle as  $\theta_D$  for the primary rainbow ray. For the tilt angle  $\gamma$  of  $20^\circ$  used in this figure, an observer positioned to view the glare of the primary rainbow ray would be in the shadow zone of the secondary rainbow (see Fig. 3) so that no rays having two internal reflections would be visible. Conversely, an observer positioned to view the glare of the secondary rainbow ray would be in the shadow zone of the primary rainbow.

### 3. Caustics and Glare Points Created by Sunlit Icicles

Sunlit icicles were observed visually and photographically in Pullman, Wash., during February 1996. Most of the observations and supporting photographs were taken in midmorning with the Sun position calculated to be  $20^\circ$  to  $25^\circ$  above the horizon. With the axis of the icicle approximated as vertical, the corresponding values for  $\gamma$  in the cylinder model range from  $20^\circ$  to  $25^\circ$ . Figures 5(a) and 5(b) show color photographs with the camera focused on the icicle such that the camera is on opposite sides of the me-

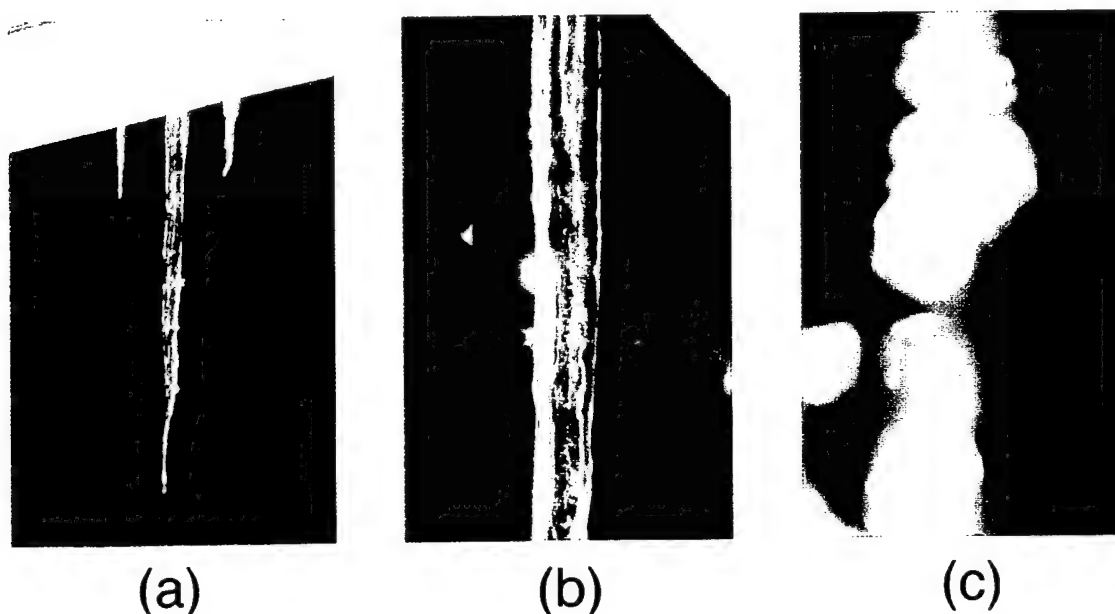


Fig. 5. (a) Descartes glare point for the primary Airy caustic of an icicle is visible as the bright patch on the right-hand side of the icicle near its center. The weak glare point visible on the left-hand side is for a twice-reflected and twice-refracted ray. (b) The same icicle viewed from the other side of the meridional plane shows a line of primary Descartes glare points on its left-hand side with a line of specular glints to the right of an imaginary center line (shown as the cylinder's bisector in Fig. 4). The lower of the two very bright glare points has a green hue. (c) Perspective as in (b) but with the camera's focal plane conjugate to the far-field scattering. The glare points are now out of focus, and some of the Descartes glare points appear to be associated with colored regions.

meridian formed by  $\mathbf{k}_i$  and the cylinder's axis. Glare points evident in these photographs were also easily observed by eye. The causes of the glare points were identified when the head of the observer moved and by comparisons with ray diagrams as shown in Fig. 4. Note that Fig. 4 is drawn for the perspective shown in Fig. 5(b) such that the Descartes ray appears on the left whereas a glint from external specular reflection appears on the right-hand side of the bisector. Corresponding features can be identified in Fig. 5(b). The lowest bright glare point has a green hue.

The glare points (or point) from the two-chord ray in Fig. 5(b) disappeared when  $\theta$  was reduced below a value that was interpreted to be  $\theta_D$  because the transition appeared to be associated with the Descartes ray. In support of this interpretation, the glare was observed to be especially bright at this Airy transition when compared with the glare of the specular reflection. Furthermore, observations with a camera defocused, so the focal plane was conjugate to the far-field scattering, manifested the color sequence of the rainbow in the defocused image of some two-chord glare points but not for the specularly reflected glint. (This viewing method displays the angular dependence of the scattered light. Other examples of out-of-focus viewing are discussed on pp. 231–234 of Ref. 7.) Figure 5(c) is a far-field image for the camera perspective as in Fig. 5(b) and Fig. 4. Vertical colored bands apparently associated with the Airy caustic of the primary rainbow rays are visible slightly below the center of the photograph, with the red side (and the corresponding shadow region of the caustic) appearing on the right-hand side of the bands. It was confirmed with a diagram based on elementary lens theory that the shadow side in the defocused photograph [Fig. 5(c)] agreed with predictions in which the illuminating rays correspond to the primary rainbow rays of the icicle. The limited extent of the colored bands in Fig. 5(c) appears to be the result of vignetting due to the lens aperture. With the observer in the two-ray region of the primary rainbow, it was difficult to resolve the distinct glints of the associated rays (labeled 1 and 2 in Fig. 2); however, that may be more a deficiency of the observer's vision (the author's) than of the actual optical phenomenon.

Although it was not feasible to measure  $\theta_D$  for natural icicles, some information about relative positions of Descartes angles is available because glare points that have the expected properties of a secondary or  $p = 3$  Descartes ray were occasionally visible by careful positioning of the observer. Such a glare point is visible on the left-hand side of the icicle in Fig. 5(a). This glare point is slightly below a primary Descartes ray glare point visible on the right-hand side. Although Fig. 4 shows that the primary and the secondary glare points, if visible, should be on opposite sides of a cylinder, the recording of both glare points with a single camera position merits some discussion. This is because inspection of Figs. 3 and 4 shows that the secondary rainbow lies in the shadow region of the primary rainbow of a cylinder except for small values of  $\gamma$ , namely,  $\gamma < 4.9^\circ$  for  $n = 1.31$ . The

simultaneous observation of the primary and the secondary glare points as in Fig. 5(a) may be indicative of deviations from a circular cross section or a cylindrical shape, both of which are visible in Fig. 5 and from direct inspection of the icicle. It is noteworthy here that the vertical spread of the glints from the  $p = 2$  ray also indicates deviations from a cylindrical shape, since it is to be expected that, for perfect cylinders, the vertically displaced glare points would spread vertically and merge into vertical lines.

Although Fig. 5 and the observations described are based on an icicle that has a diameter of approximately 1 cm, the qualitative features of the primary Descartes ray were confirmed with observations based on clusters of smaller icicles in December 1996.

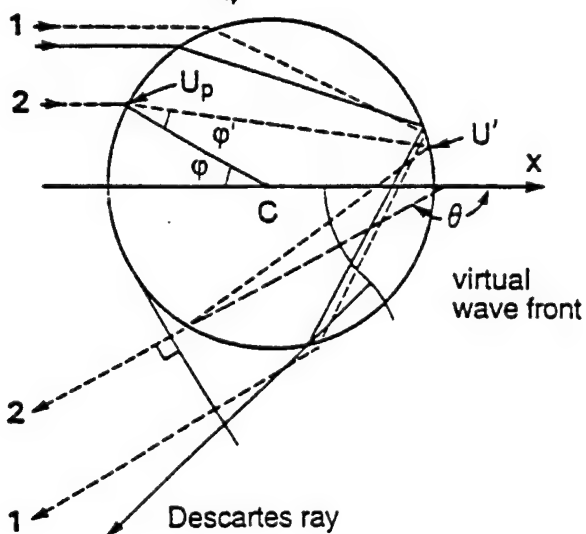
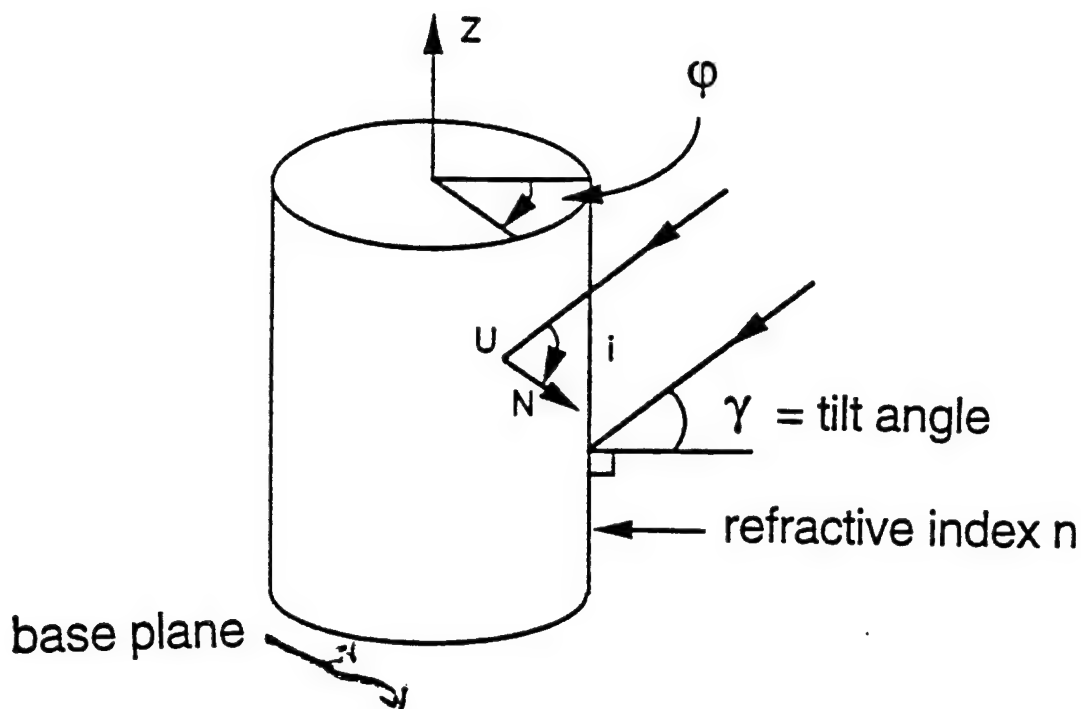
#### 4. Discussion and Conclusions

The observations summarized in Section 3 show that rays within icicles can produce bright glare points that have the expected properties of primary and secondary Descartes (or rainbow) rays. The observations were first made with the unaided eye and are supplemented by color photographs, as shown in Fig. 5. The observations were made without the aid of a polarizer and with solar illumination unobscured by clouds.

A model was presented for how the projected scattering angle for primary and secondary Descartes rays varies with the tilt angle  $\gamma$  of the illumination relative to the axis of the icicle for the situation in which the icicle has the shape of a circular cylinder. This model neglects the taper and the shape variations of real icicles. For a sufficiently small value of  $\gamma$ , corresponding to nearly horizontal illumination, the model suggests that a sufficiently distant observer could lie within the two-ray region of both the primary and the secondary caustics. The observations were made with  $\gamma$  only somewhat larger than the crossover value (predicted to be at or near  $4.9^\circ$ ) so that the apparent simultaneous observation of primary and secondary rainbow glare points in Fig. 5(a) may be due to deviations from the simplified shape assumed.

The above-mentioned model also predicts that at a critical value of tilt  $\gamma_c$  (predicted to be at  $60.75^\circ$  for ice) the primary rainbow Descartes rays from both sides of the cylinder merge in the meridional plane. The expression for  $\gamma_c$ , Eq. (A3), may be derived either with the Bravais effective refractive index, Eq. (1), or from a direct calculation of the appropriate principal curvature of the refracted internal wave front in the meridional plane, Eq. (A5). Although it is not currently known whether the caustic-merging transition (CMT) is observable for naturally sunlit icicles, if it is observable for icicles, the transition glare point should be very bright. This consequence of the enhanced focusing is discussed in Appendix A. The merging and the general brightness enhancement of the two-chord glare point were both confirmed by the viewing of a long tilted polished glass rod illuminated with light from a distant slide projector. Unfortunately, conditions favorable for such observations

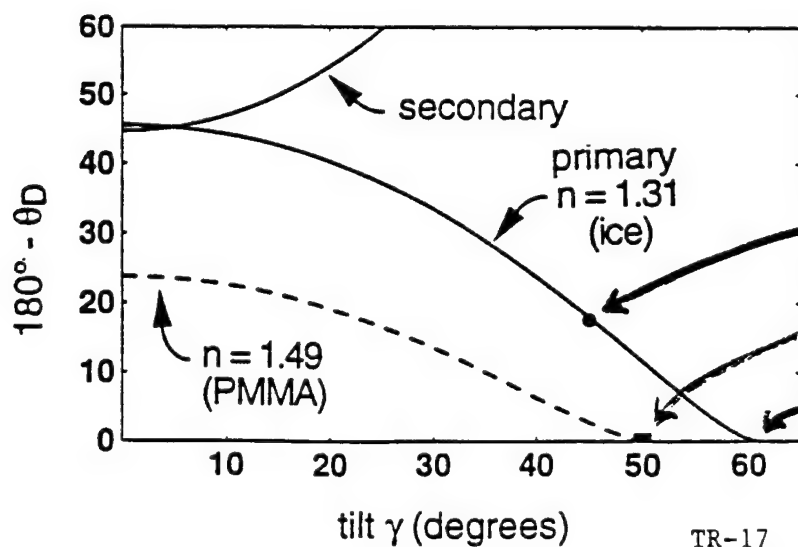




The projection is like rays through a cylinder at normal incidence having a refractive index of

$$n'(\gamma) = \frac{\sqrt{n^2 - \sin^2 \gamma}}{\cos \gamma}$$

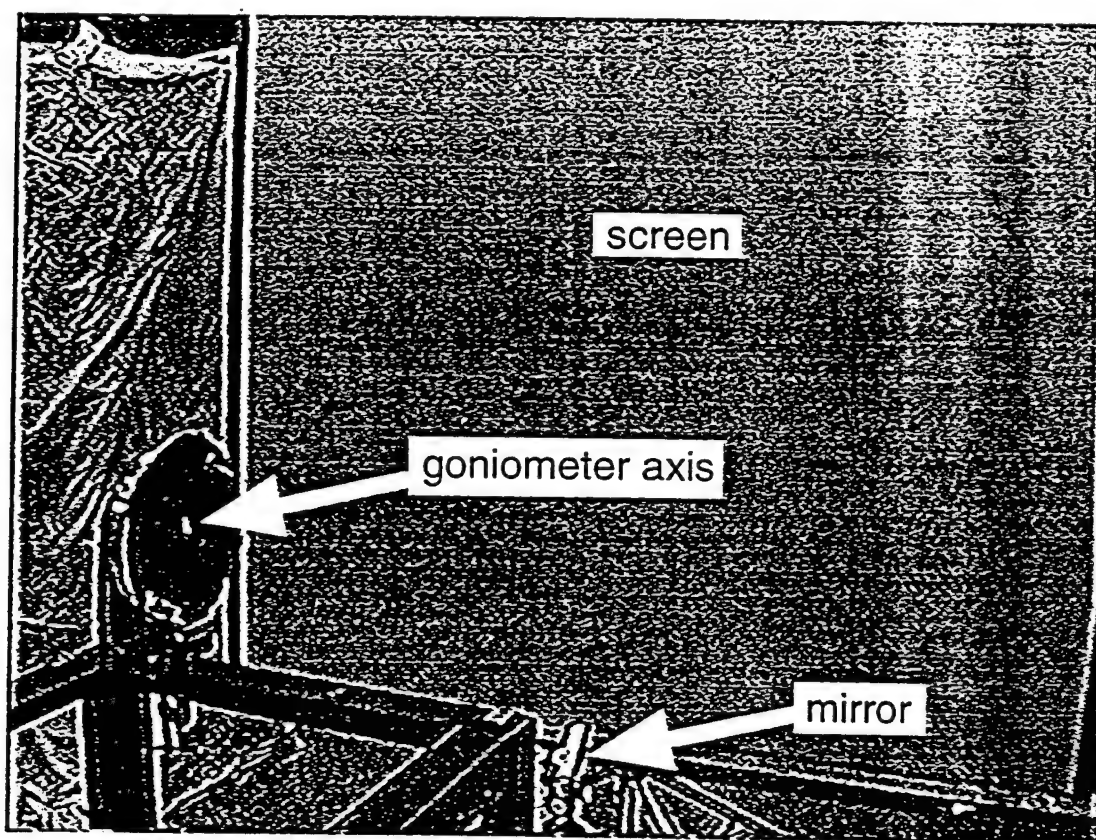
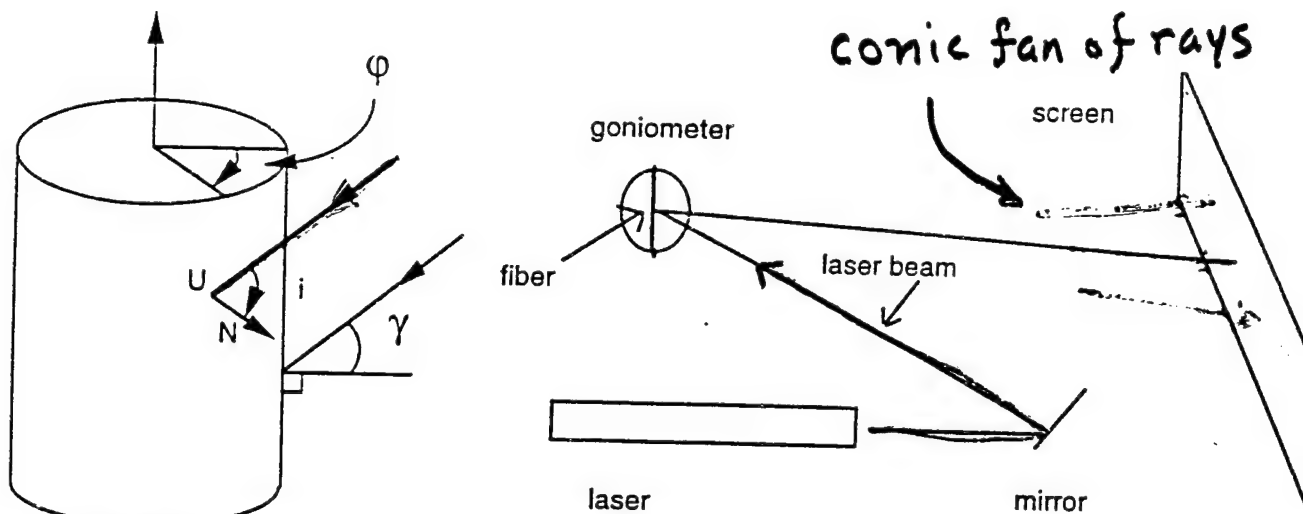
(Bravais law of refraction of skew rays)



halo simulation  
Takano + Tanaka (1980)

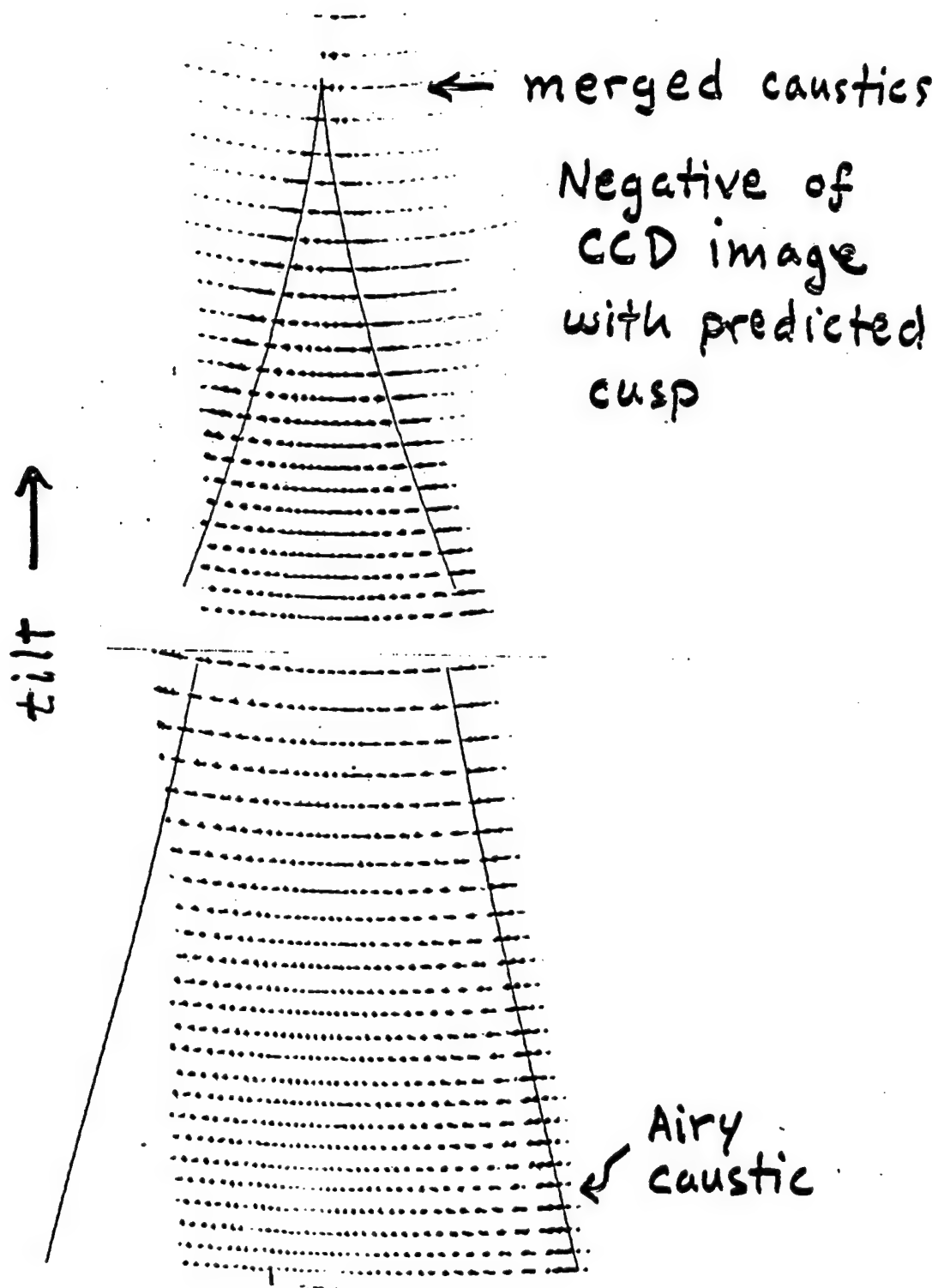
C. Mount (data)

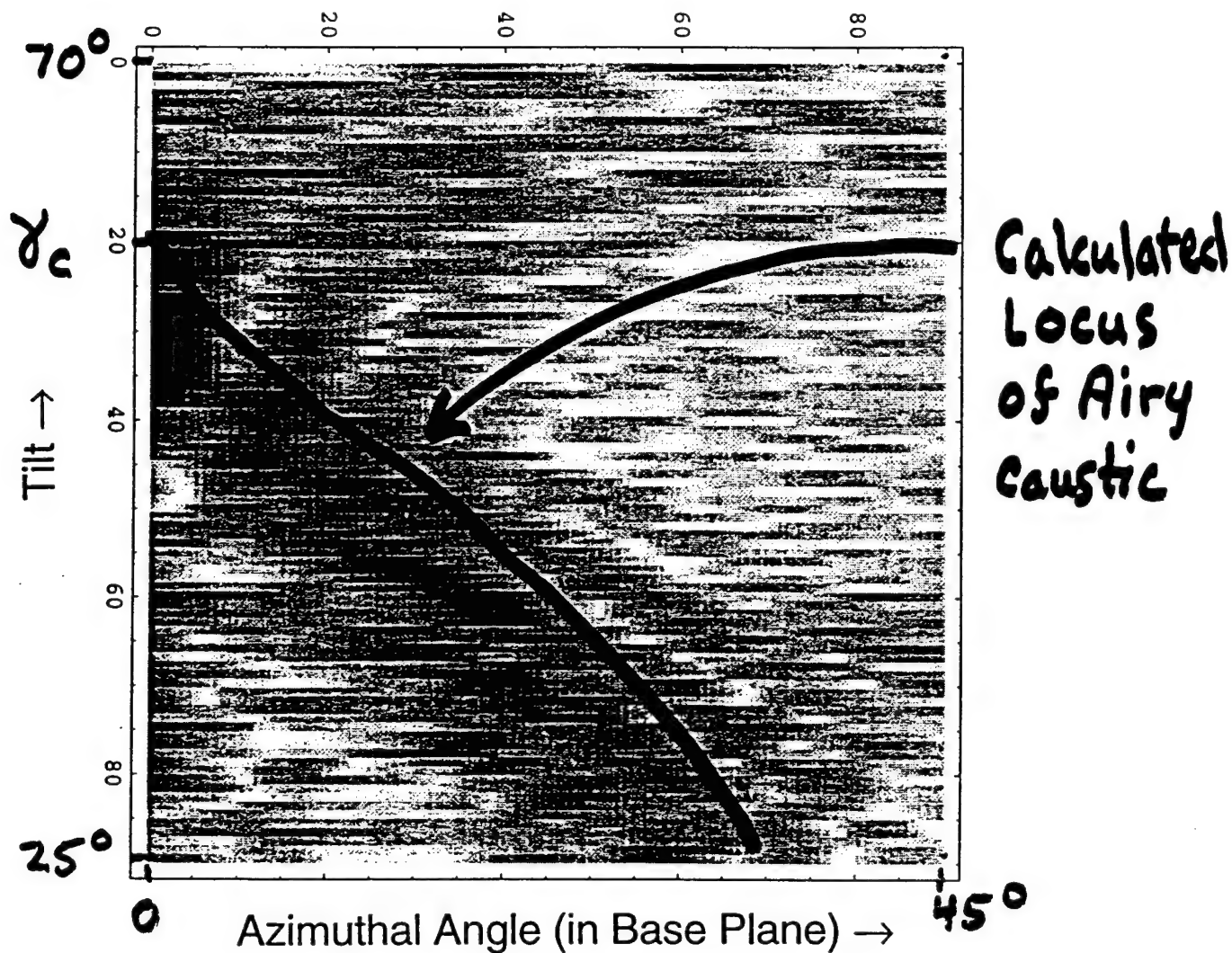
$\gamma_c$



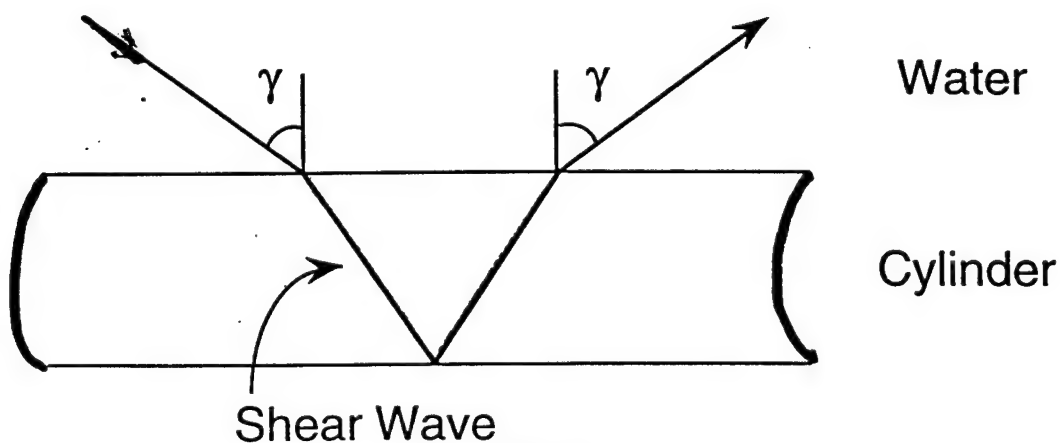
**Scattering observations for tilted transparent fibers: evolution of Airy caustics with cylinder tilt and the caustic merging transition**

Catherine M. Mount, David B. Thiessen, and Philip L. Marston APPLIED OPTICS / Vol. 37, No. 9 20 March 1998



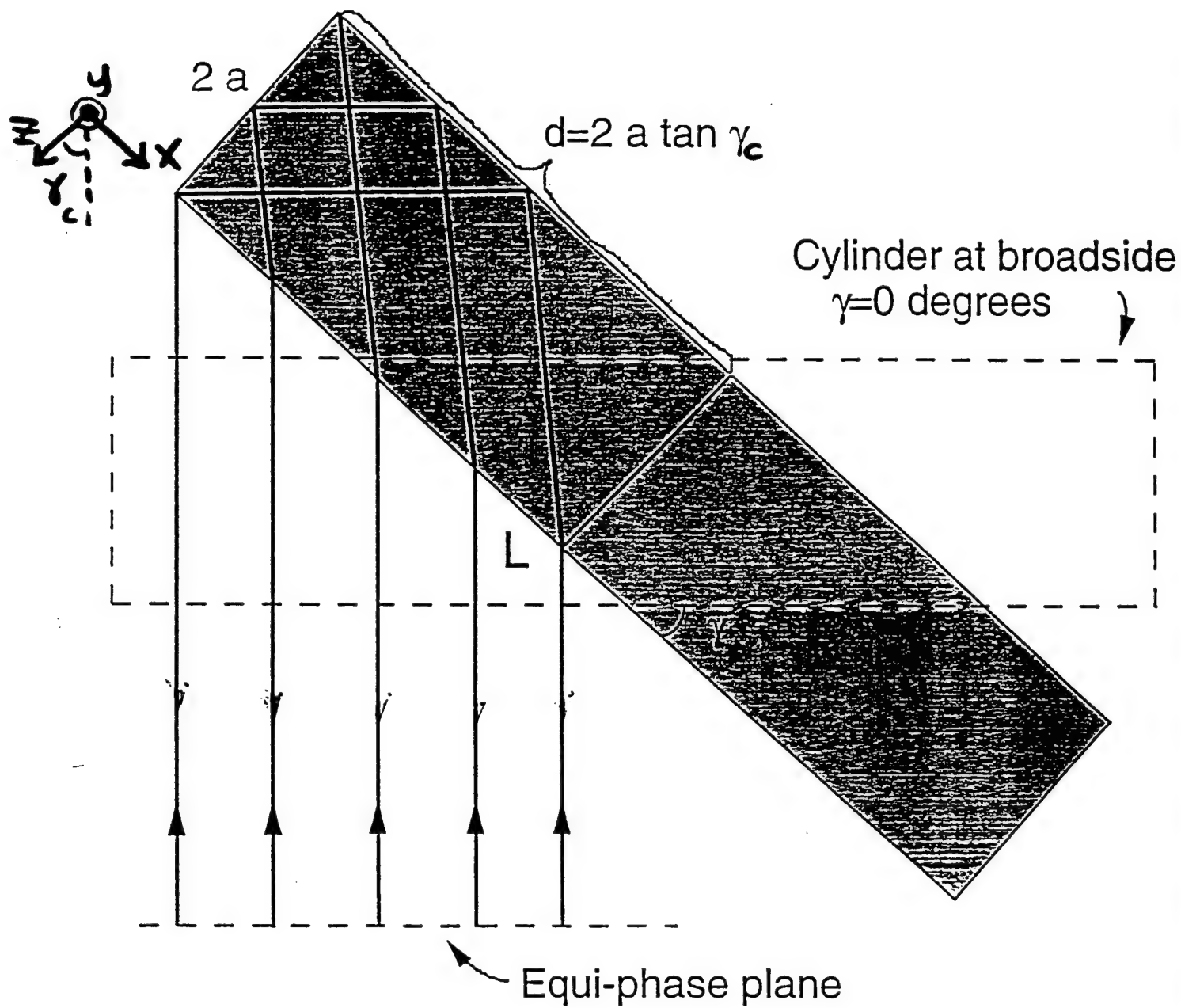


Calculated for infinite polystyrene (elastic) cylinder in water for sound with  $ka = 50$ .



TR-20

In the Meridional Plane...



$$W(x, y=0) = x \sin \gamma \quad (0 < x < d)$$

## New Backscattering Enhancement Mechanisms for Solid Polymer Targets

1. For filled targets (such as solid cylinders): "New" transmitted rays for generating flat scattered wavefronts.

Examples: Subsonic shear waves (PMMA or Polystyrene)  
Subsonic longitudinal waves (RTV)

Geometry: Tilted bluntly truncated cylinder

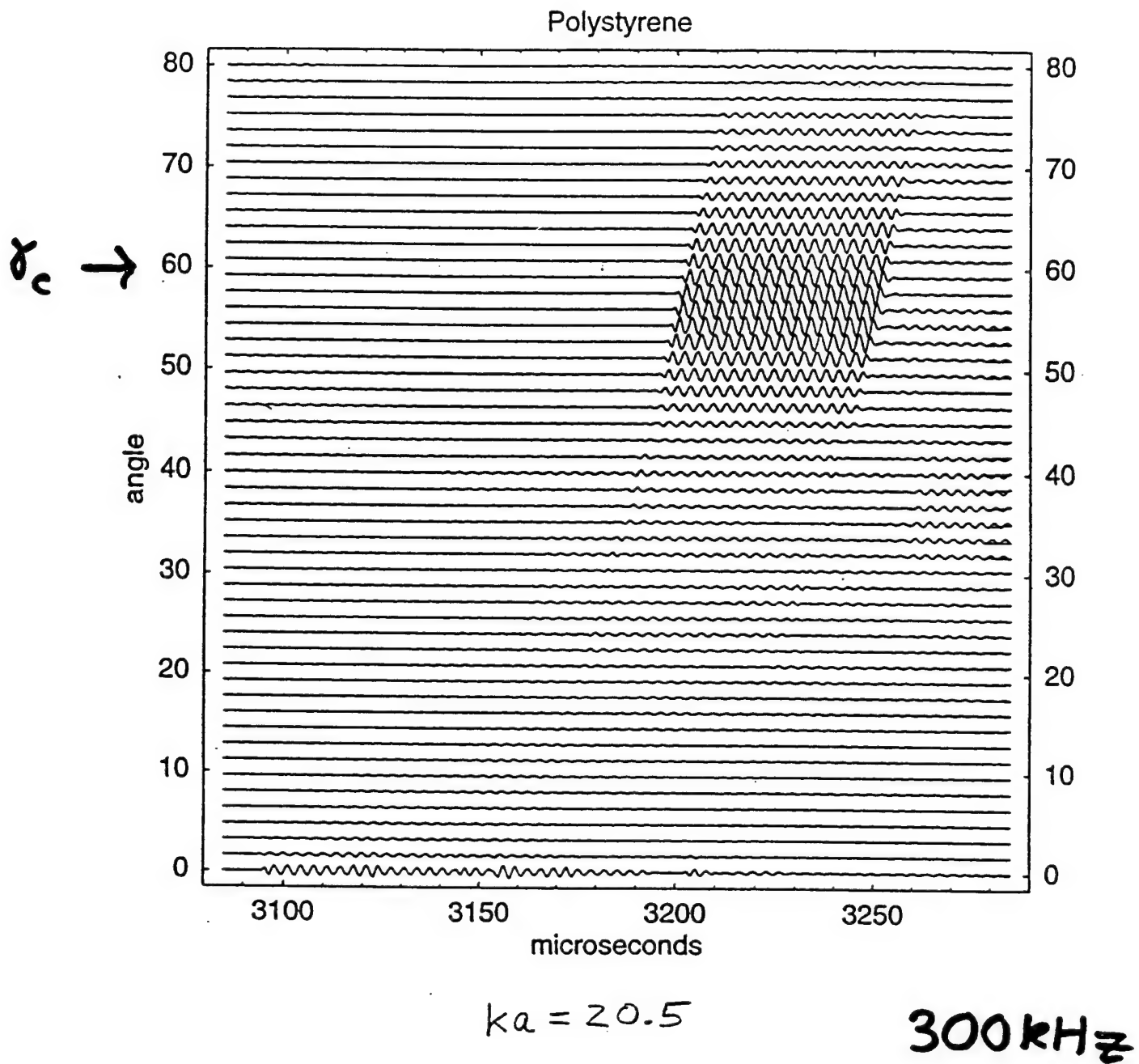
Caustic merging transition at critical tilt:

$$\gamma_c = \arccos \left[ \left( (N^2 - 1) / 3 \right)^{1/2} \right], \quad N > 1,$$

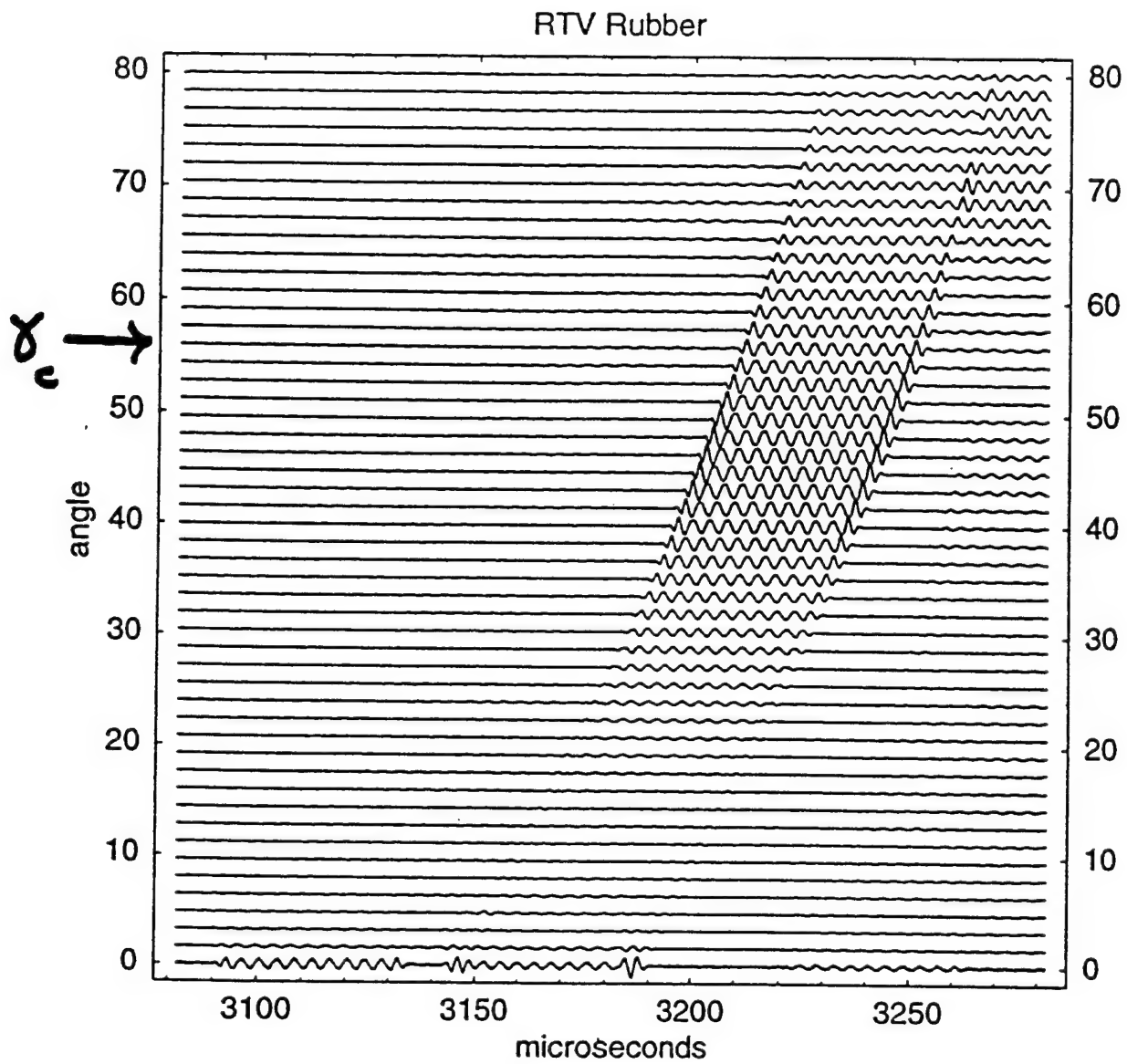
Refractive Index  $N = c / c_T$  or  $c / c_L$

Marston, Applied Optics **37**, 1551-1556 (1998)

Blonigen and Marston, JASA (abstract) **102**, 3088 (1997)



TR-23

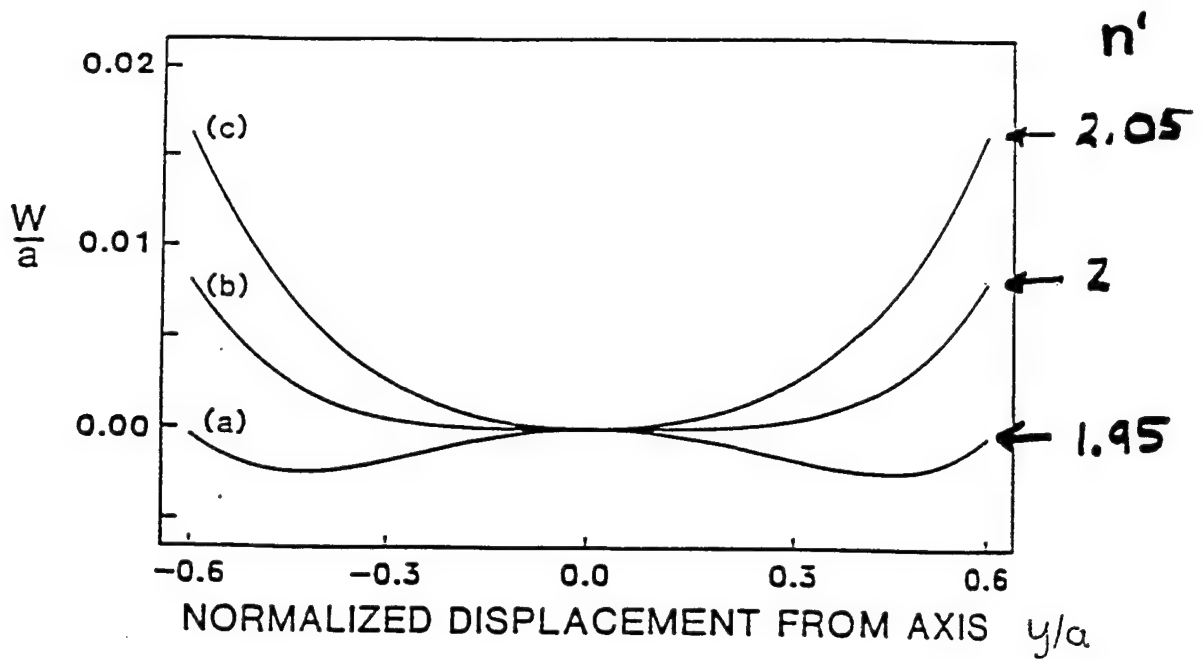


$$ka = 14.3$$

250 kHz

TR-24

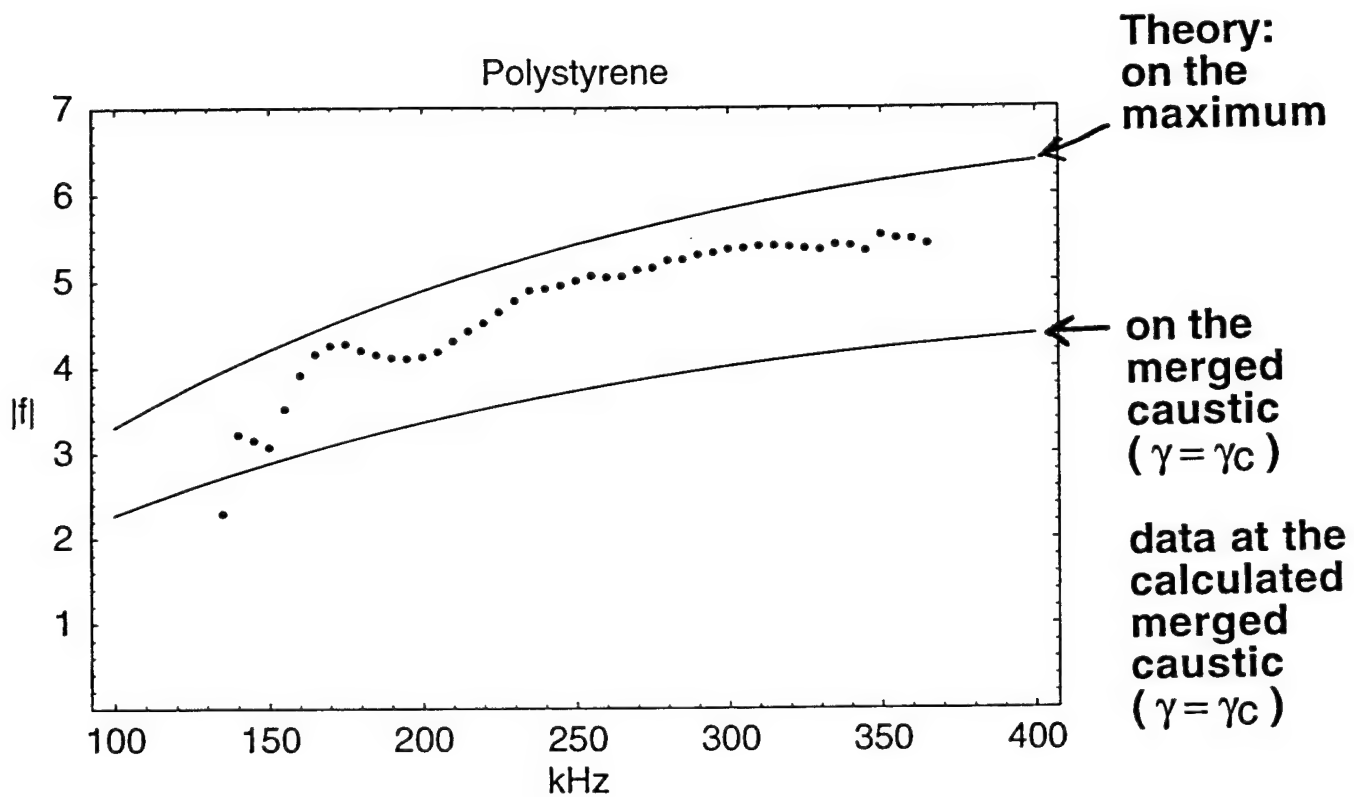




$$W(x=0, y) = a_4 y^4 - a_2 y^2$$

$$\text{For } n' \approx 2 \left\{ \begin{array}{l} a_2 = \frac{1}{2\alpha} \approx \frac{(2-n')}{\alpha n'} ; \quad a_4 \approx \frac{2}{\alpha^3 n'^3 (2+n')} \end{array} \right.$$

$$\Rightarrow W(0, y) = \frac{y^4}{16 \alpha^3}$$



## Pearcey Function

$$P_{\pm}(w_1, w_2) = \int_{-\infty}^{\infty} ds \exp[\pm i(s^4 + w_2 s^2 + w_1 s)]$$

$$P_{\pm}(0, 0) = \frac{1}{2} \Gamma(\frac{1}{4}) e^{\pm i\frac{\pi}{8}}$$

Backscattered Pressure:

$$P = P_i \frac{a}{2r} f e^{ikr}$$

where  $f$  = form function

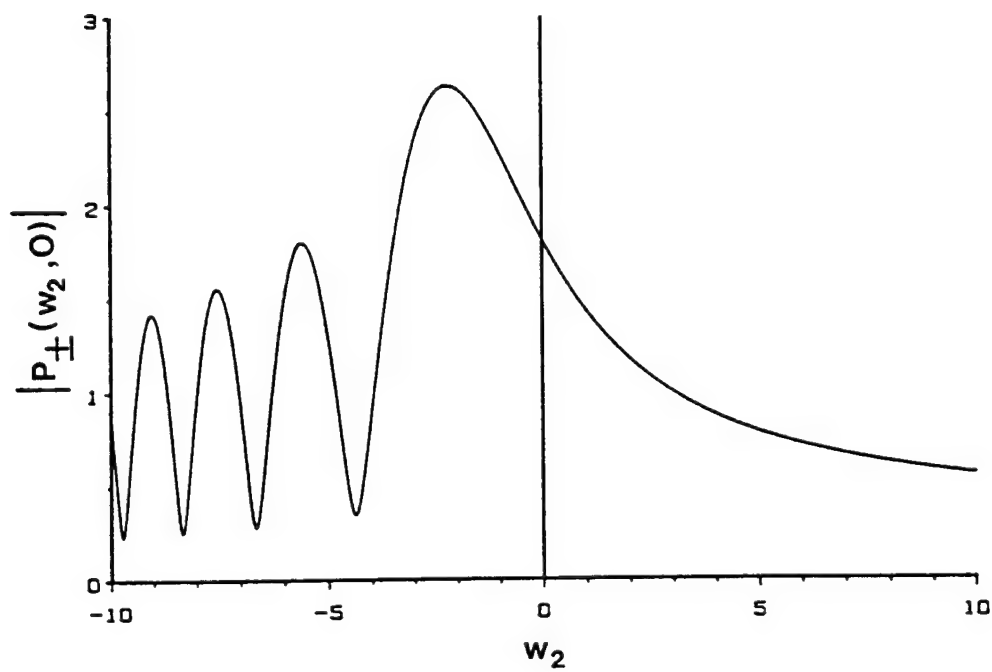
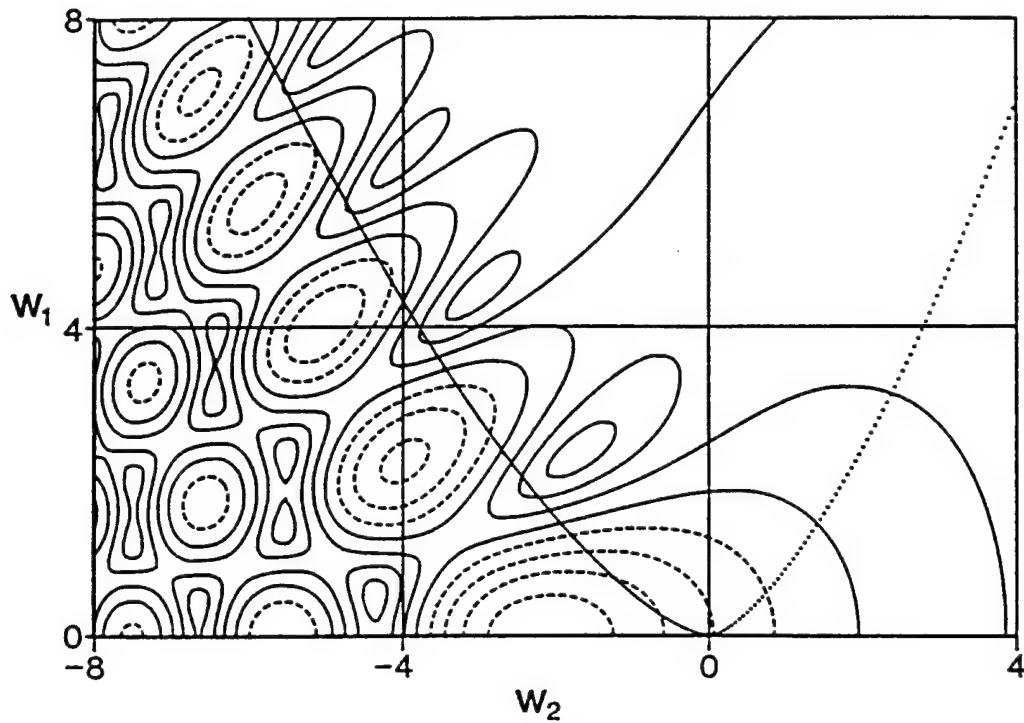
hard sphere:  $|f| = 1$

tilted cylinder: On the caustic  $w_1 = w_2 = 0$ ,

$$|f| = \frac{2}{\pi} \Gamma(\frac{1}{4}) |B| (ka)^{3/4} \sin \chi / (\cos \chi)^{1/2}$$

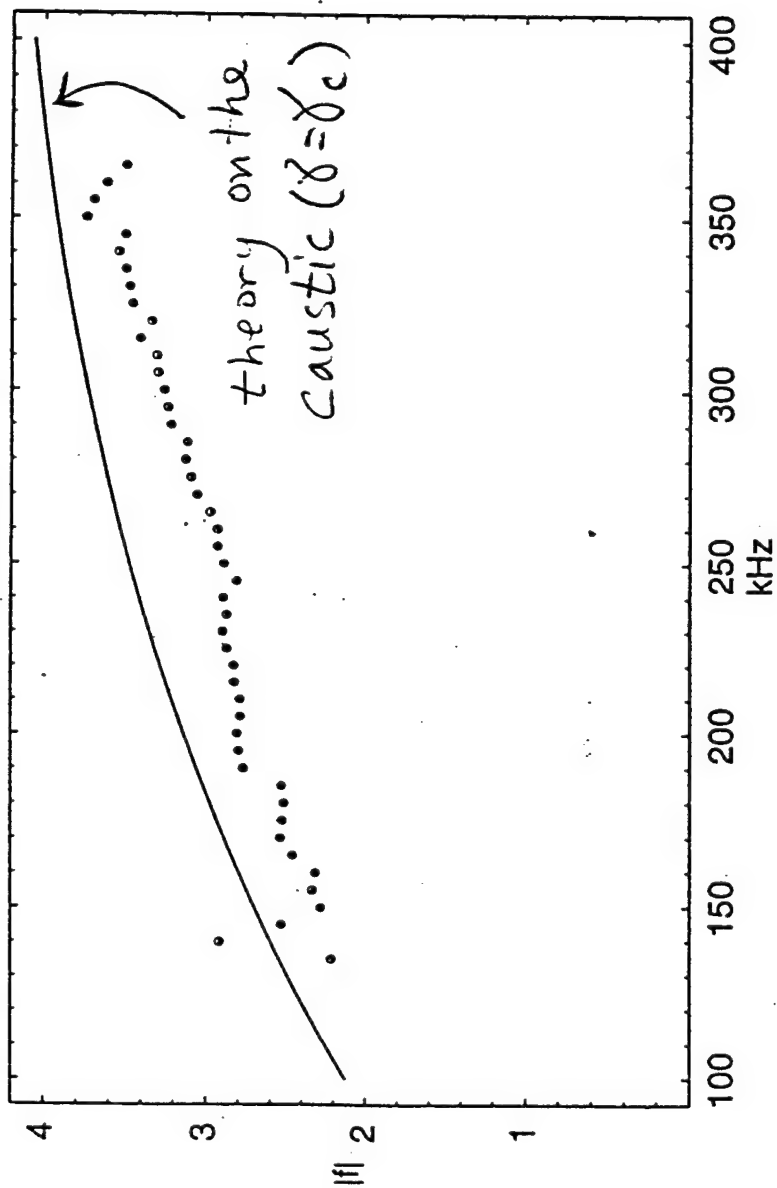
# Pearcey Function

$$P_{\pm}(w_2, w_1) = \int_{-\infty}^{\infty} \exp[\pm i(s^4 + w_2 s^2 + w_1 s)] ds$$

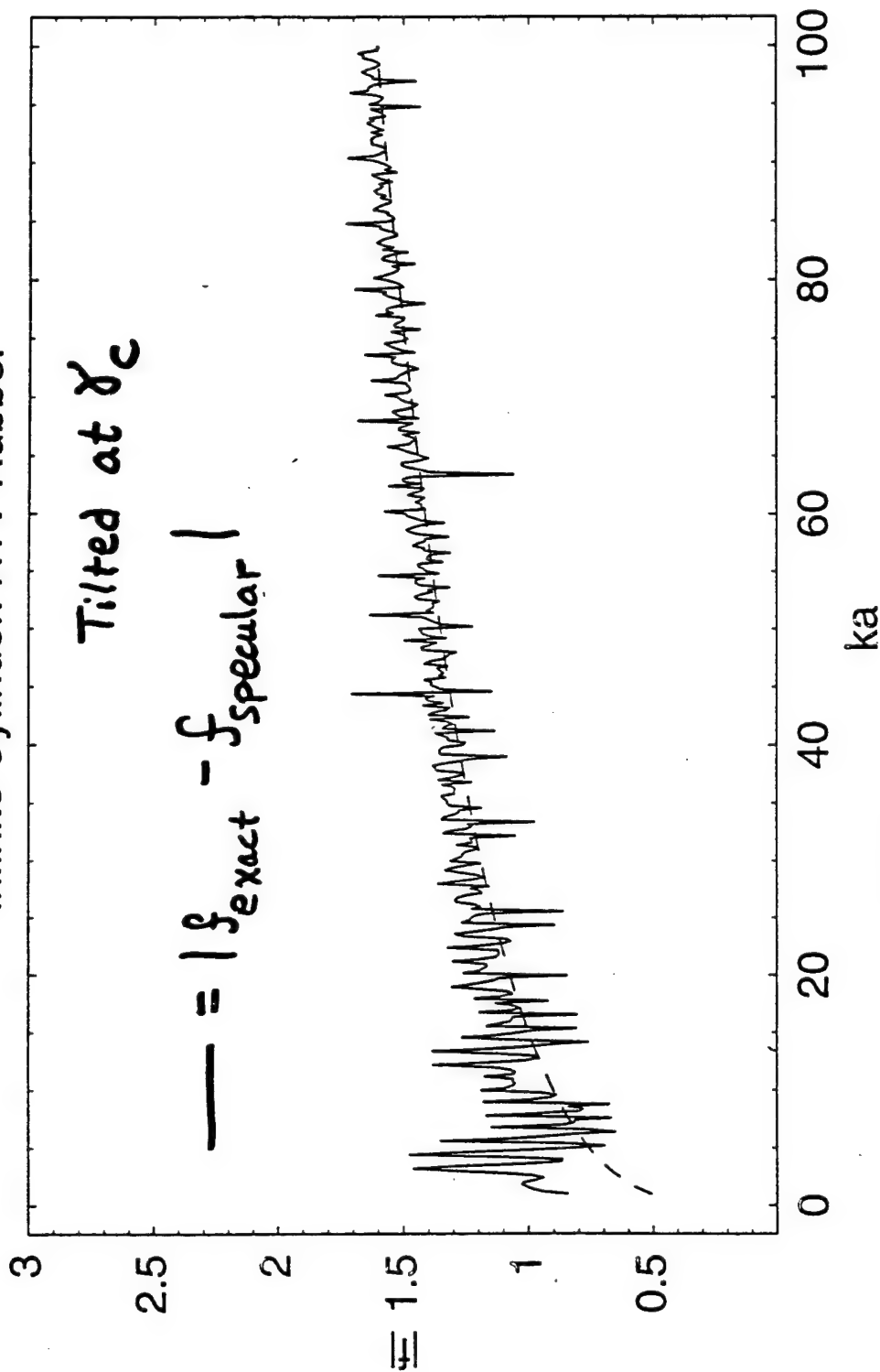


TR-28

# RTV Rubber



# Infinite Cylinder: RTV Rubber

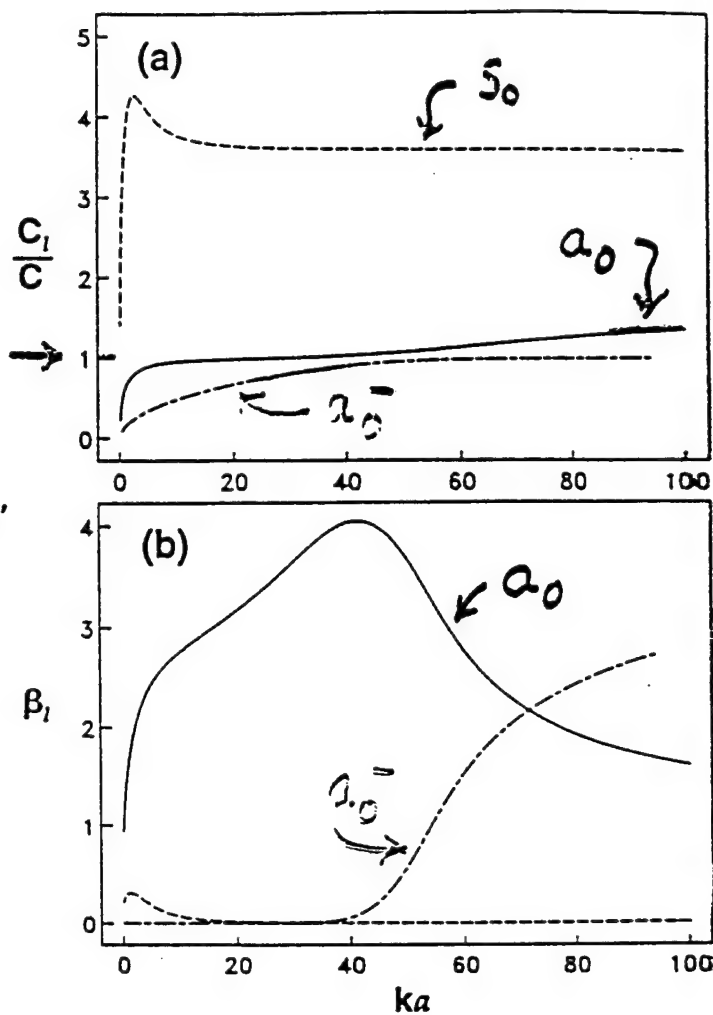
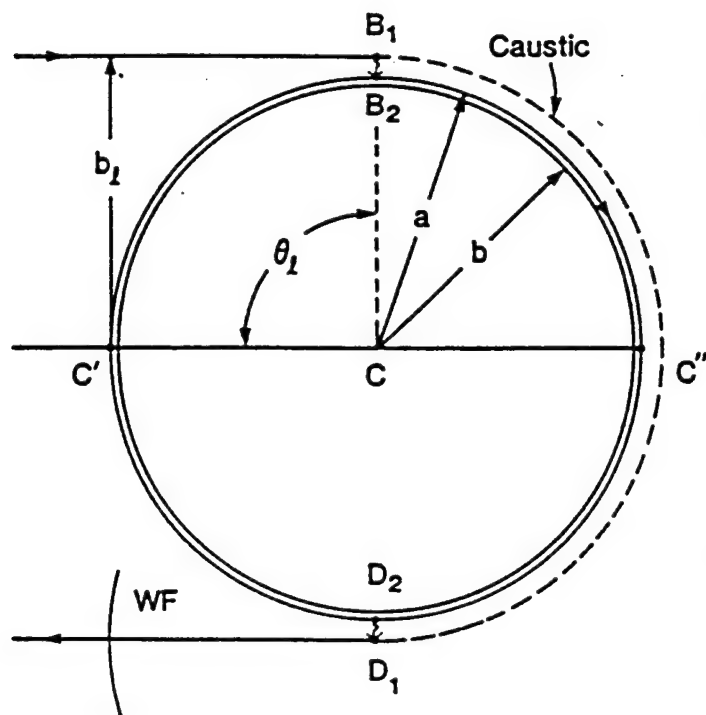


TR-30

$$|p_{2d}| = p_i \sqrt{\frac{a}{2\gamma_i}} |f|$$

---- Ray theory:  $|f_{\text{ray}}| = \frac{2|B|}{\sqrt{\pi}} (ka \cos \gamma_c)^{1/4} |P_-(0,0)|$

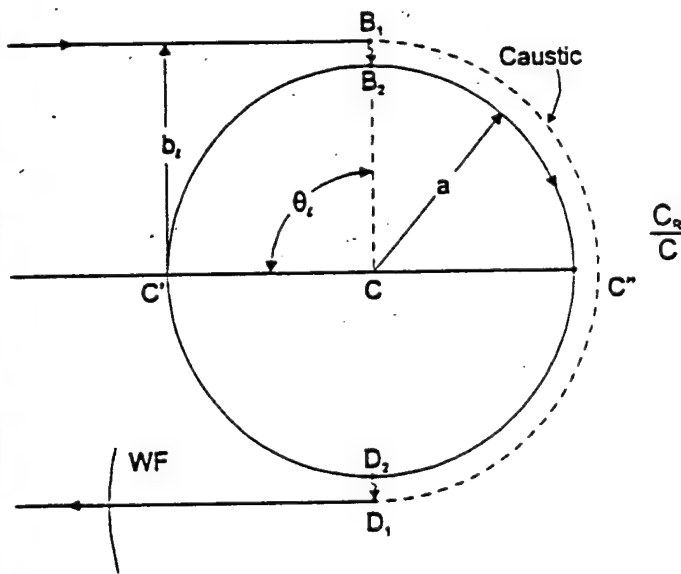
- 1. Coincidence frequency of  $a_0$  Lamb wave and tunneling model:  
 [Zhang, Sun, & Marston (1992), Kaduchak & M(1993), M &  
 Sun (1995) JASA]



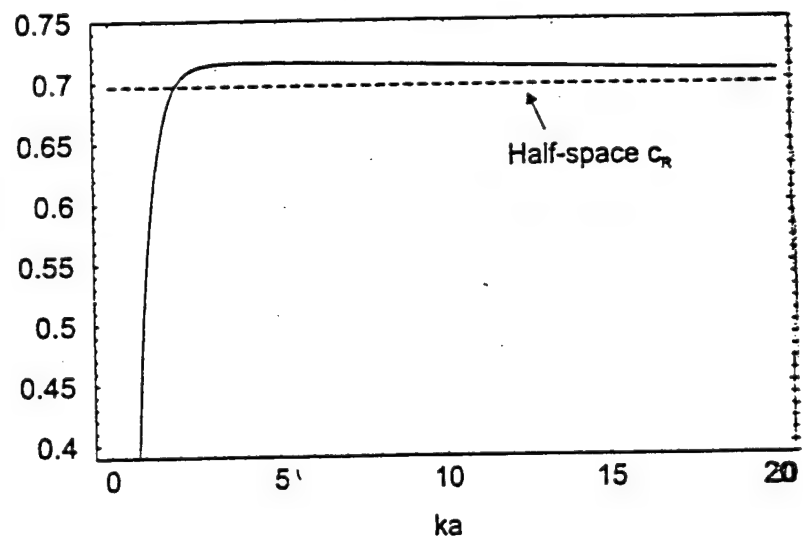
## New Backscattering Enhancement Mechanisms for Solid Polymer Targets

### → 2. Tunneling enhancements for "new" subsonic waves

- Example: Rayleigh wave on PMMA or Polystyrene becomes subsonic with respect to water or sediment loading.
- Ray Theory: Same as Zhang, Sun, & Marston (1992) but with Rayleigh wave parameters and attenuation.
- Experimental confirmation for solid spheres: B. T. Hefner.



Rayleigh wave velocity for an acrylic sphere in water.



### → 3. New coincidence frequency and backwards wave enhancements for shells.

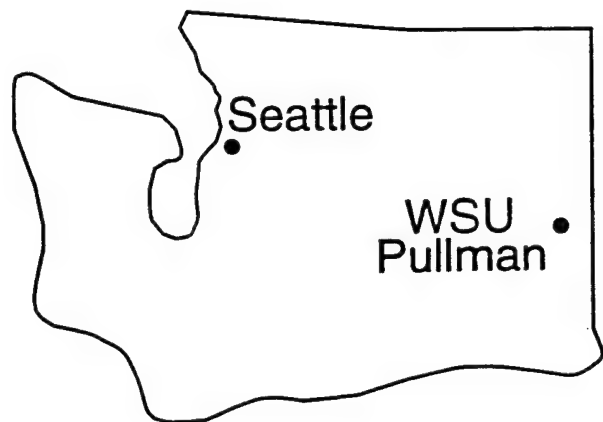


## Summary

- Quantitative leaky ray theory has been confirmed for meridional ray backscattering enhancements for truncated steel shells and solid cylinders (Morse and Gipson).
- In addition to leakage by supersonic  $a_0$  waves, reflection from the end of a shell near a mode threshold causes radiation due to the collective motion of a cut-off  $a_1$  wave (Morse).
- Plastic targets in water or sediment require rethinking of enhancement mechanisms.
- For blunt tilted solid cylinders of polystyrene and RTV a backscattering enhancement due to the merging of farfield caustics of transmitted rays has been observed (Blonigen).
- PMMA spheres were observed to give a backscattering enhancement due to tunneling to a subsonic Rayleigh wave (Hefner).
- We modeled the resulting Rayleigh wave resonance scattering by extending previous ray-tunneling models of subsonic  $a_0$  waves on steel shells [Zhang, Sun, & Marston JASA (1992)].

Subsonic Rayleigh wave resonances on solid polymer spheres in  
water and backscattering enhancements associated with  
tunneling: experiments, models, and the relative significance of  
material and radiation damping

Brian T. Hefner  
and  
Philip L. Marston  
Dept. of Physics  
Washington State University  
Pullman, WA 99164-2814



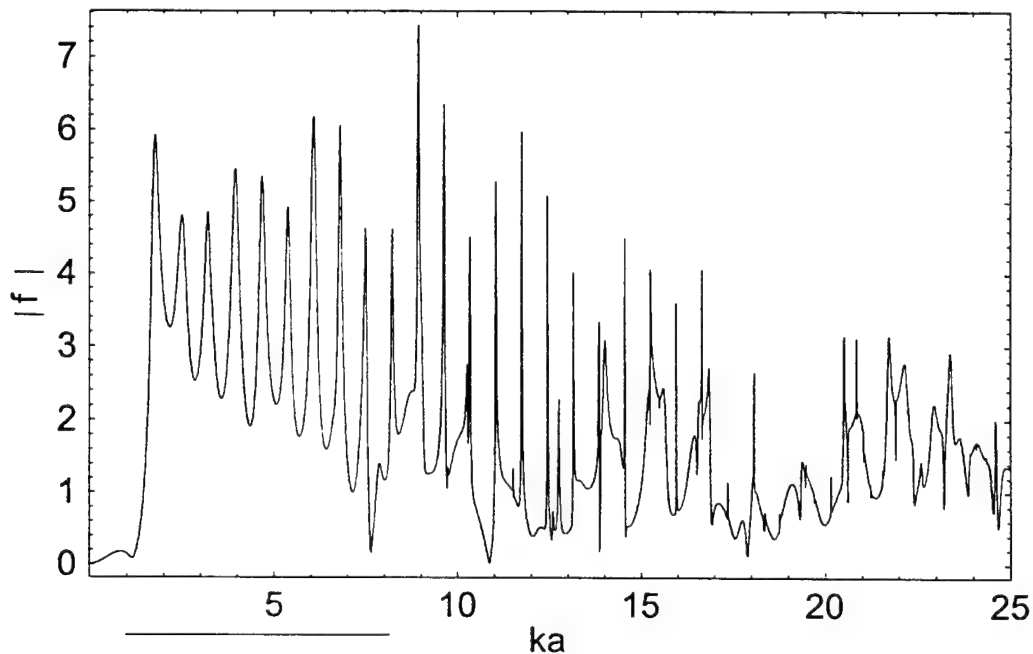
Supported by the Office of Naval Research.

# Backscattering from solid spheres

## Material properties:

	Longitudinal velocity (mm/ $\mu$ s)	Shear velocity (mm/ $\mu$ s)	Rayleigh velocity (mm/ $\mu$ s)	Density (g/mm <sup>3</sup> )
Stainless Steel	5.675	3.141	2.903	7.57
<b>Acrylic (PMMA)</b>	<b>2.690</b>	<b>1.340</b>	<b>1.250</b>	<b>1.19</b>
Water	1.479			1.00

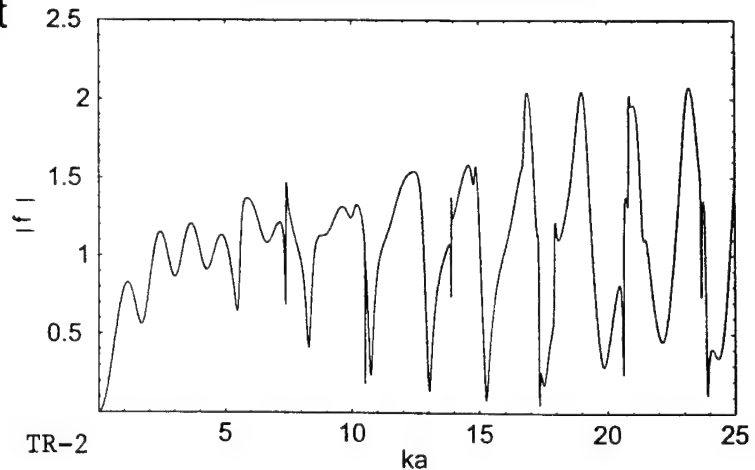
$|f|$  for Solid Acrylic Sphere



Low frequency enhancement  
associated with subsonic  
Rayleigh wave resonances.

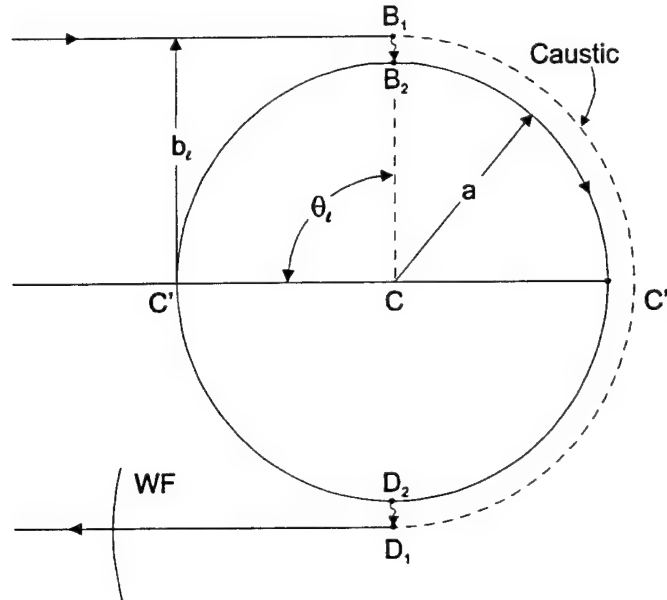
Enhancement is absent  
from  $|f|$  for steel.

$|f|$  for Solid Stainless Steel Sphere



TR-2

Coupling of incident field to subsonic Rayleigh wave:



Partial wave series (PWS) solution :

$$p_{sca} = p_{inc} (a/2r) f e^{i(kr - \omega t)} \quad f^{PWS} = \frac{2}{ika} \sum_{n=0}^{\infty} (-1)^n (2n+1) \frac{B_n(ka)}{D_n(ka)}$$

Sommerfeld Watson Transform (SWT):

$$D_{v_l}(x) = 0$$

$c_l$  : phase velocity for the  $l$  th surface wave

$$v_l = \alpha_l + i\beta_l$$

$\beta_l$  : damping coefficient

$$\frac{c_l}{c} = \frac{x}{\alpha_l + \frac{1}{2}}$$

Rayleigh wave contribution:

$$f_{mR} = -G_R \exp[-\pi\beta_R - 2\pi m\beta_R + i(\eta_R - m\pi + 2\pi m x c/c_R)]$$

$$f_R = \sum_{m=0}^{\infty} f_{mR} = \frac{-G_R \exp(-\pi\beta_R + i\eta_R)}{[1 + \exp(-2\pi\beta_R + i2\pi x c/c_R)]}$$

$$|G_R| \approx 8\pi\beta_R c/c_R \quad \eta_R = \pi k a c/c_R$$

# Incorporating material absorption into the backscattering formulations.

Assume both longitudinal and shear wave velocities have linear absorptions

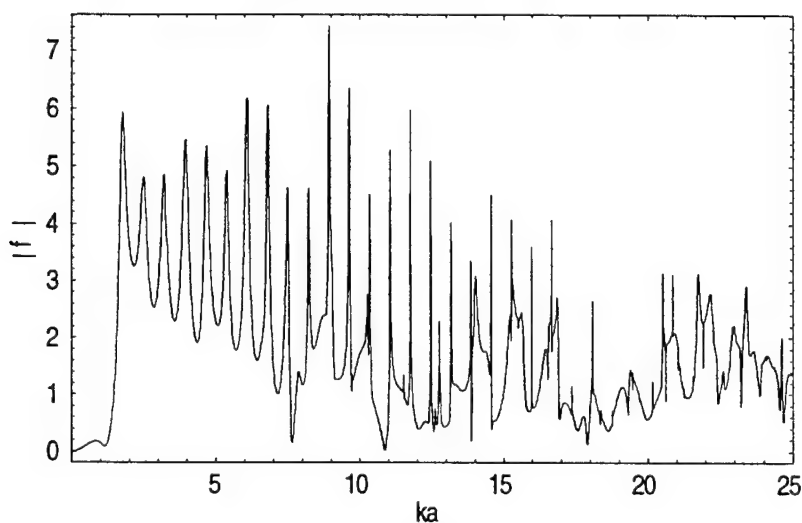
$$k_L \Rightarrow k_L(1 + i\alpha_L)$$

$$k_S \Rightarrow k_S(1 + i\alpha_S)$$

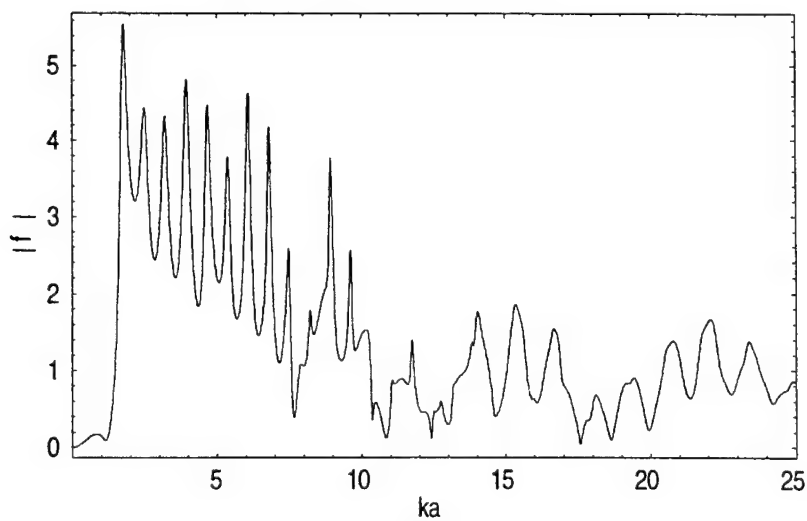
For Acrylic:  $\alpha_L = 0.0034$   
 $\alpha_S = 0.0053$

In the exact  $|f|$ -

$|f|$  for Solid Acrylic Sphere

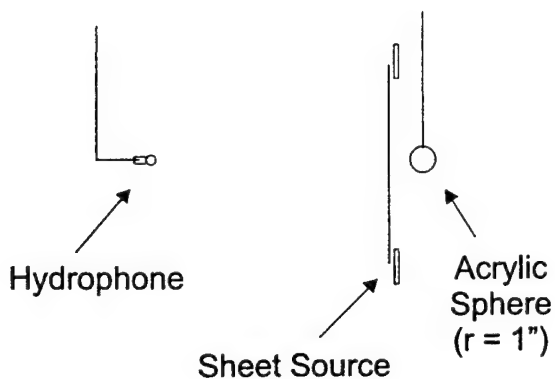


damped  $|f|$  for Solid Acrylic Sphere



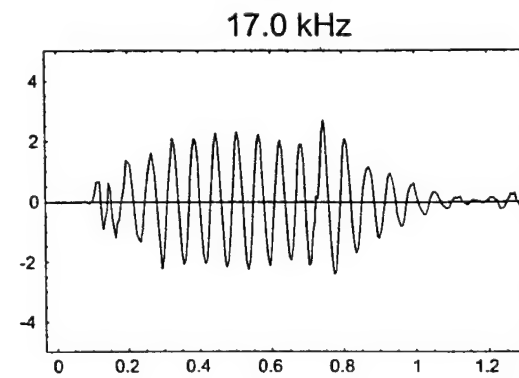
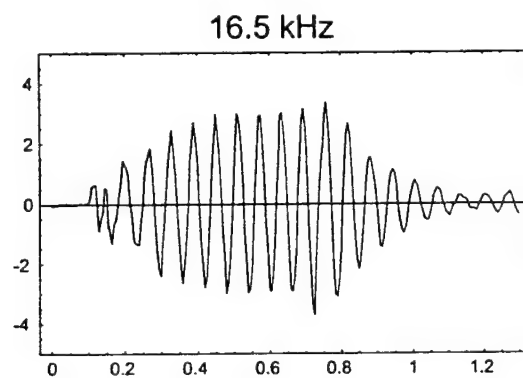
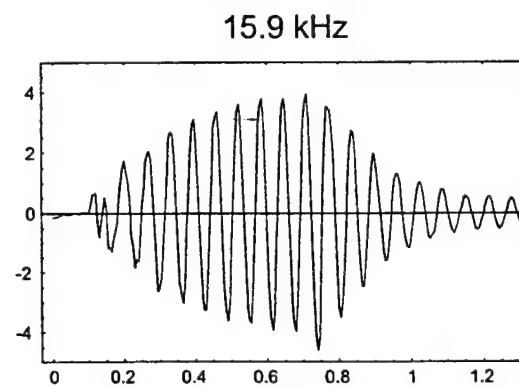
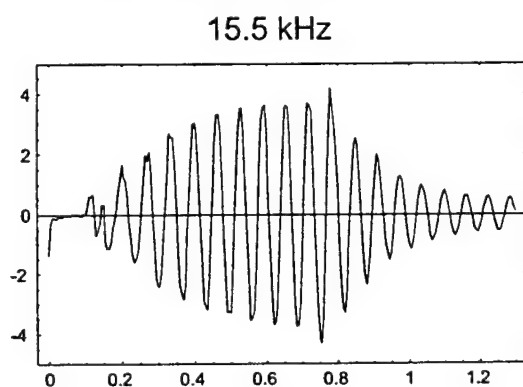
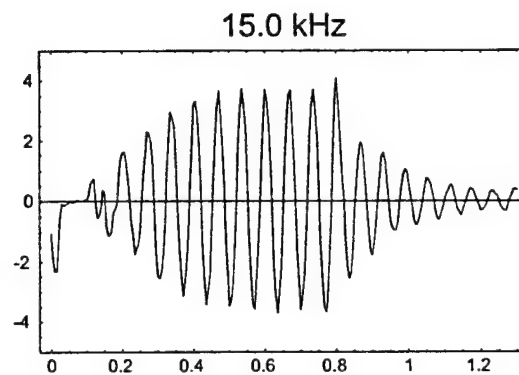
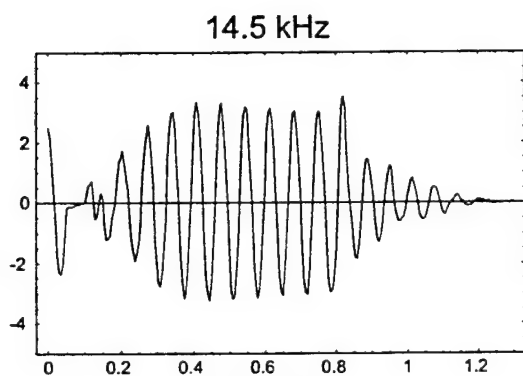
TR-4

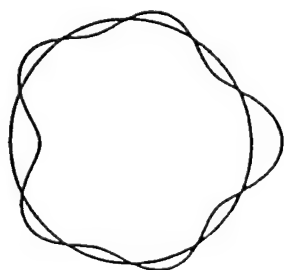
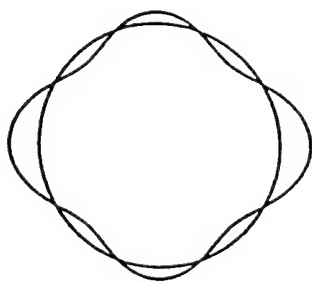
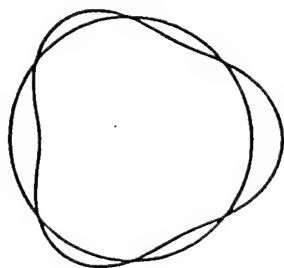
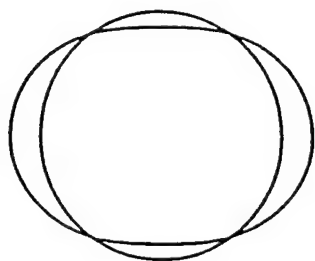
# 10-cycle tone burst backscattered from acrylic sphere.



For calculated resonance  
at 15.9 kHz

Measured:  $Q = 9.31$





TR-6

## Absorption in the approximate Rayleigh wave contribution.

$|G_R| \approx 8\pi\beta_R c / c_R$  depends on the radiation damping parameter,  $\beta_R$

In the remaining expression for  $f_{ml}$ , the damping parameter must include both radiation damping and material absorption.

The equation  $D_{v_R}(x) = 0$  is then solved with the complex longitudinal and shear wavenumbers to yield  $v'_R = \alpha'_R + i\beta'_R$  and

$$f_R = \sum_{m=0}^{\infty} f_{mR} = -G_R \frac{\exp(-\pi\beta'_R + i\eta_R)}{[1 - \exp(-2\pi\beta'_R + i2\pi x c / c_R)]}$$

Possible to approximate  $\alpha'_R$  and  $\beta'_R$  by using results for fluid-loaded PMMA half space:

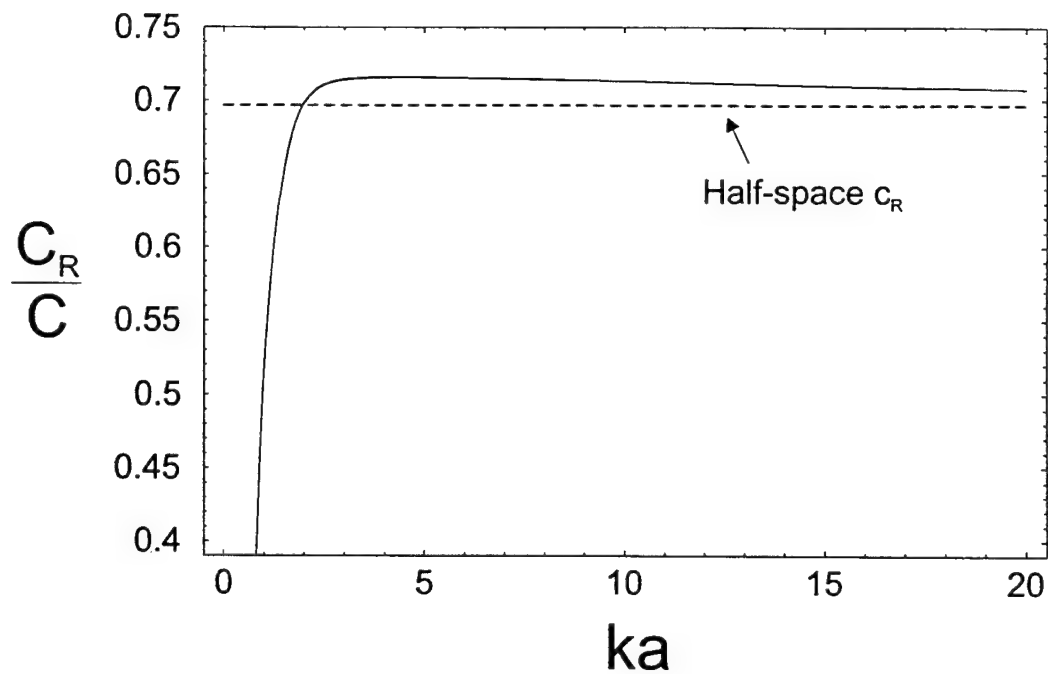
$$k'_R = k_R(1 + i\gamma_R) \Rightarrow \gamma_R = 0.00149$$

Using this value and the values calculated for the sphere without material absorption:

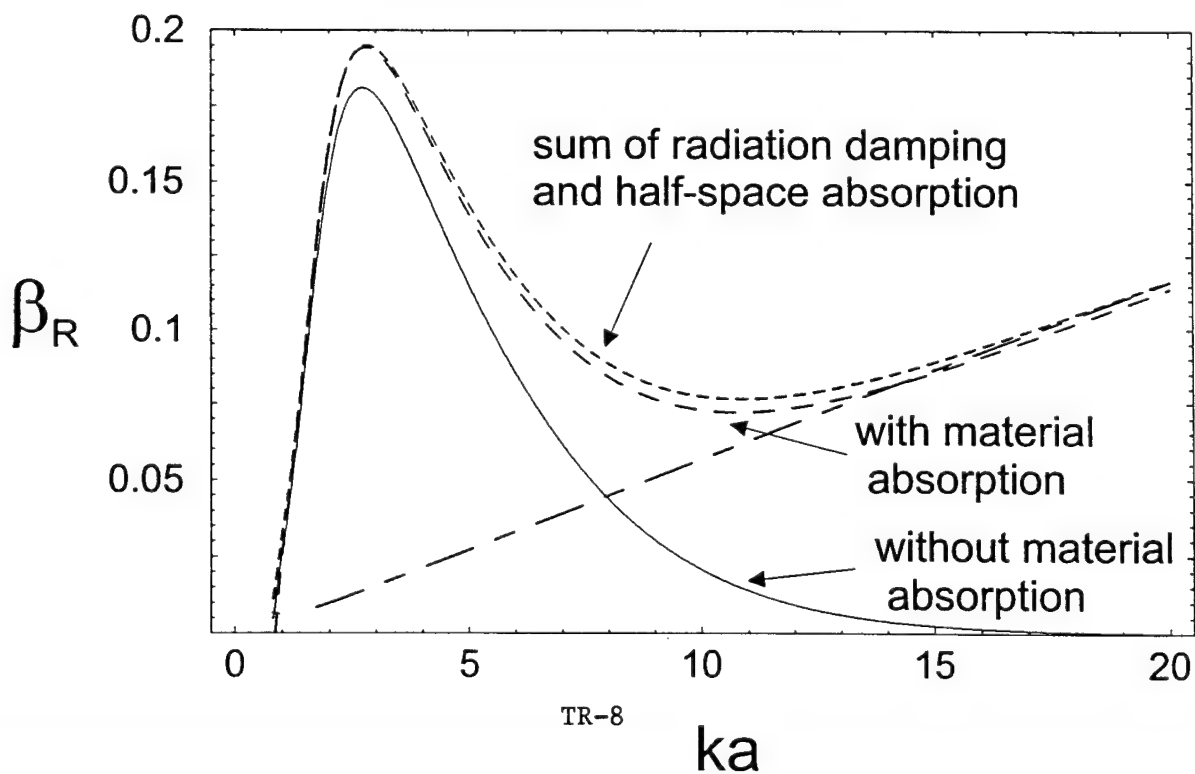
$$\begin{aligned} \alpha'_{R(approx)} &\approx \alpha_R \\ v'_{R(approx)} &\approx \alpha_R \left( 1 + i \left( \frac{\beta_R}{\alpha_R} + \gamma_R \right) \right) \\ \beta'_{R(approx)} &\approx \beta_R + \gamma_R \left( \frac{x}{c_R / c} - \frac{1}{2} \right) \end{aligned}$$



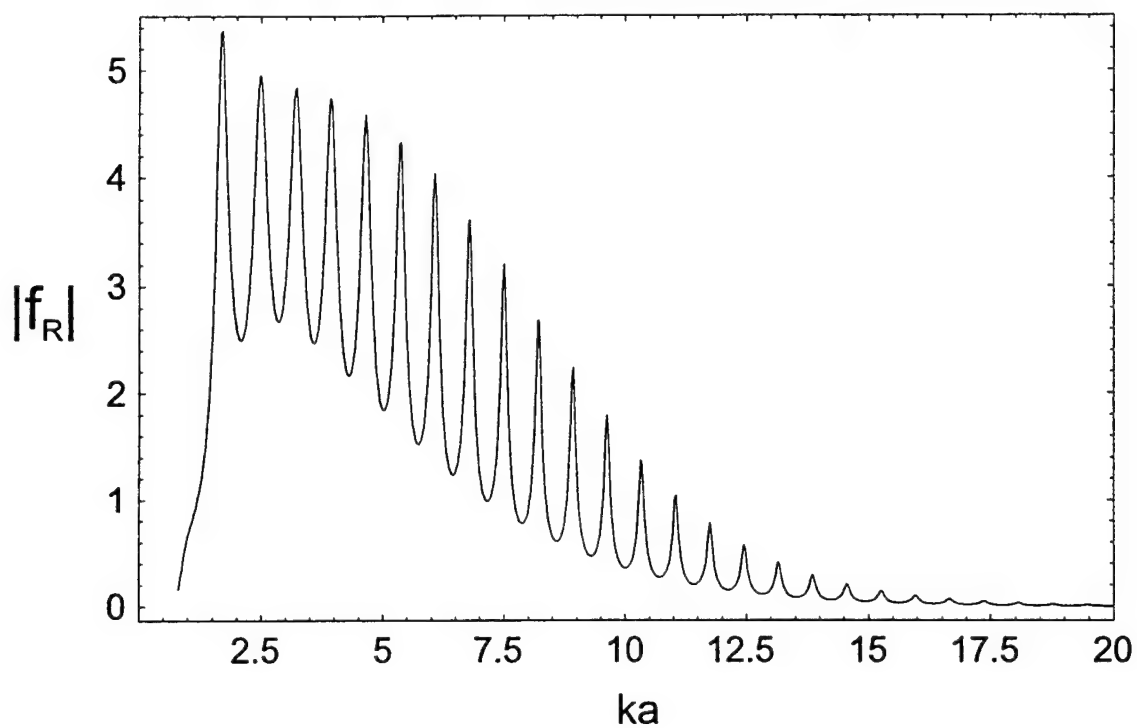
# Rayleigh wave velocity for an acrylic sphere in water.



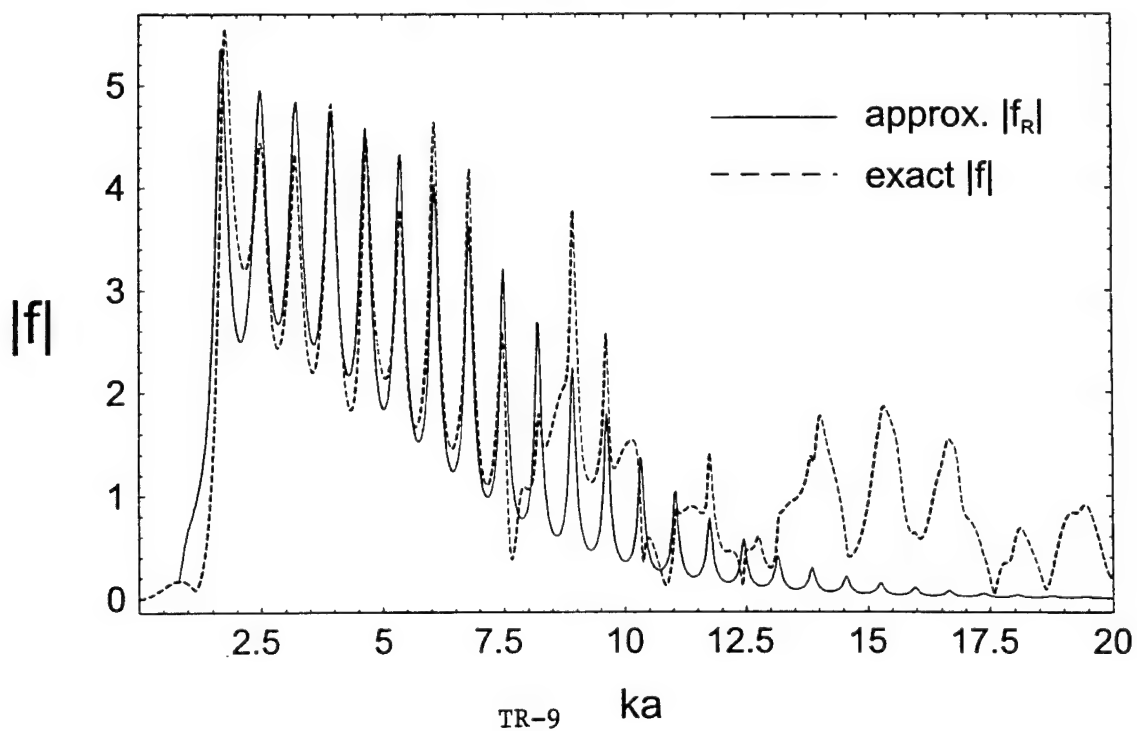
# Rayleigh wave damping for an acrylic sphere in water.



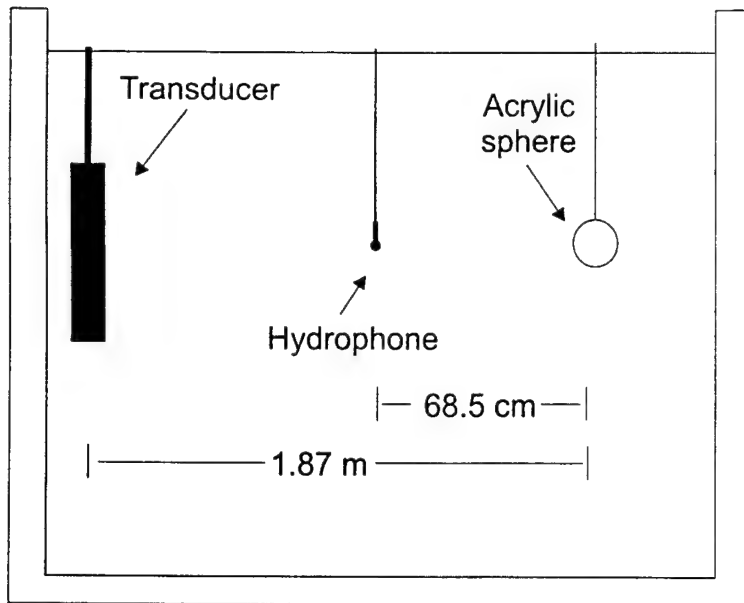
Approximate ray synthesis with material attenuation



Approximate ray synthesis and exact form function  
with material absorption



# Measurements of form function for acrylic sphere.



30-cycle tone bursts  
from a 3-1 composite  
transducer.

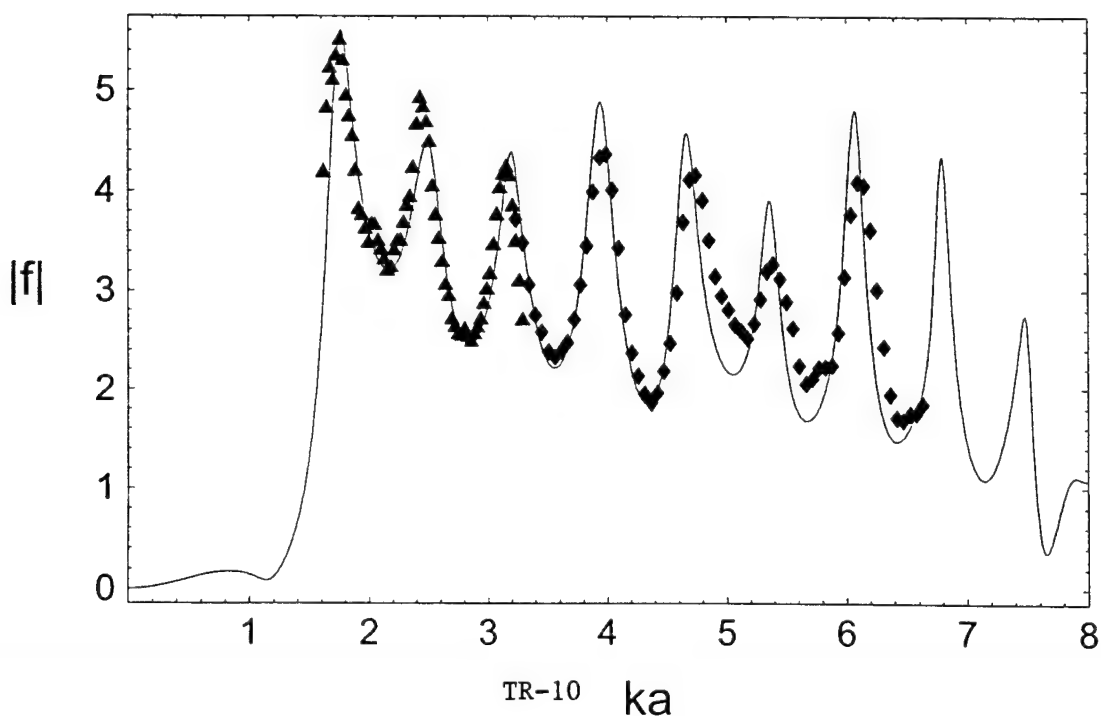
The response was  
normalized by measuring  
the signal when the  
hydrophone was placed  
at the sphere location.

Data taken for two spheres  
in the frequency range  
30 - 60 khz

▲  $a = 1.27$  cm

◆  $a = 2.54$  cm

Comparison of measured and calculated  $|f|$



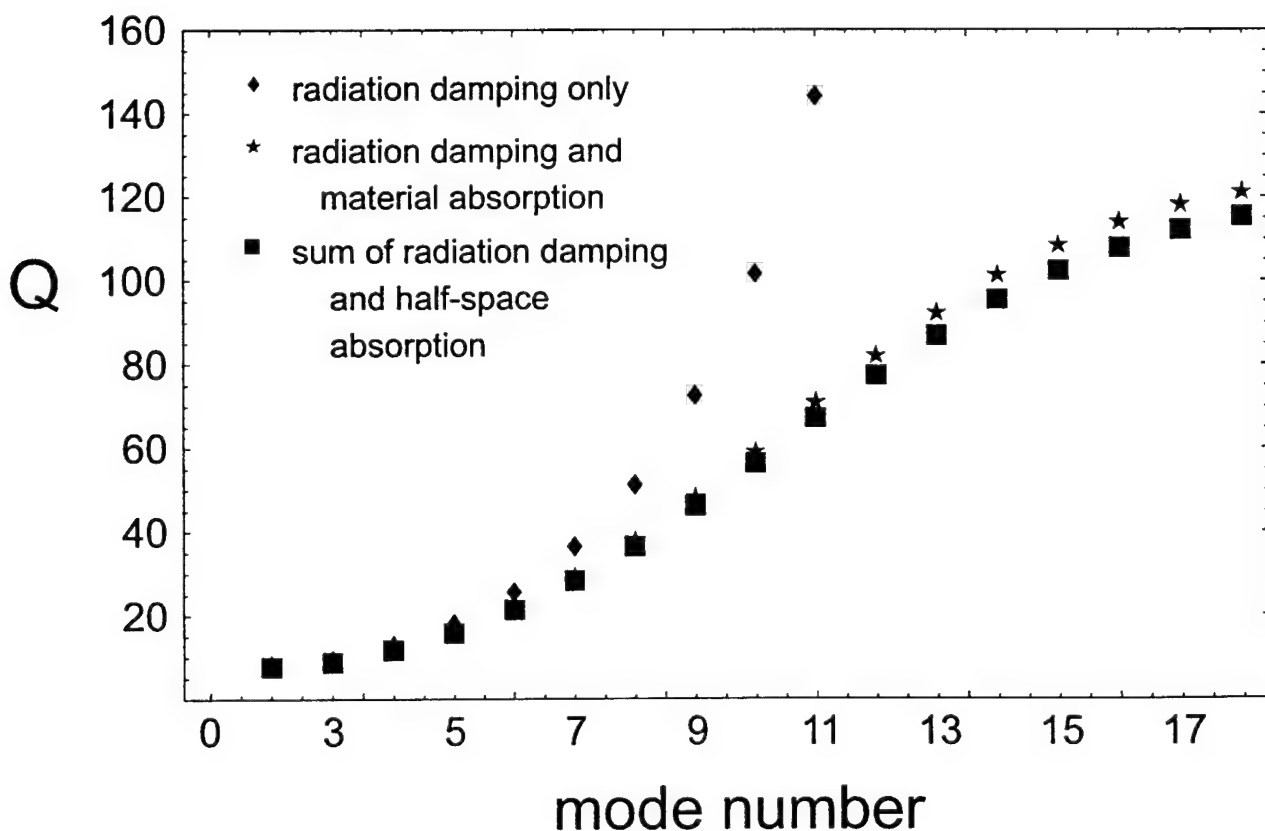
# Calculation of quality factors for Rayleigh wave resonances of a fluid-loaded PMMA sphere.

Quality factors of resonances can be calculated from decay rate of echo amplitudes (L. R. Dragonette, et. al. 1996)

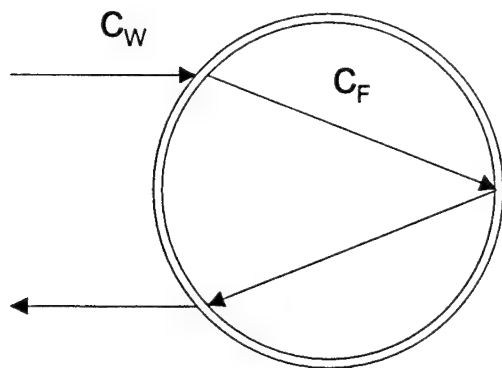
$$\text{mth echo} \rightarrow A_m \approx B \exp(-T\beta_R c_{gR}/c)$$

$$T \equiv (ct - R)/a$$

$$Q \approx ka/(2\beta_R c_{gR}/c)$$



## Target Strength for PMMA spheres.



Thin fluid-filled shells  
typically used as  
passive sonar targets.

Glory scattering

$$\underline{c_W < c_F}$$

CFC's have typically been used as filler fluid however  
alternatives are now required due ban on CFC's.

(G. Kaduchak and C. M. Loeffler, 1998)

For a PMMA sphere at largest resonance

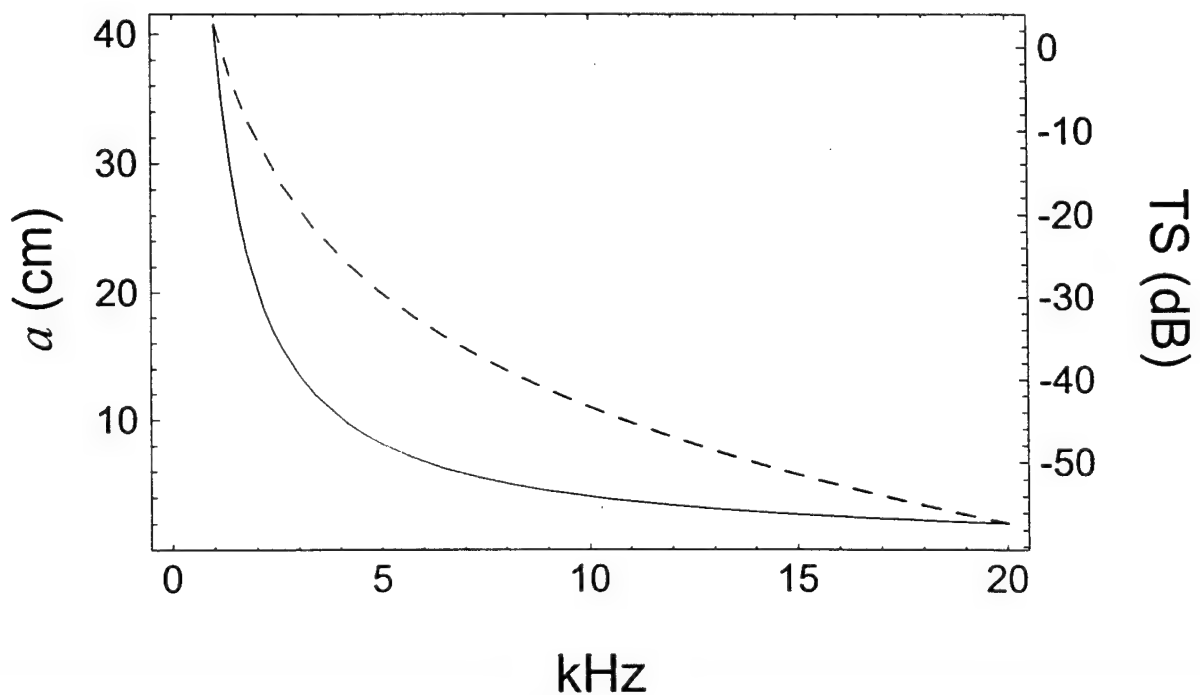
$$(ka = 1.73)$$

$$TS = 20 \log \left( |f| \frac{1.73c}{4\pi R_{ref} F} \right)$$

$$a(m) = 407/F(Hz)$$

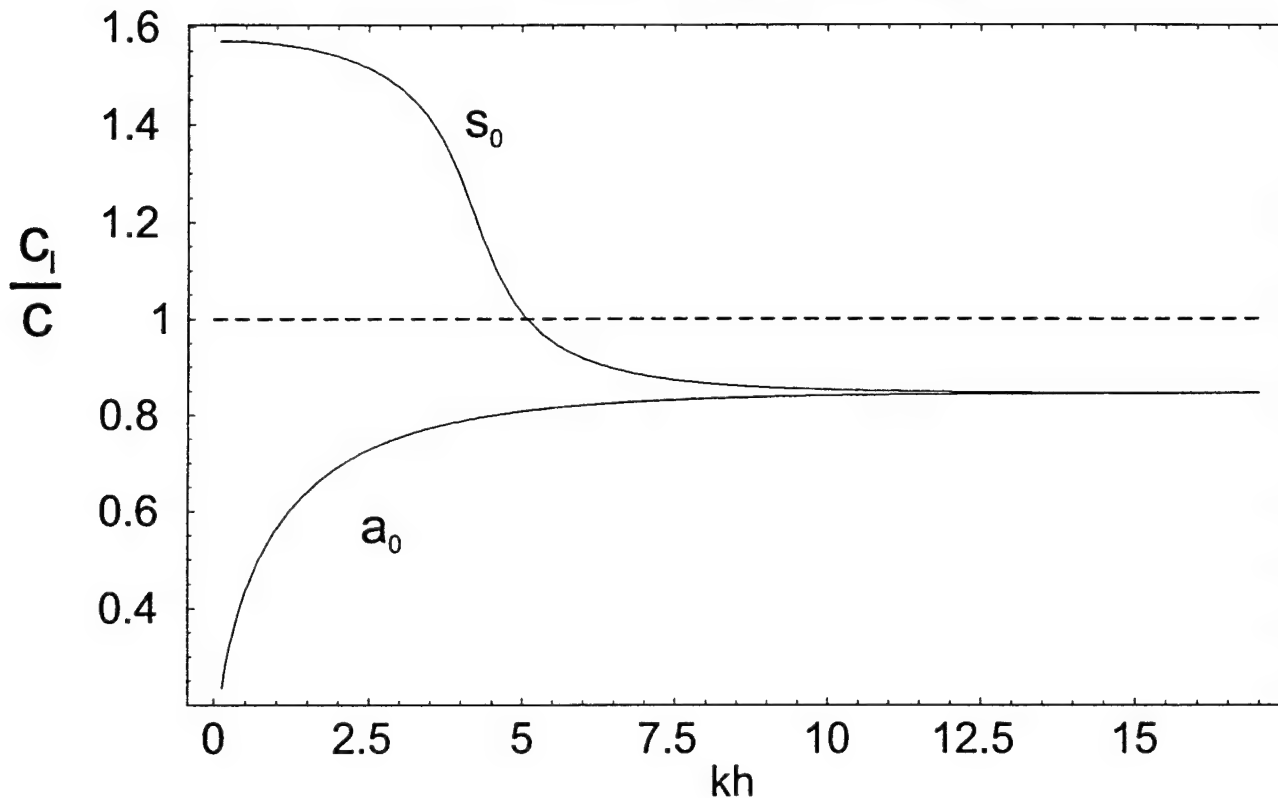
$$R_{ref} = 1 \text{ m}$$

$$|f| = 5.63$$



# Lamb waves on an acrylic plate.

Phase velocities for 2mm thick acrylic plate  
without fluid-loading



Characteristic equation for symmetric waves on a plate immersed in water.

$$\text{Vacuum term } \left[ (\xi^2 + \beta^2) \cosh(\alpha h/2) \sinh(\beta h/2) - 4\xi^2 \alpha \beta \cosh(\beta h/2) \sinh(\alpha h/2) \right] +$$

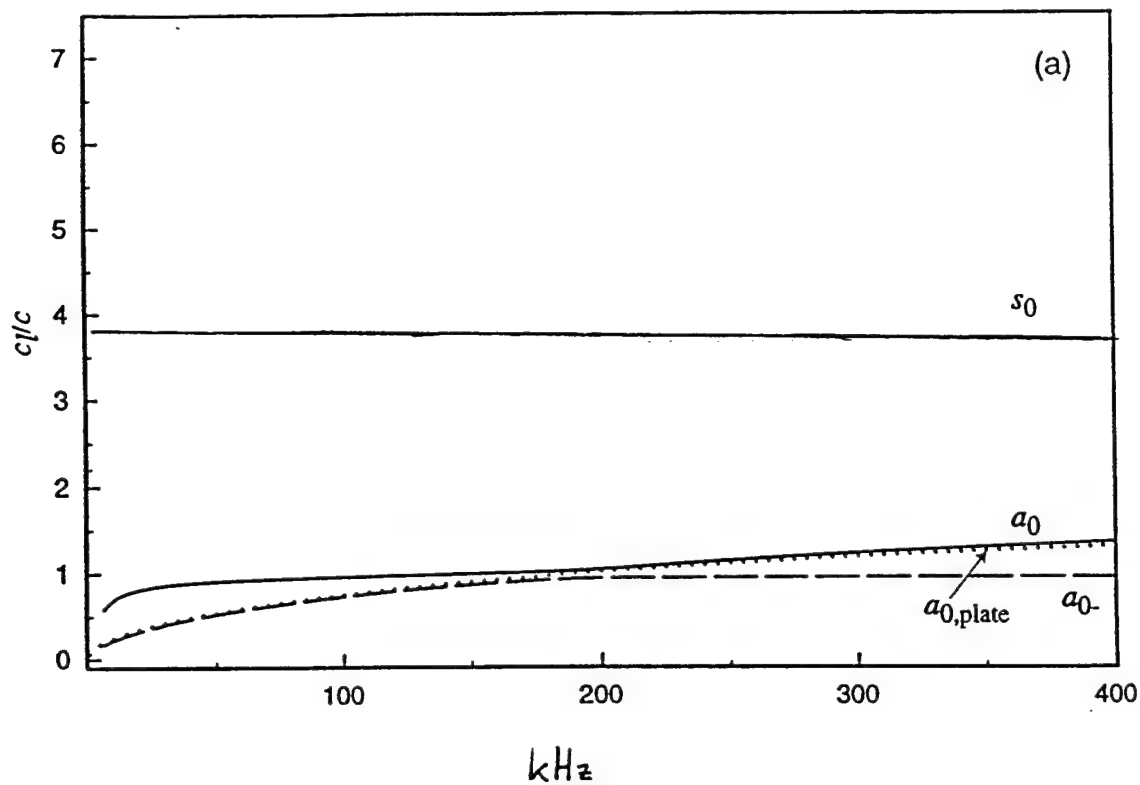
$$\text{Fluid - loading } \left[ \frac{\rho c^2}{\rho_e c_s^2} \frac{\eta^2 - \xi^2}{\eta} \alpha (\beta^2 - \xi^2) \sinh(\beta h/2) \sinh(\alpha h/2) \right] = 0$$

Two properties of acrylic which significantly affect  
the fluid-loaded Lamb wave spectrum:

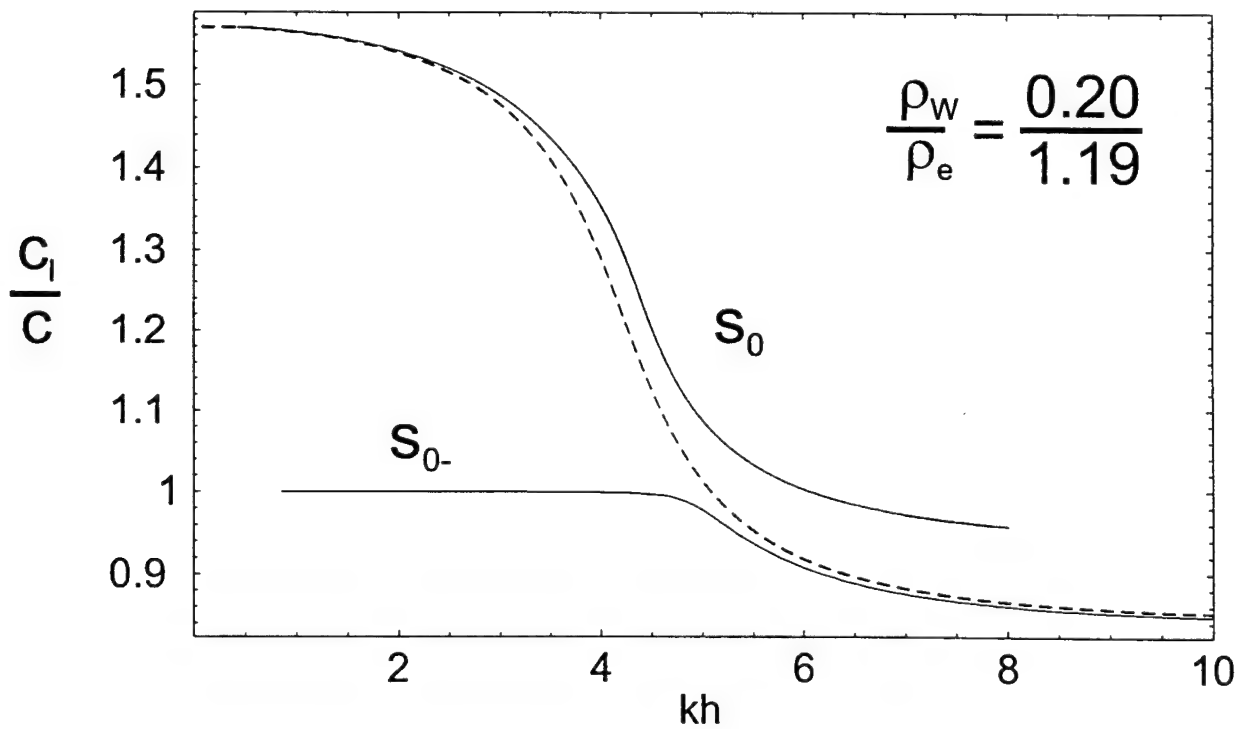
$$1) \quad c_R < c_W$$

$$2) \quad \rho_E \sim \rho_W$$

Phase velocities for 1.44 mm thick SS304 plate



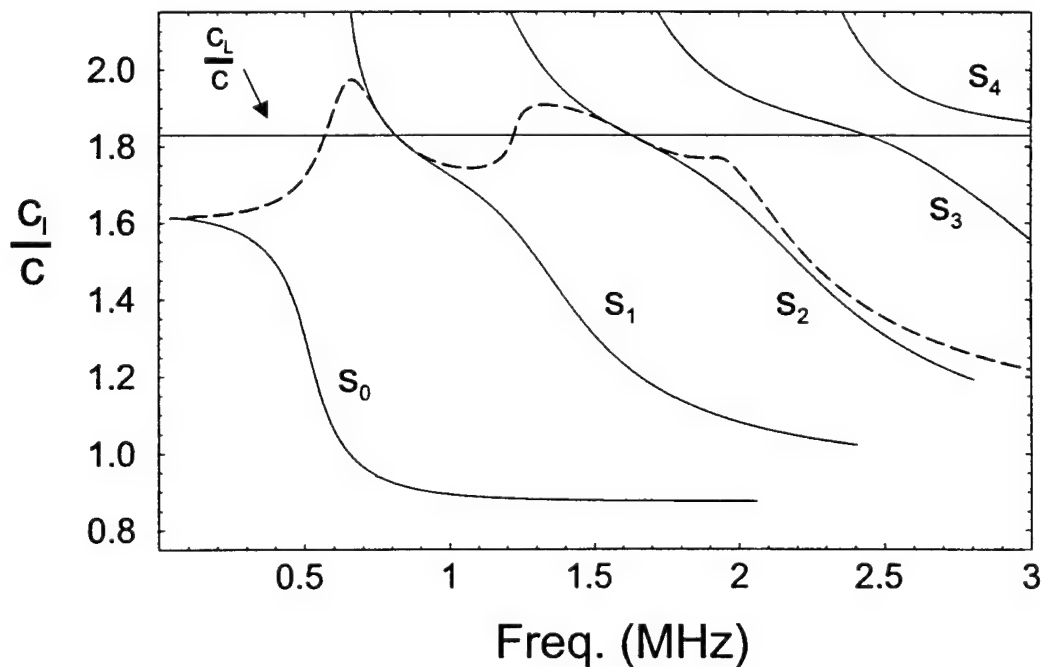
Phase velocities for fluid-loaded acrylic plate



For Acrylic:  $\frac{\rho}{\rho_e} = \frac{1.00}{1.19} = 0.84$

For Stainless Steel:  $\frac{\rho}{\rho_e} = \frac{1.00}{7.57} = 0.13$

For large density ratios, fluid-loading is no longer a small perturbation to Lamb wave spectrum.



As  $\rho_w/\rho_e$  increases, the Lamb wave spectrum transitions to a plate with mixed boundary conditions.

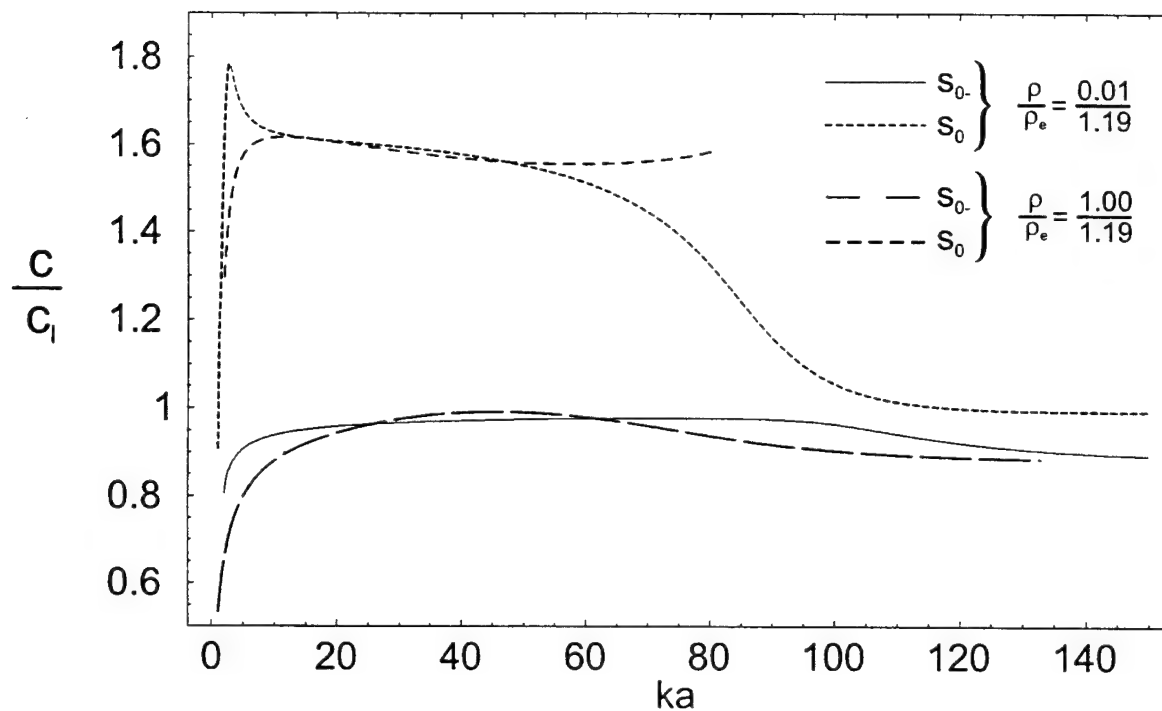
S.I. Rokhlin et.al., J. Acoust. Am. **85**(3), March 1989

A. Freedman, J. Acoust. Am. **99**(6), June 1996

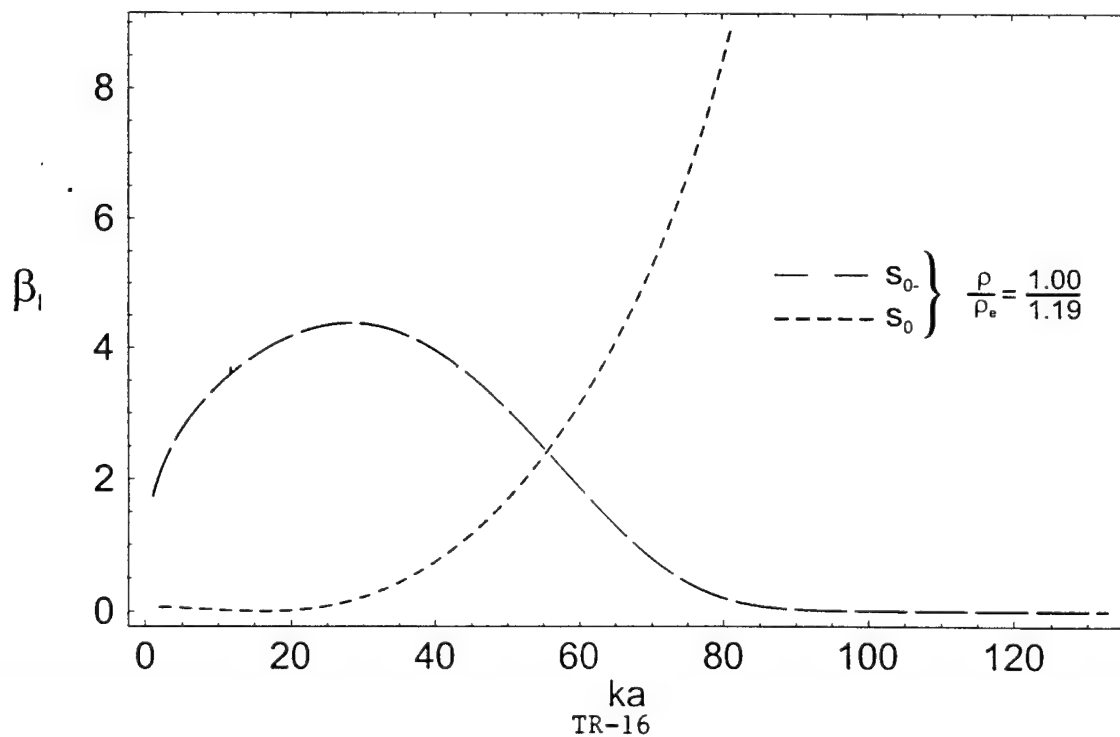


# Surface waves on a 5% acrylic shell

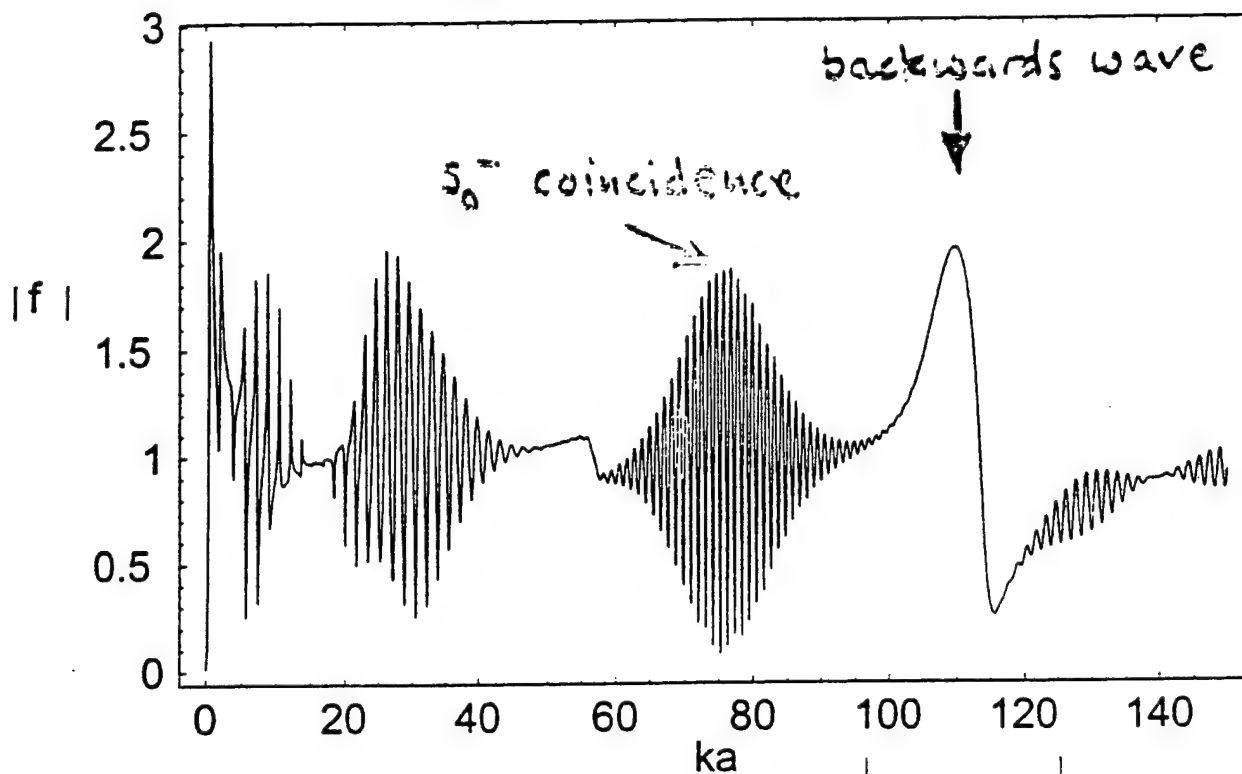
Phase velocities for an empty fluid-loaded acrylic shell



Damping parameters for an empty fluid-loaded acrylic shell



# Backscattering form function for 5% acrylic shell with material absorption.

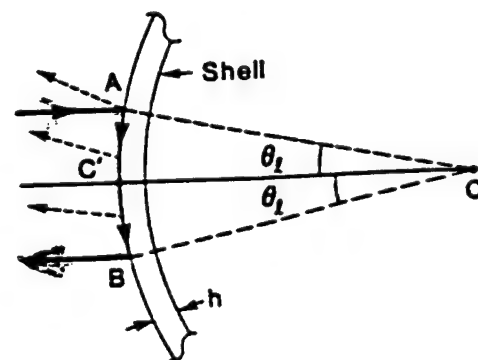
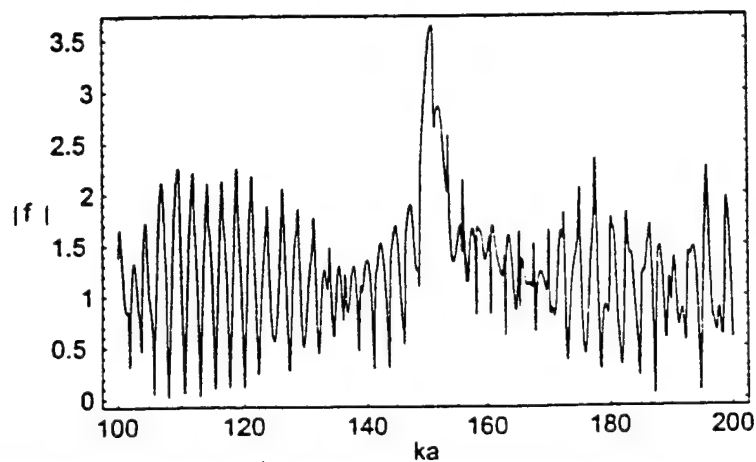


Possible backwards wave enhancement

This type of enhancement was previously observed in SS304 shells.

G. Kaduchak et. al., J. Acoust. Am. **96** (6), 1994

$|f|$  for 7.5% SS304 shell



TR-17

## Summary

- PMMA spheres were observed to give a backscattering enhancement due to tunneling to a subsonic Rayleigh wave.
- We have modeled the resulting Rayleigh wave resonance scattering by extending the previous ray-tunneling models of subsonic  $a_0$  waves on steel shells [Zhang, Sun, & Marston JASA (1992)].
- Radiation damping and intrinsic material absorption are similar in magnitude and are additive.

### Computational study for shells:

- Because the Rayleigh wave velocity is subsonic, there are two branches associated with the lowest symmetric mode, the  $s_0$  and  $s_{0-}$  waves which give rise to associated backscattering enhancements.
- Although the intrinsic material absorption may suppress enhancements associated with waves which travel around the shell, it may be possible to detect enhancements associated with a backwards wave on the shell.

# **Remote Ultrasonic Classification of Fluids using Acoustic Resonance Characteristics of the Container**

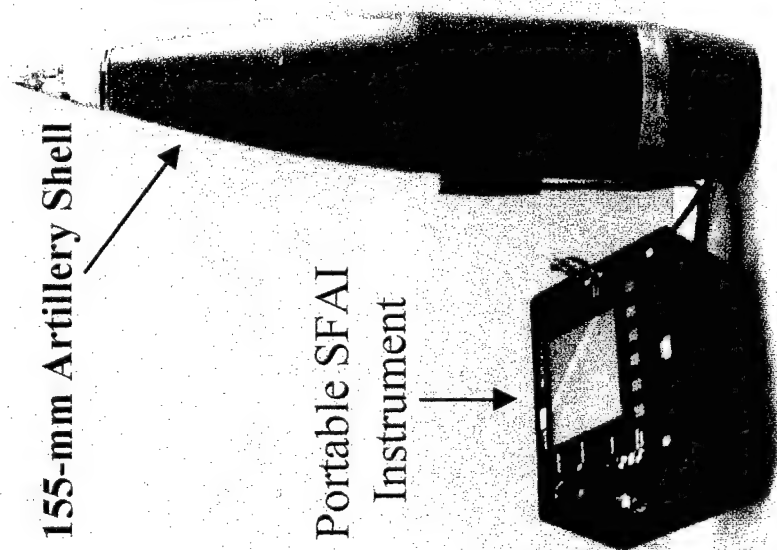
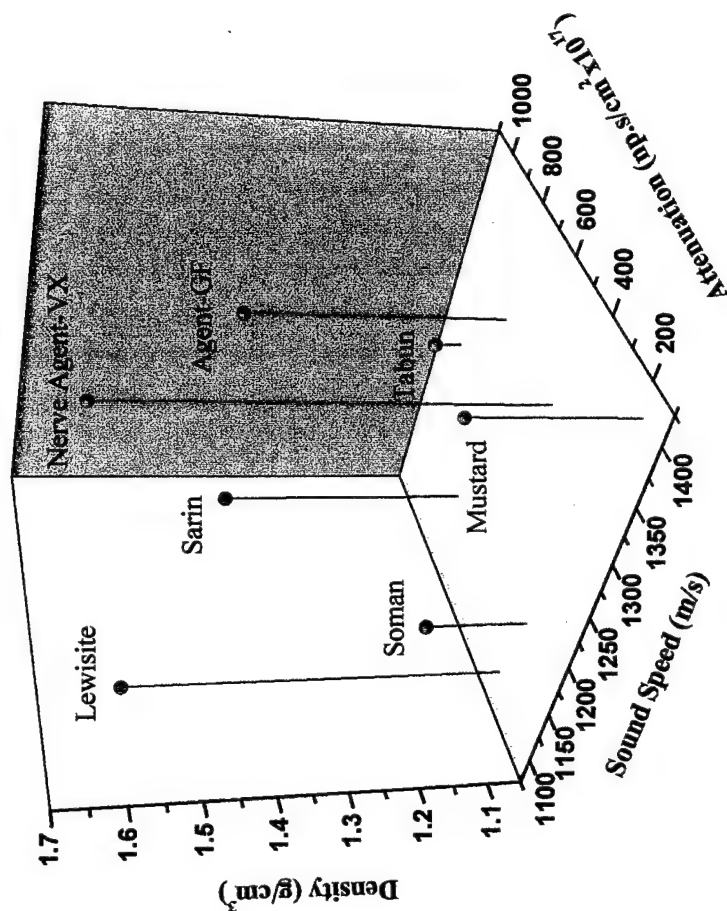
TR-1

**Dipen N. Sinha, Gregory Kaduchak and  
Michael J. Keleher**

**Los Alamos National Laboratory  
Electronic and Electrochemical Materials and Devices Group**

# Noninvasive Identification of CW Agents

SFAI Measurements of CW Agent  
Physical Properties



# Remote Classification of Fluids

## **Why needed:**

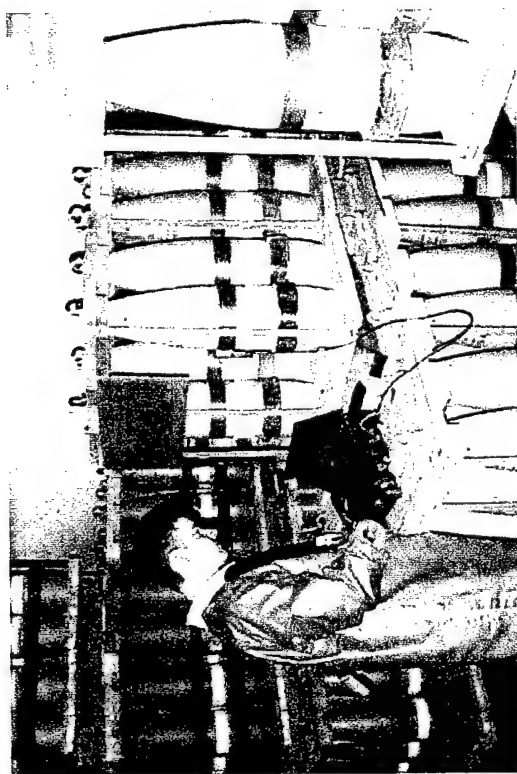
Determining information on contents of containers in hazardous situation (leaking toxic vapors, radioactivity, etc.,) or inaccessible areas.

## **Where:**

- Waste storage facilities (Rocky Flats, WIPP)
- Law enforcement
- Chemical weapons compliance monitoring
- Chemical and other industry
- Firefighters, HAZMAT

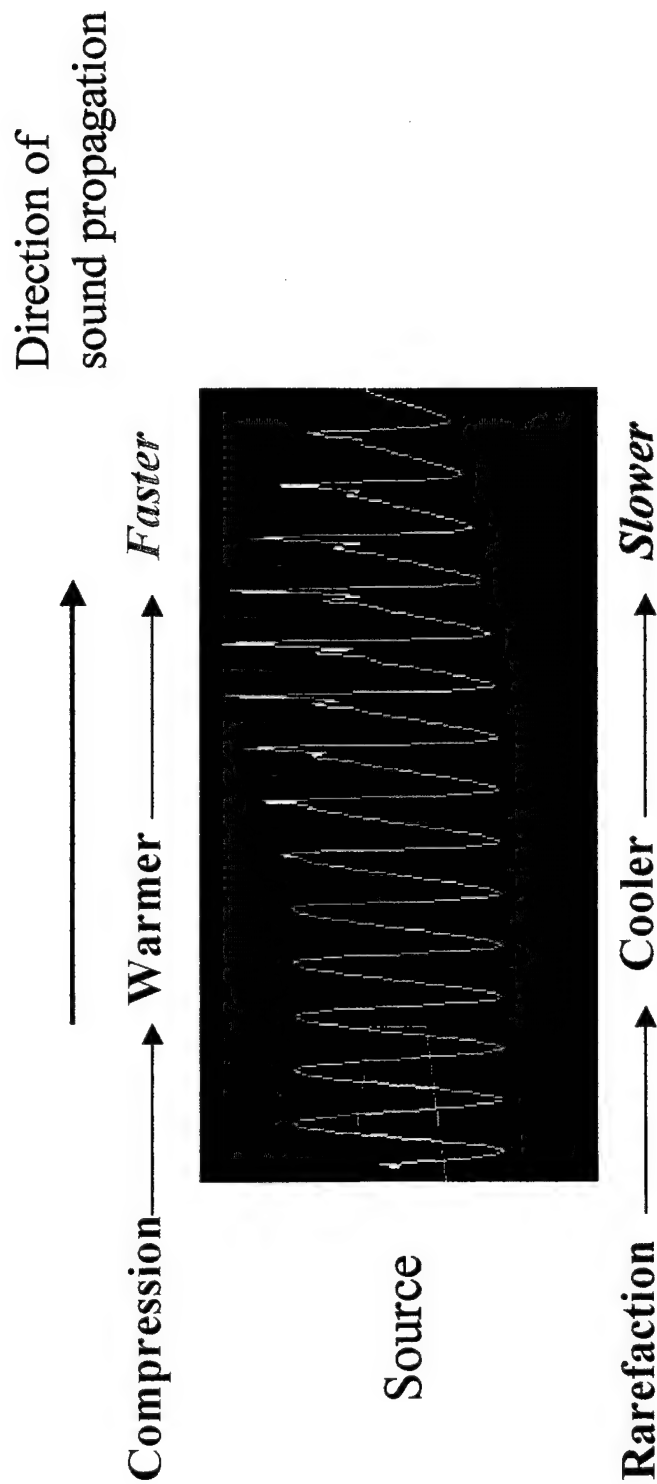
## Noninvasive Identification of Chemical Agents using Ultrasonic Interferometry

- Identification is based on physical properties of chemicals
- Requires physical contact between transducer and item being tested
- Need a technique that does not require operator to be exposed to hazardous situation (e.g., chemical vapor, radioactive, etc.,)



TR-4

# Nonlinear Sound Propagation in Air



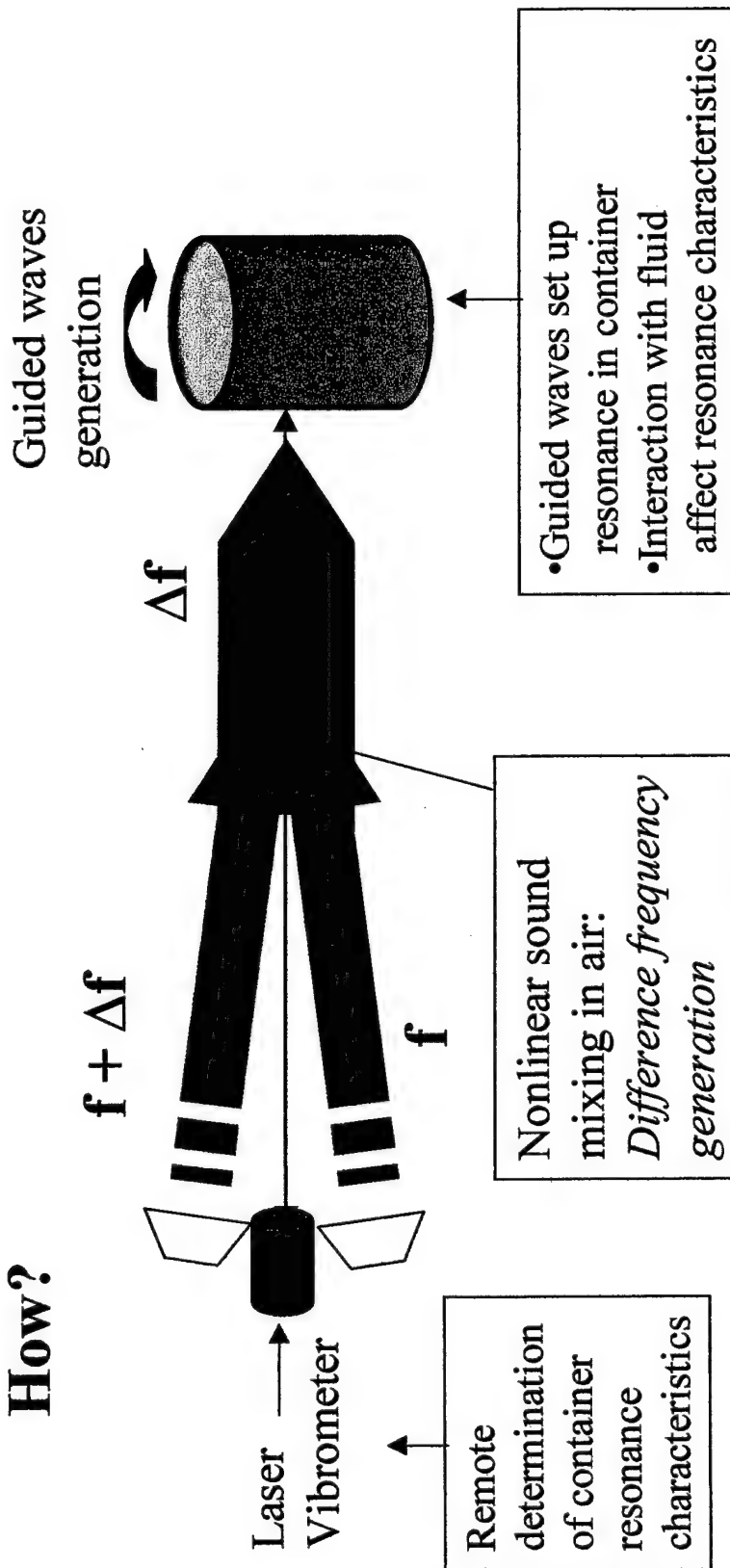
**For high intensity sound waves, the waves start to distort as they travel, generating harmonics, 'self-demodulation', rectification, and shocks.**

(Figure: Courtesy of F. Joseph Pompei, MIT Media Lab)



# Remote Excitation of Vibration

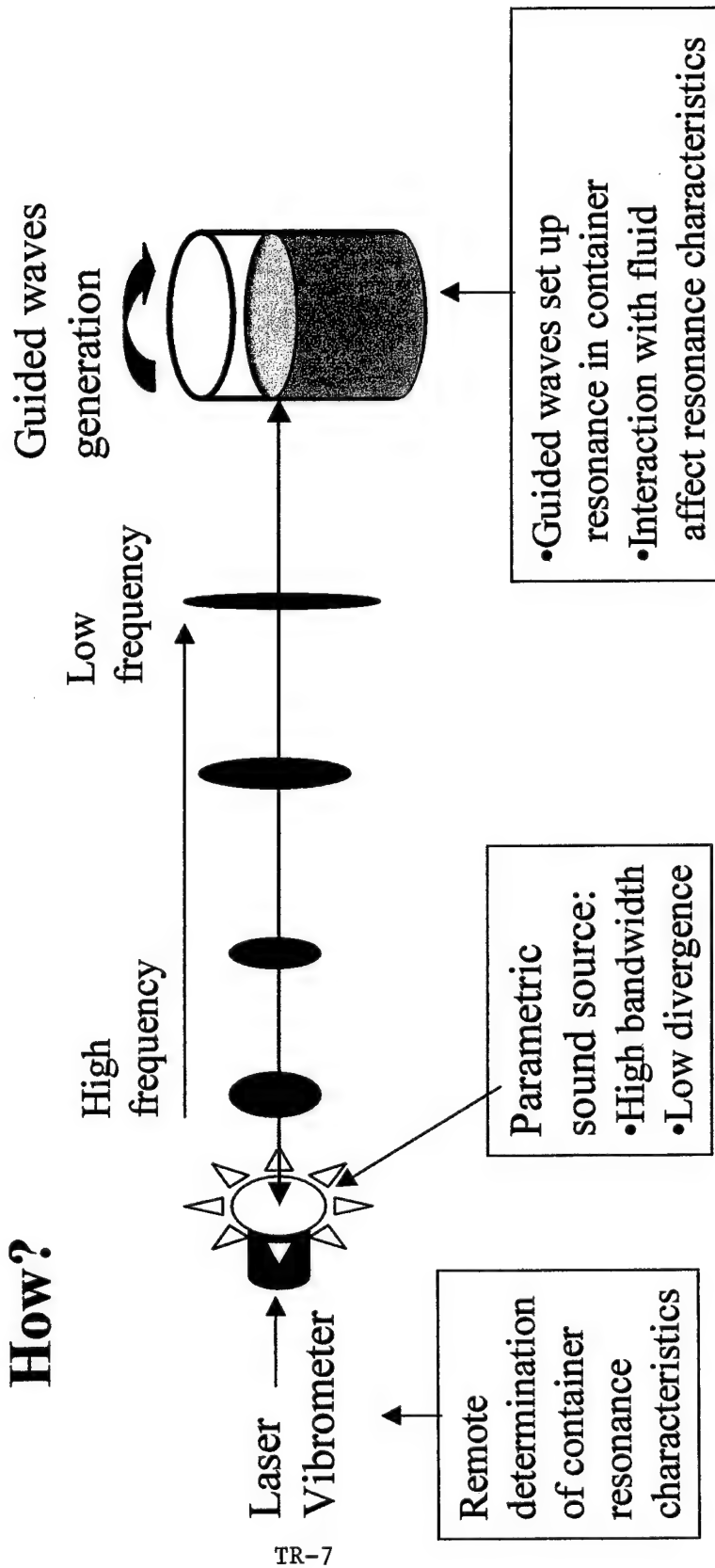
How?



TR-6

# Remote Classification of Fluids

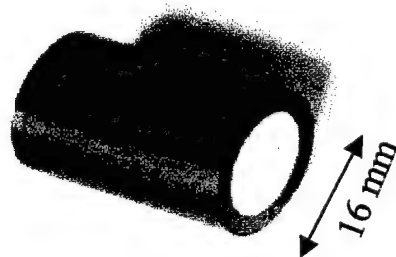
How?



# Parametric Array

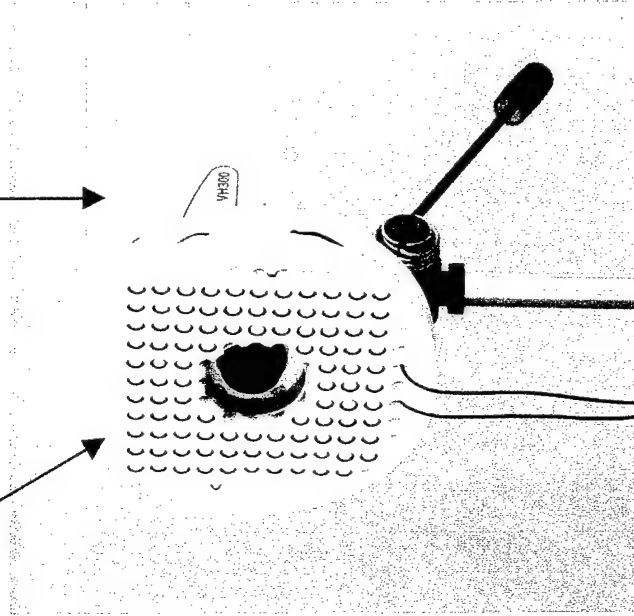
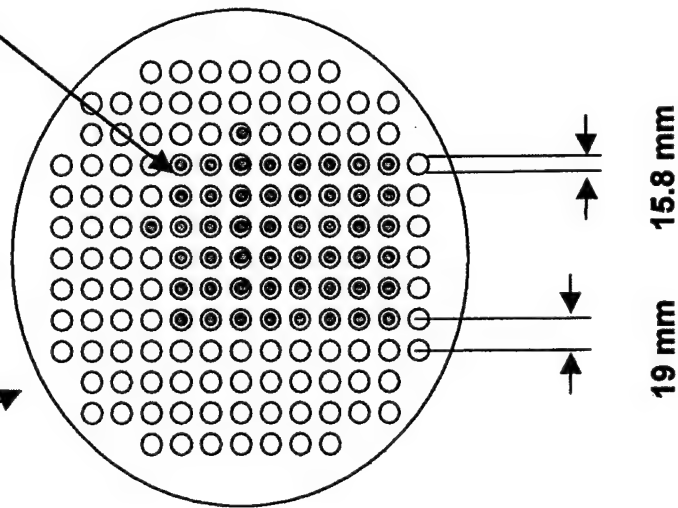
Delrin Housing  
99 Air-Coupled Transducers  
(Resonance Frequency = 200 kHz)

Laser  
vibrometer

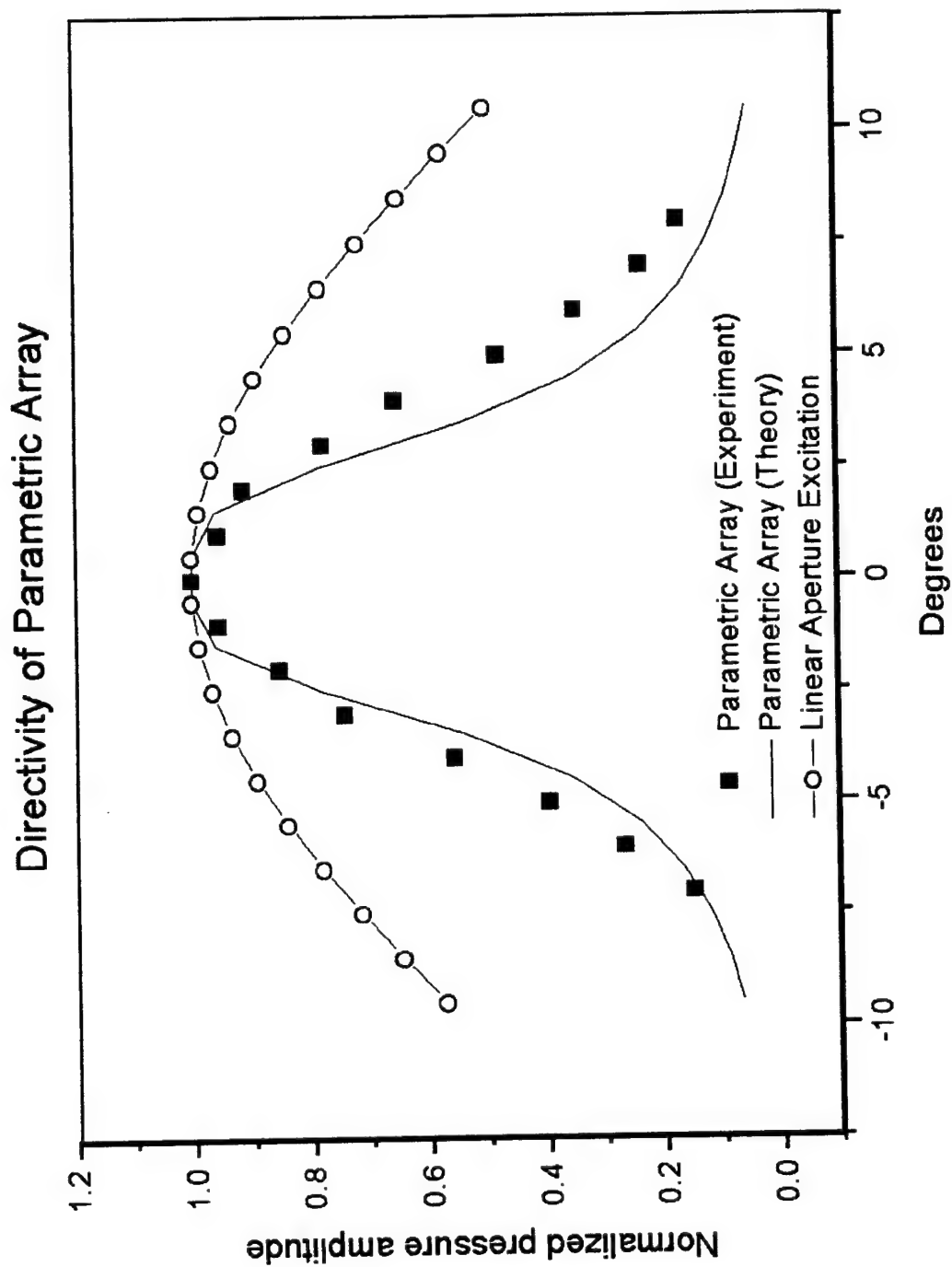


AIRMAR  
AT200

$Q \sim 25$   
Beamwidth  $\sim 7^\circ$



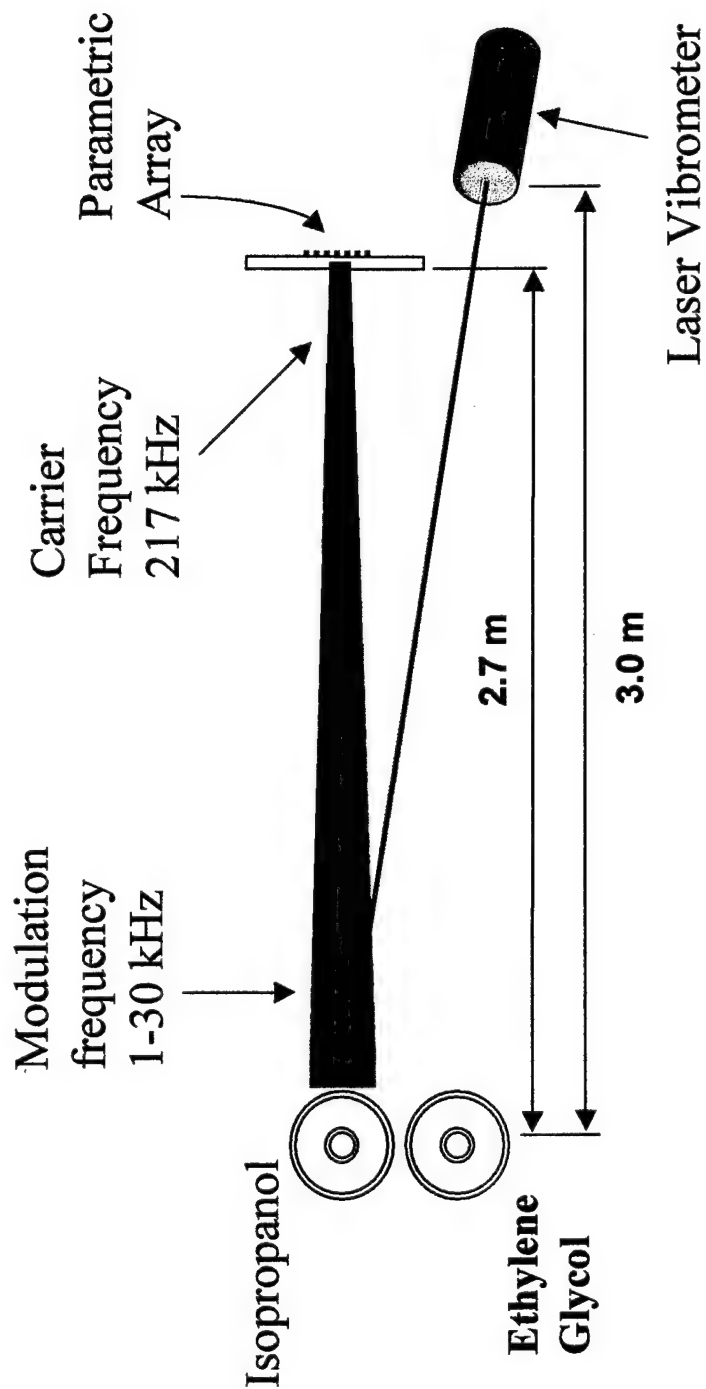
TR-8



TR-9

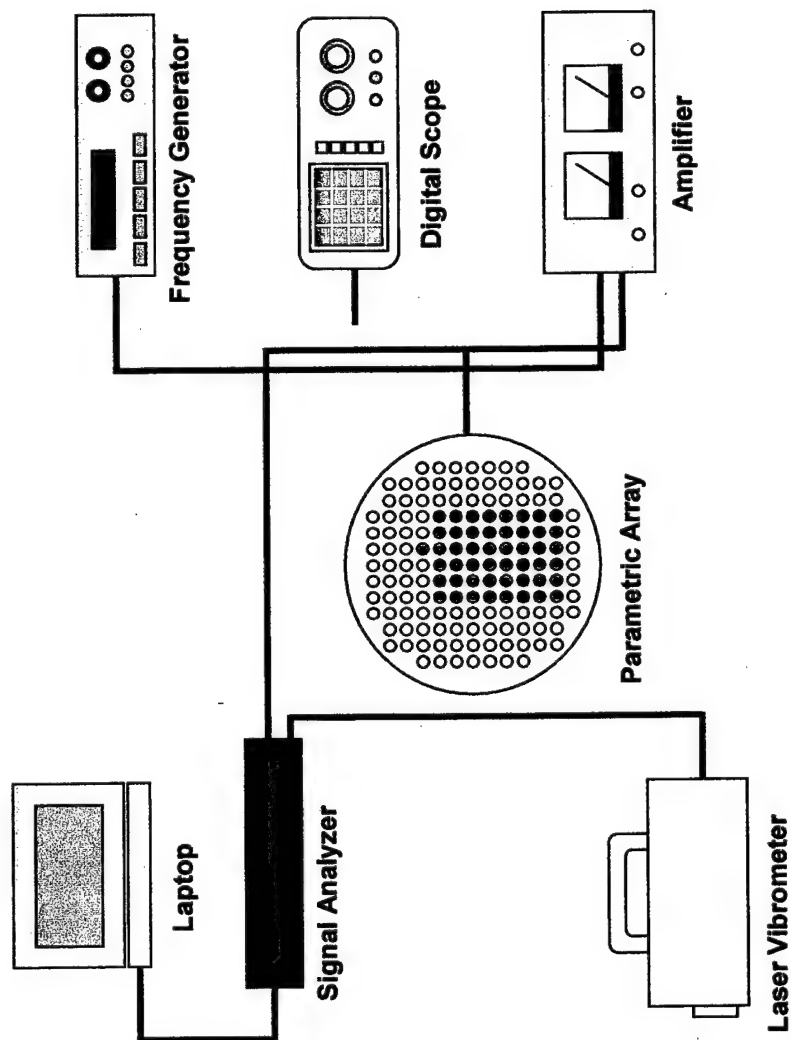
# Experimental Configuration

## AM Modulated Beam



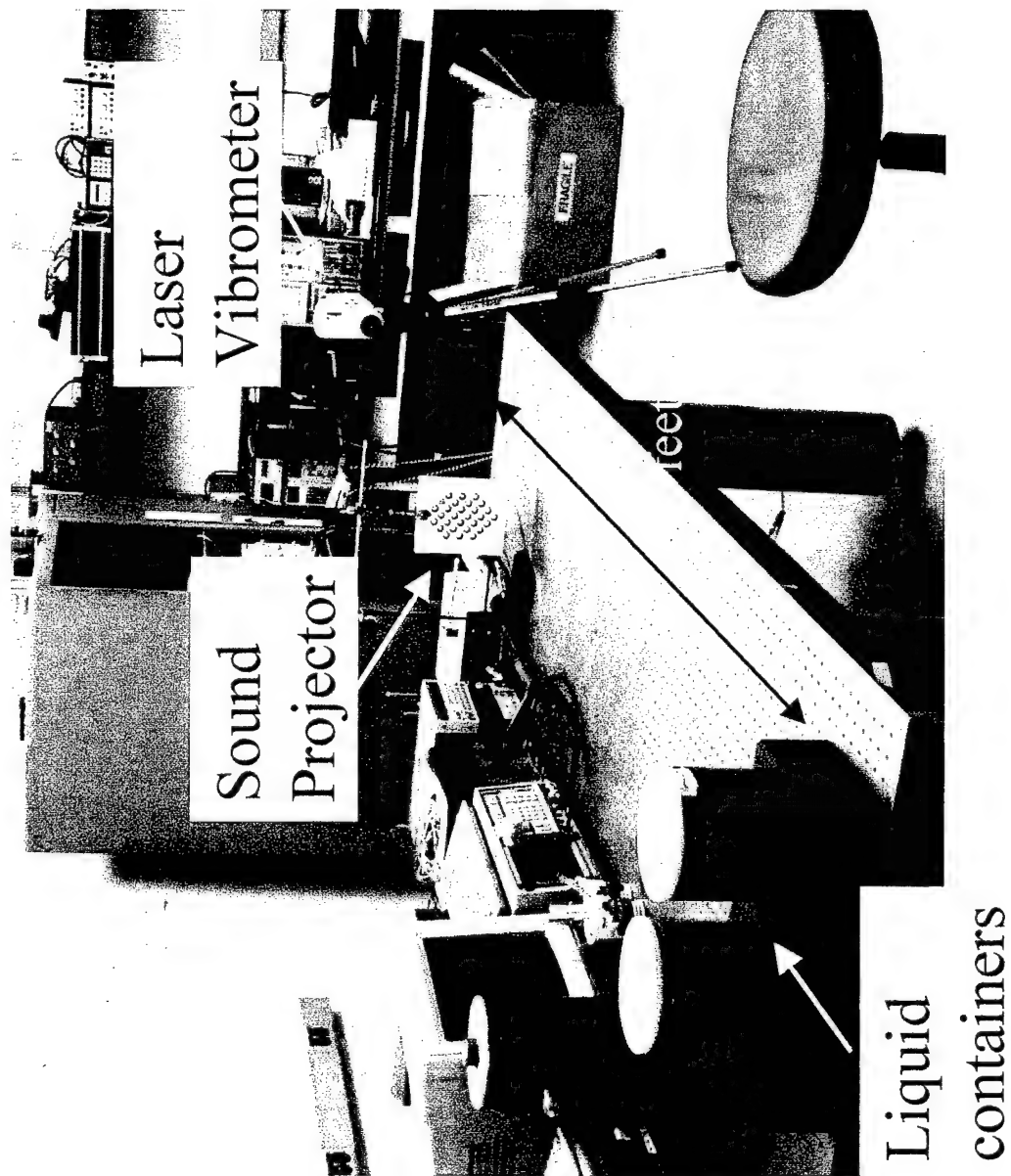
TR-10

# Instrument Setup



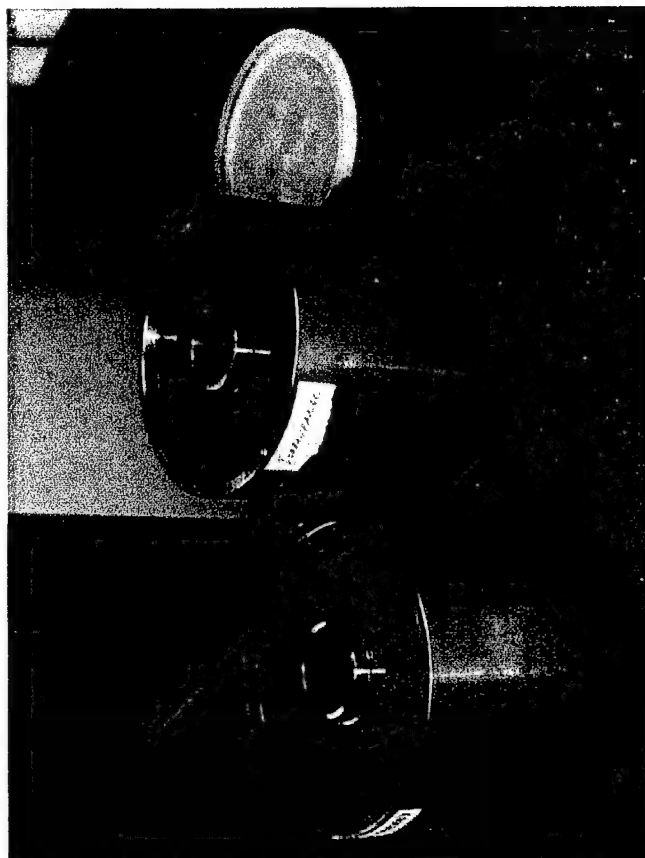
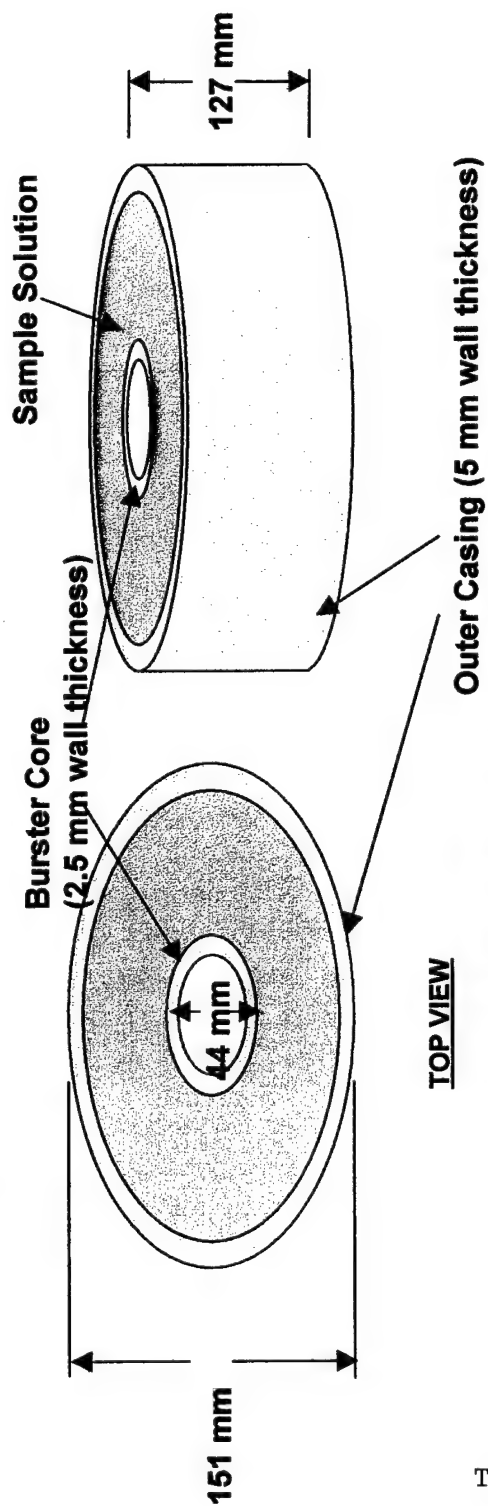
TR-11

# Ultrasonic Remote Assay of Munitions



TR-12

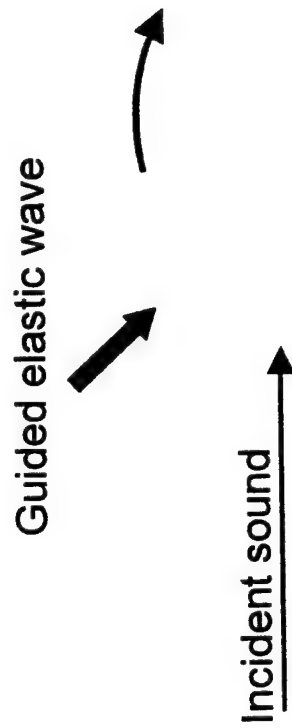
# Liquid-filled Steel Containers



TR-13



# Guided waves



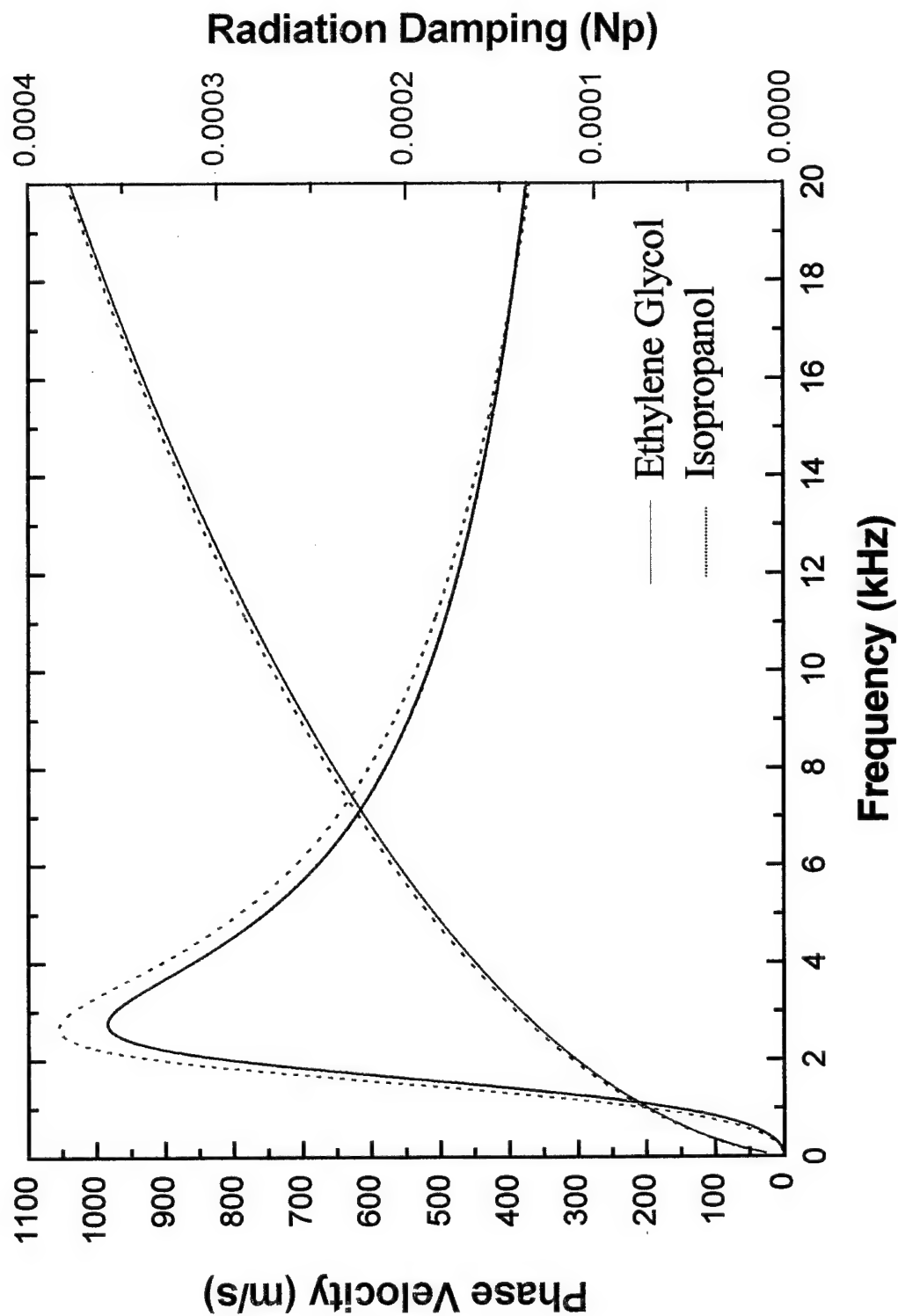
Interior fluid

Resonance occurs at  $f_{\text{res}} = n \lambda_{\text{gw}}$

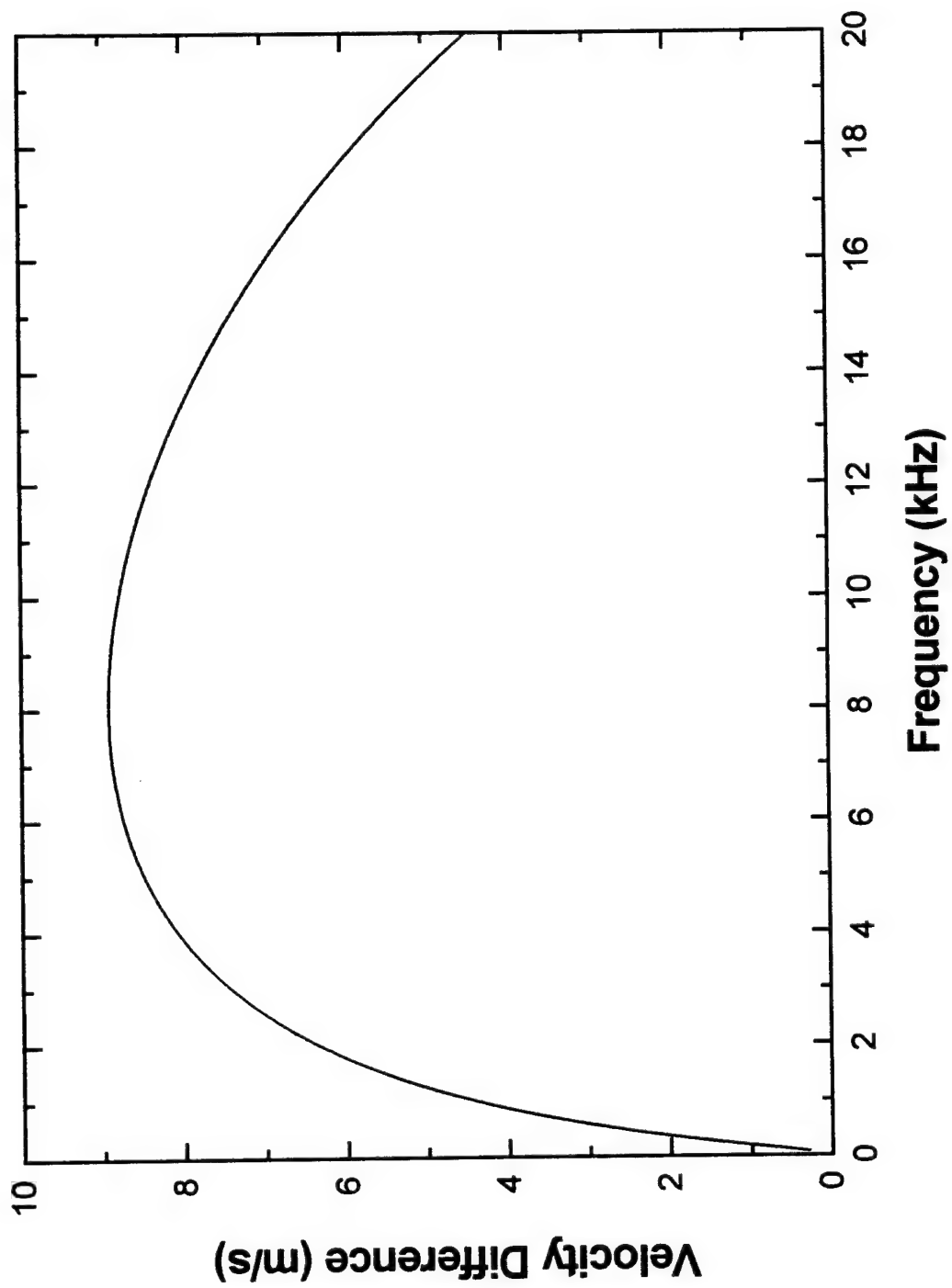
Guided wave properties may be used to determine:

- interior fluid sound speed
- interior fluid density
- interior fluid attenuation
- shell diameter
- shell thickness or material composition

# Calculated Phase Velocity and Radiation Damping for $a_0$ Lamb Wave

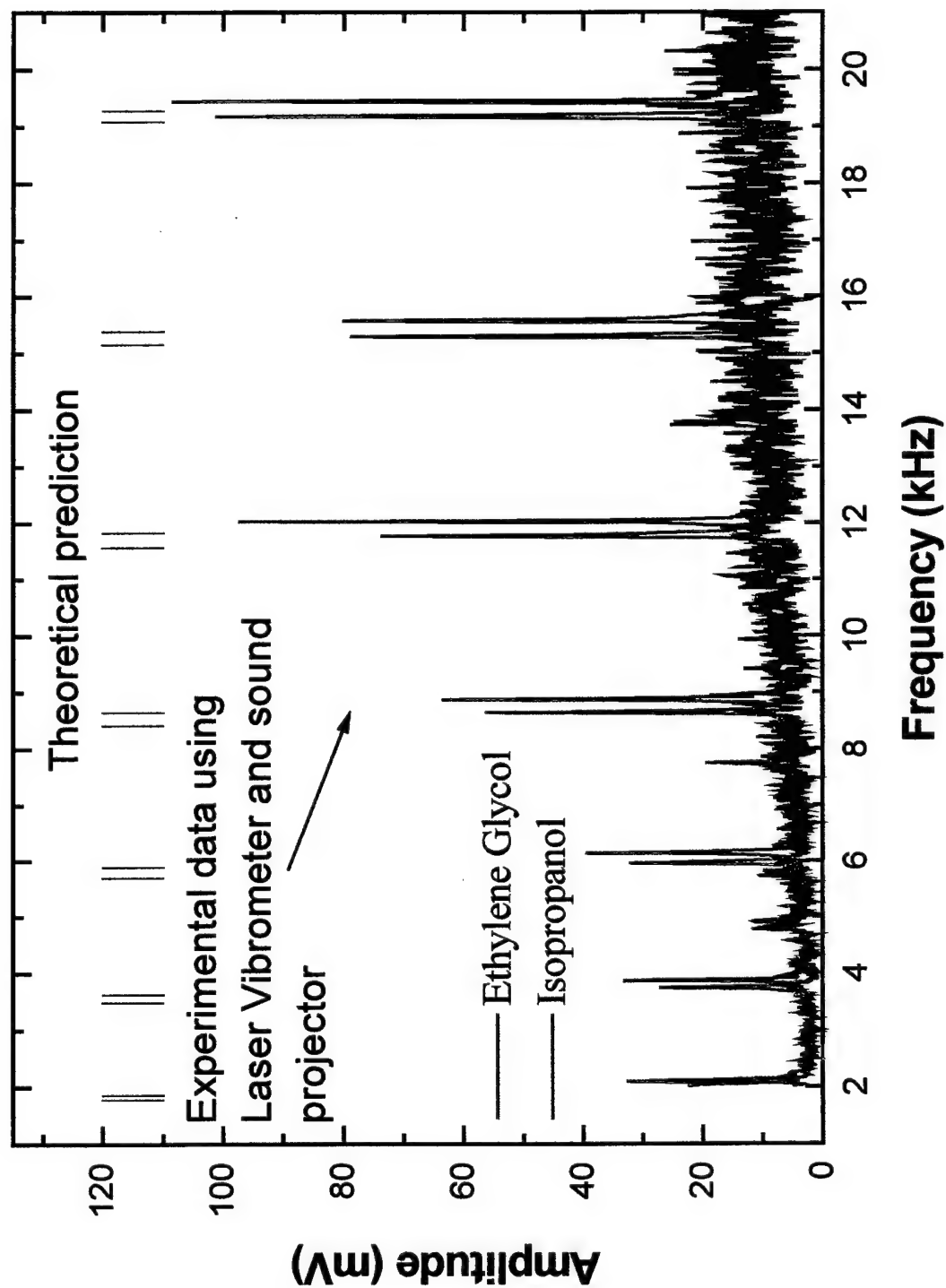


# Calculated Phase Velocity Difference

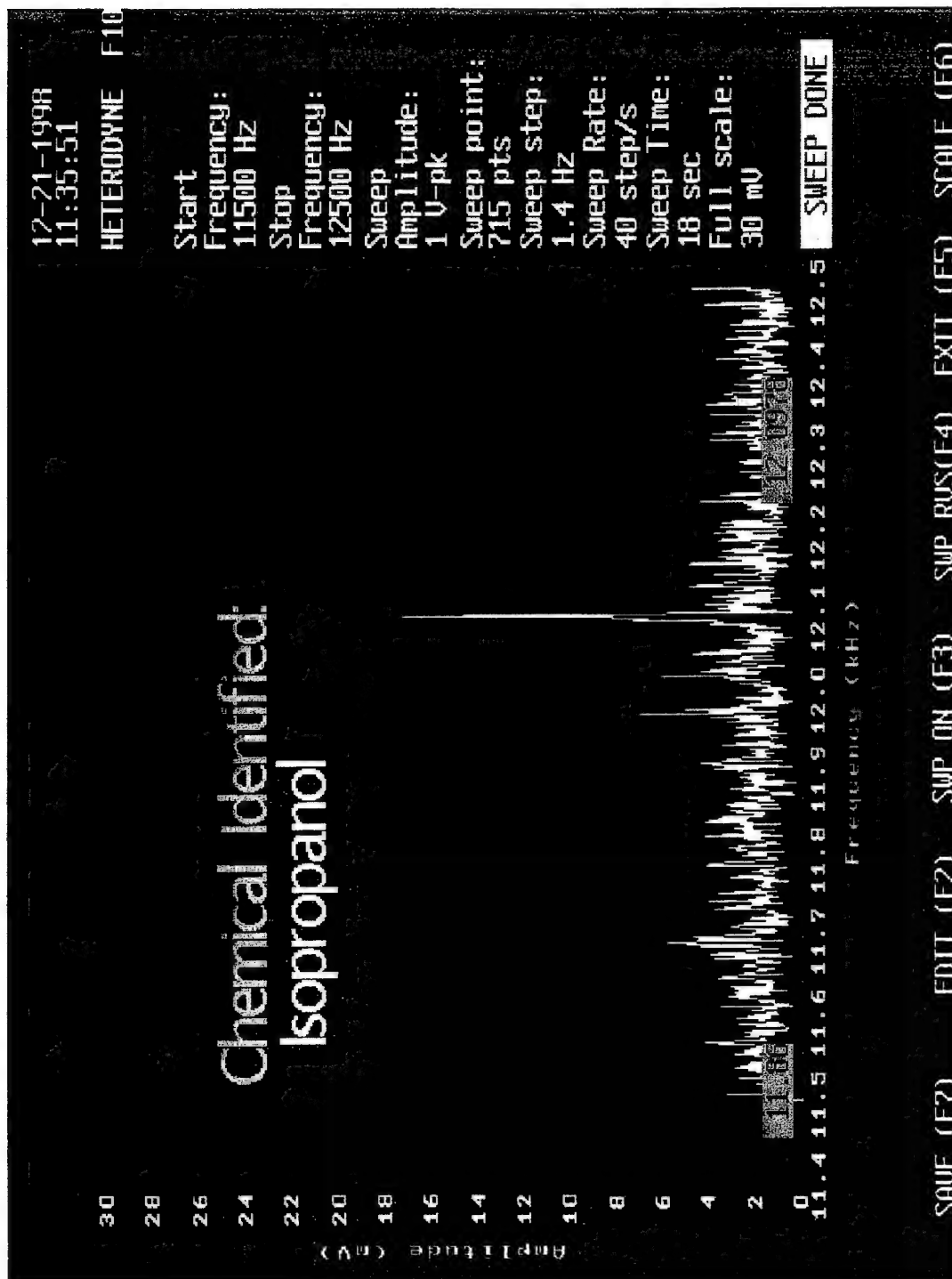


TR-16

# Resonance Spectrum of Two Liquid-filled Steel Containers



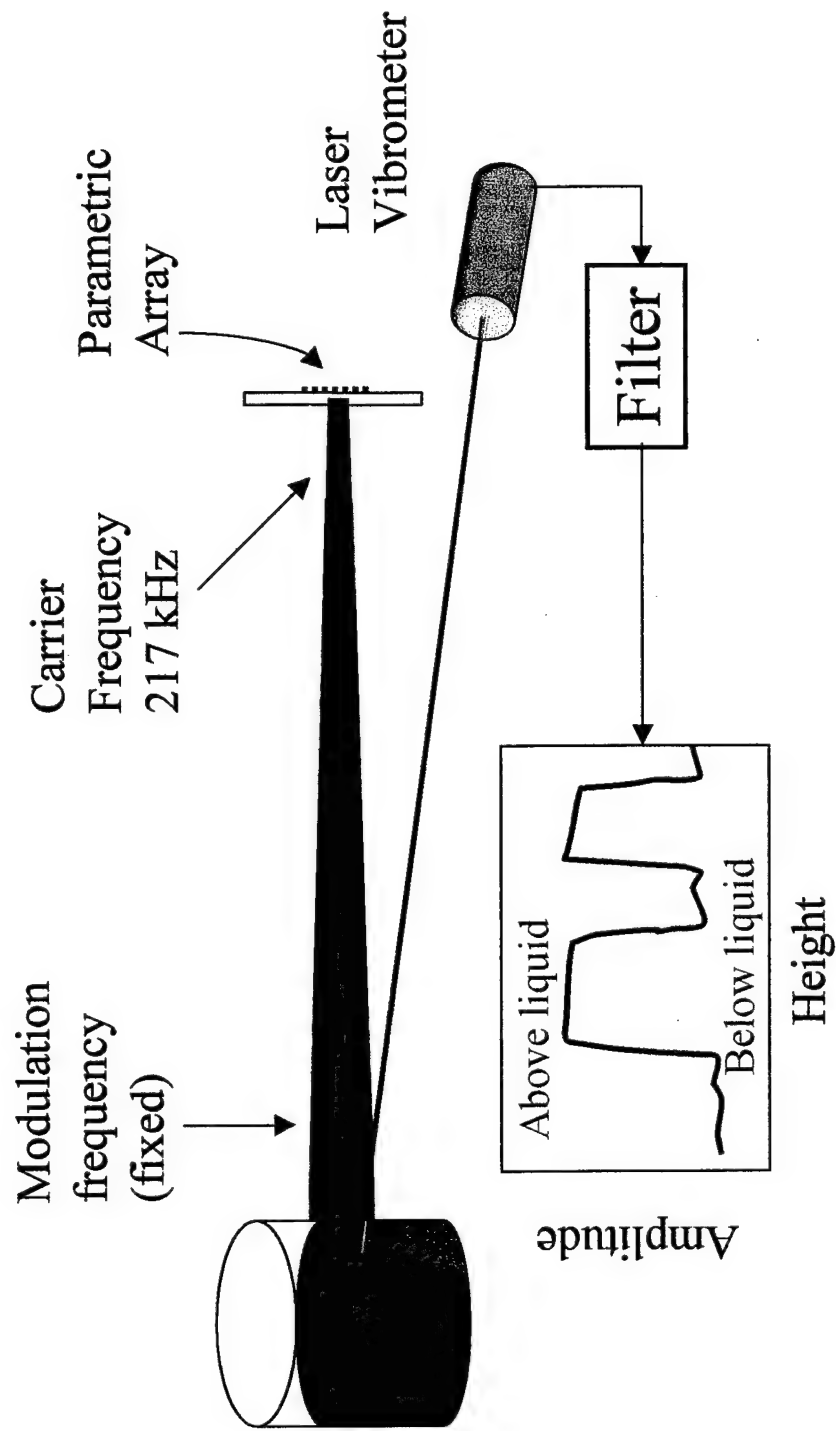
TR-17



TR-18

# Liquid Level Determination

## AM Modulated Beam



# CONCLUSIONS

- It is possible to excite guided waves in liquid-filled containers remotely using a parametric sound source
- A simple and inexpensive parametric source in air is demonstrated
- Guided wave-liquid interaction can be used to determine liquid physical property for liquid identification
- It is possible to monitor liquid level remotely
- It is also possible to determine wall integrity
- This remote monitoring approach has many applications

TR-21

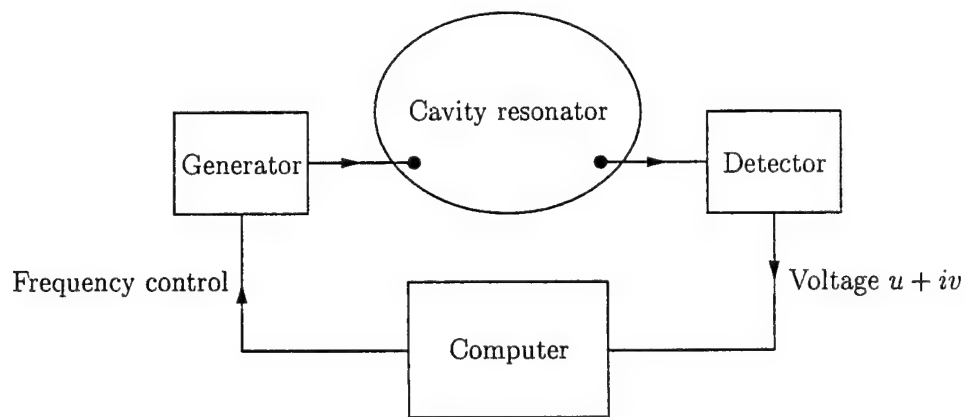
**Acoustic Gas Resonators for Measurements  
of Thermophysical Properties and  
Thermometry**

**James B. Mehl**

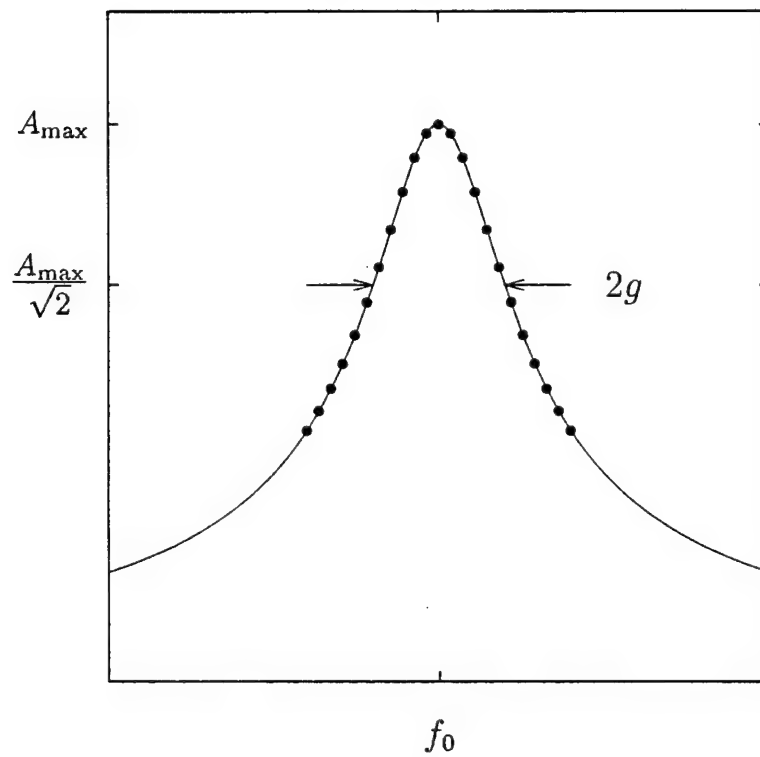
**Resonance Meeting, 1999  
University of Mississippi**

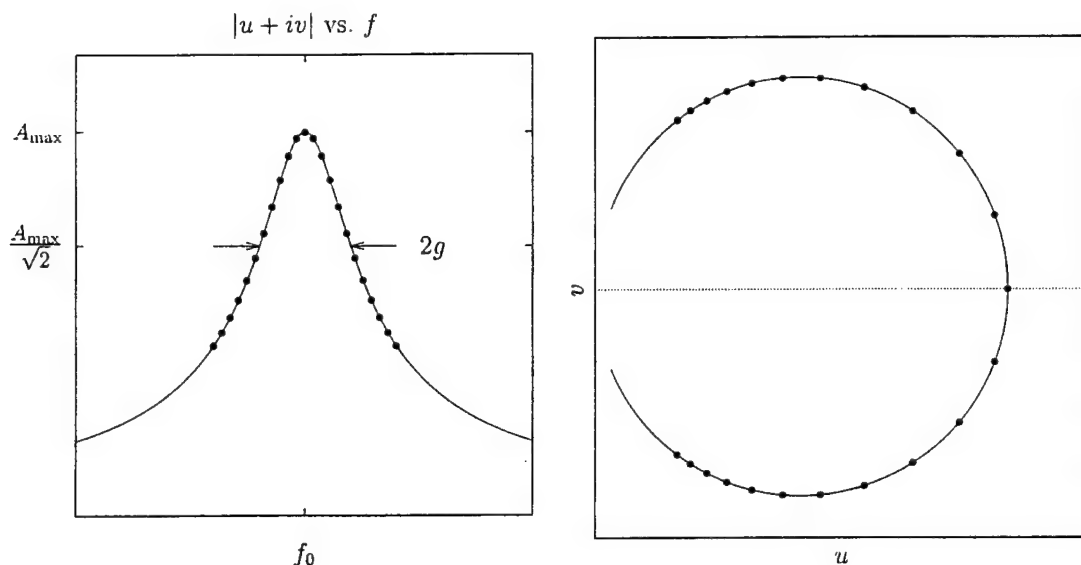
**University of Delaware**





$|u + iv|$  vs.  $f$





Acoustic pressure at point  $\mathbf{r}$  in an enclosure, with source  $S$  at point  $\mathbf{r}'$  [Morse-Ingard (9.5.14)]:

$$p = \frac{i\rho\omega S}{V} \sum_N \frac{\Psi_N(\mathbf{r})\Psi_N(\mathbf{r}')}{\Lambda_N(k^2 - K_N^2)}$$

Signal detected:

$$u + iv = \sum_N \frac{if\mathbf{A}_N}{f^2 - F_N^2}$$

For a single mode of interest:

$$u + iv = \frac{if\mathbf{A}}{f^2 - (f_0 + ig)^2} + \mathbf{B} + \mathbf{C}(f - \tilde{f}) + \dots$$

Fit: Pick  $\mathbf{F}$ ,  $\mathbf{A}$ ,  $\mathbf{B}$ , ... to minimize

$$\sum [(u_i - u(f_i))^2 + (v_i - v(f_i))^2]$$

Interesting parameters:  $\mathbf{F} = f_0 + ig$ .

$$u + iv = \frac{i\mathbf{A}f}{f^2 - \mathbf{F}^2} + \text{background}$$

Insert extra factor of  $f$  in numerator:

$$f = \mathbf{F} + (f - \mathbf{F}) = \mathbf{F} + \Delta f$$

Then

$$\begin{aligned} u + iv &= \frac{i\mathbf{A}f\mathbf{F}}{f^2 - \mathbf{F}^2} + \frac{i\mathbf{A}f\Delta f}{(f + \mathbf{F})\Delta f} + \text{background} \\ &= \frac{i\mathbf{A}'f}{f^2 - \mathbf{F}^2} + \frac{i\mathbf{A}\left(1 + \frac{\Delta f}{\mathbf{F}}\right)}{1 + \frac{\Delta f}{2\mathbf{F}}} + \text{background} \\ &= \frac{i\mathbf{A}'f}{f^2 - \mathbf{F}^2} + \text{modified background} \end{aligned}$$

Similar relations will handle, *e.g.*

$$\mathbf{A}(f) = \mathbf{A}(\mathbf{F}) + \frac{d\mathbf{A}}{df}\Delta f + \dots$$

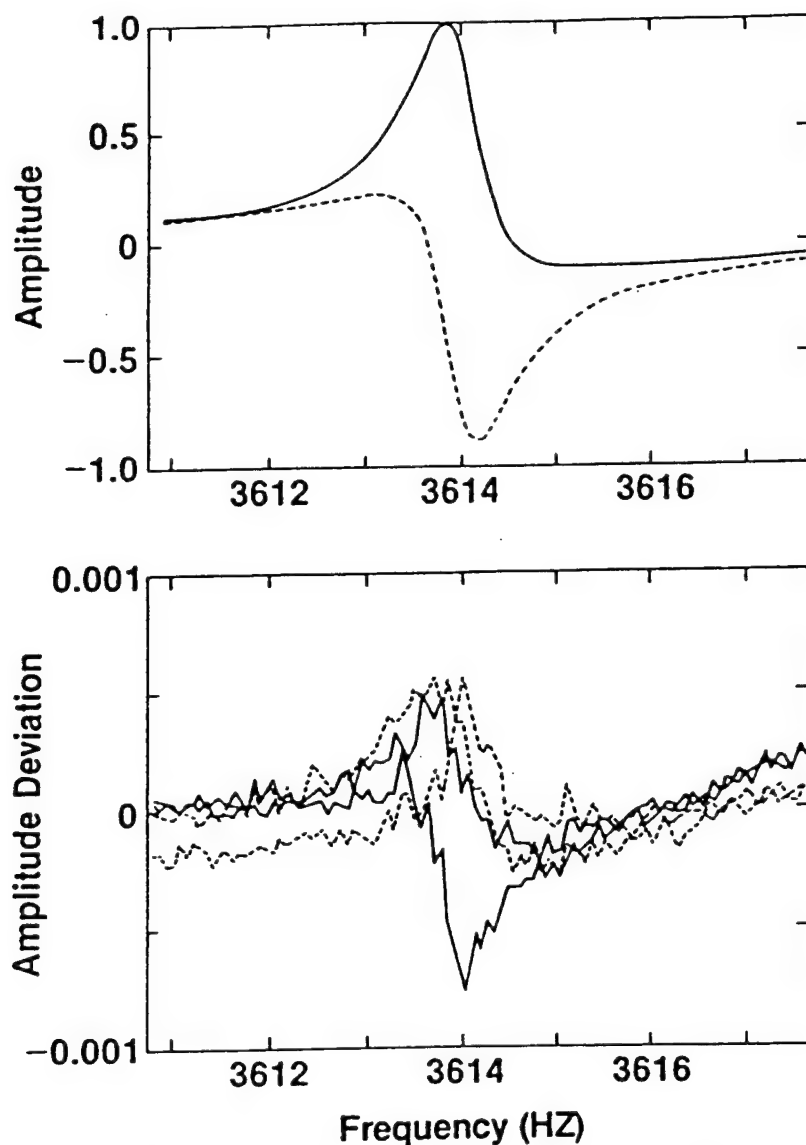


FIG. 4. Top: In-phase (solid curve) and quadrature (dashed curve) voltages from the detector as a function of frequency near the (0,2) resonance in argon at 0.4032 MPa and 296.309 K. Bottom: Measured voltages minus calculated voltages [Eq. (75)] with the fitted parameters  $f_{02} = 3613.9970$  Hz,  $g_{02} = 0.4275$  Hz,  $A = 0.7535 - 0.5831i$ ,  $B = 0.0048 + 0.0068i$ , and  $C = 0.0001 + 0.0001i$ . Data were taken at intervals of 0.07 Hz, first from the lowest to the highest frequency and then back to the lowest frequency. Separate deviation plots for the upward and downward frequency sweeps show the effects of drifts in the lock-in amplifier.

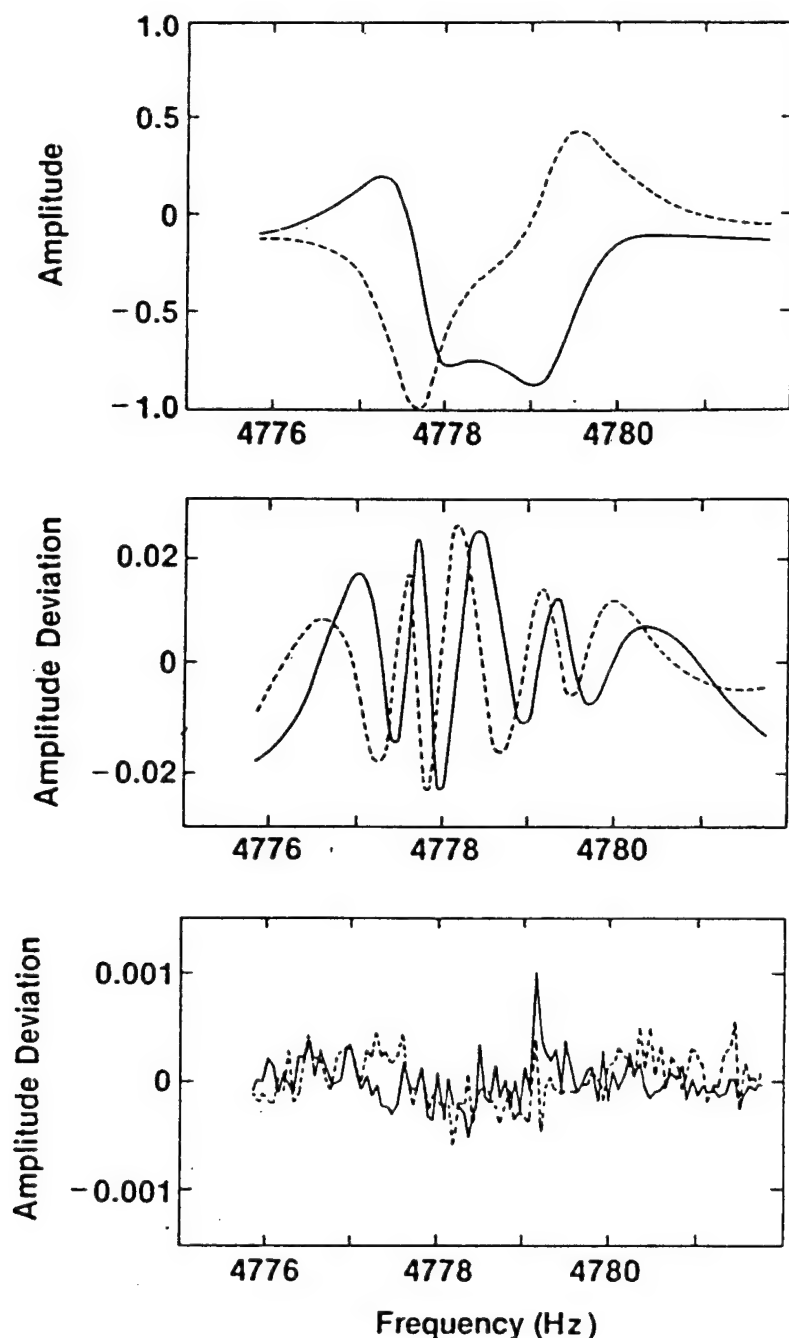
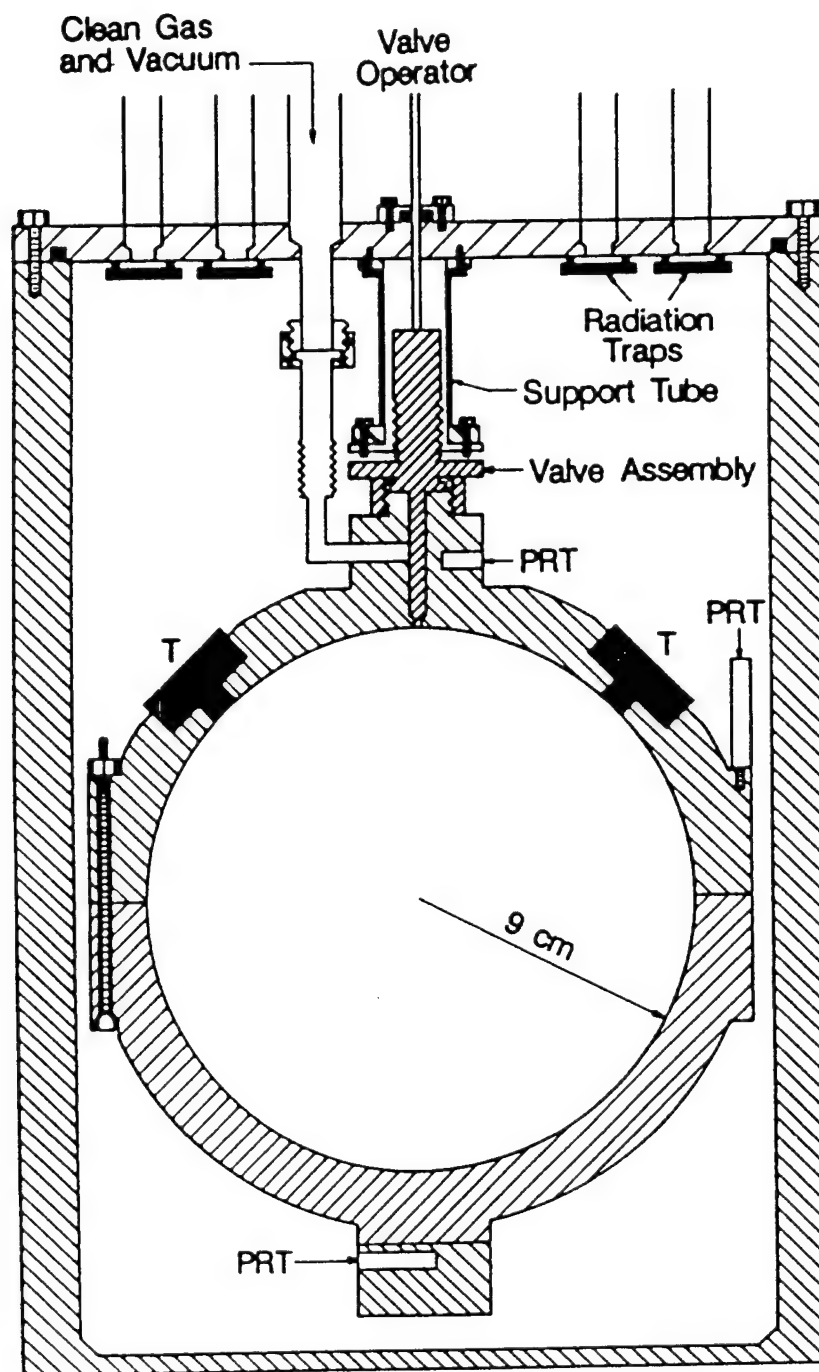


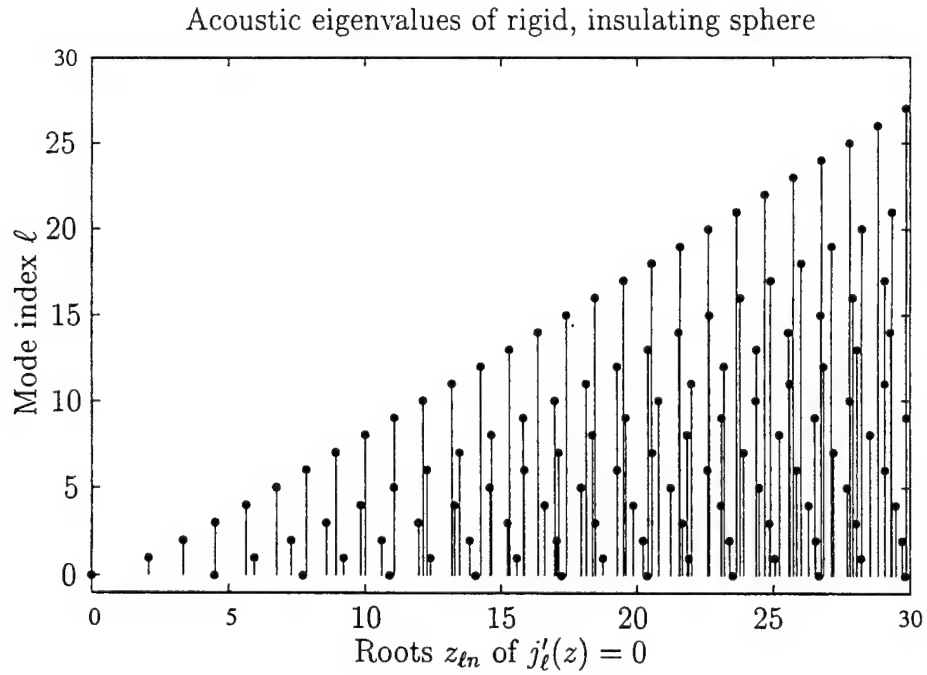
FIG. 5. Top: In-phase (solid curve) and quadrature (dashed curve) voltages from the detector as a function of frequency near the (1,2) resonance in argon at 0.4032 MPa and 296.309 K. Middle: Measured voltages minus two-resonance trial function. Note that the deviations are systematic although the trial function has twelve parameters [eight parameters specify resonances at 4777.63 and 4779.38 Hz, and four specify the constant and linear background terms in Eq. (75)]. Bottom: Measured voltages minus fitted function. The fitted function has sixteen parameters [twelve parameters specify resonances at 4777.693, 4777.903, and 4779.351 Hz with half-widths of 0.550, 0.546, and 0.555 Hz. The remaining four parameters specify  $B$  and  $C$  in Eq. (75)].

TR-5



**Figure 5.** Cross-section of resonator and pressure vessel. The transducer assemblies are indicated by "T," and the locations of the capsule thermometers are indicated by "PRT."

TR-6



A gas filled, acoustic resonator with rigid, thermally insulating walls, has acoustic resonances at the frequencies

$$f_{ln} = \frac{z_{ln}c}{2\pi a} \approx (0.577 \text{ kHz})z_{ln},$$

where the numerical value is for argon at room temperature ( $c \approx 322\text{m/s}$ ) in the 8.89 cm-SS spherical resonator. The acoustic pressure in the sphere is proportional to

$$j_l(z_{ln}r/a)Y_{ln}(\theta, \phi).$$

The modes are  $(2\ell + 1)$ -fold degenerate for perfect spherical geometry.

## **Theory of spherical acoustic resonator**

Temperature, pressure, particle velocity fields

PDEs (Kirchhoff 1868)

Navier Stokes: longitudinal

Navier Stokes: transverse

Fourier: Heat flow

Conservation of mass

Conservation of energy

4th order PDE in temperature

Prediction: two wave modes

$\Delta p$ ,  $\Delta T$ , particle velocity,  $\propto e^{i\omega t}$

acoustic mode ( $k_p \approx u/\omega$ )

thermal mode ( $k_t \approx (1 - i)/\delta_t$ )

### **Boundary conditions at gas-shell boundary:**

velocity of gas = velocity of shell

heat flow continuous

temperature continuous (hydrodynamic region)

### **Shell motion:**

Classical elastodynamic theory exactly soluble

for isotropic, isolated, spherical shell

### **Complete problem:**

**Exactly soluble for spherical geometry**



## Acoustic experiment

1. Measure resonance frequencies  $f_{lnm}$  and half-widths  $g_{lmn}$
2. Correct for
  - (a) thermal boundary layer
  - (b) viscous boundary layer (non-radial modes)
  - (c) shell motion
  - (d) imperfect geometry
3. Results:

$$f_{lnm} = \frac{uz_{ln}}{2\pi a} + \Delta f_t + \Delta f_v + \Delta f_{sh} + \Delta f_{geom}$$

$$g_{lmn} = g_t + g_v + g_{bulk}$$

Thermal and viscous boundary layer effects

$$\frac{\Delta f_t + ig_t}{f} = -\frac{\gamma - 1}{2} \frac{\delta_t}{a} \frac{1}{1 - l(l+1)/z_{ln}^2}$$

$$\frac{\Delta f_v + ig_v}{f} = -\frac{\gamma - 1}{2} \frac{\delta_v}{a} \frac{l(l+1)}{1 - l(l+1)/z_{ln}^2}$$

$$\delta_t = \sqrt{\frac{D_t}{\pi f}}; \quad \delta_v = \sqrt{\frac{D_v}{\pi f}}$$

Boundary shape perturbation:

$$r = a [1 - \epsilon \mathcal{F}(\theta, \phi)]$$

$\epsilon$  = deformation parameter.

For deformations that conserve the volume:

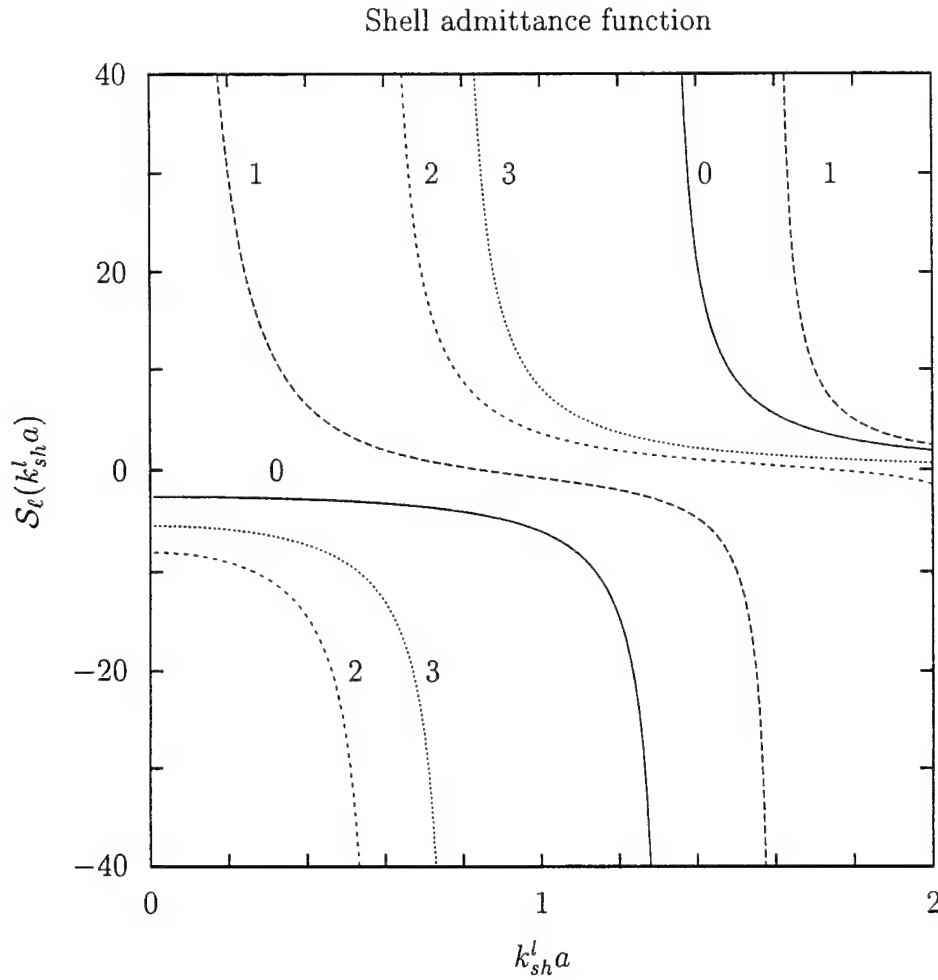
$$\left( \frac{\Delta f}{f} \right)_{0n}^{\text{geom}} = O(\epsilon^2)$$

$$\left( \frac{\Delta f}{f} \right)_{\ell n}^{\text{geom}} = O(\epsilon)$$

For nonradial acoustic and electromagnetic modes:

Average over  $2\ell + 1$  components of a multiplet.

$$\left\langle \frac{\Delta f}{f} \right\rangle_{\ell n}^{\text{geom}} = O(\epsilon^2)$$



Shell admittance function for the stainless-steel resonator with  $a = 8.89$  cm and  $b/a = 1.215$  ( $a$  = inner radius,  $b$  = outer radius). The curves are labeled with the mode index  $\ell$ , for acoustic pressures proportional to  $P_\ell(\cos \theta)$ . The fractional frequency perturbation due to shell motion is

$$\left( \frac{\Delta f}{f} \right)_{\ell n} \approx \frac{\rho_g c_g^2}{\rho_s c_{s,l}^2} \frac{S_\ell(k_{sh}^l a)}{1 - \ell(\ell + 1)/z_{\ell n}^2}.$$

Here  $k_{sh}^l = \omega/c_{sh,l}$ , with  $c_{sh,l}$  the longitudinal speed of sound in the shell.

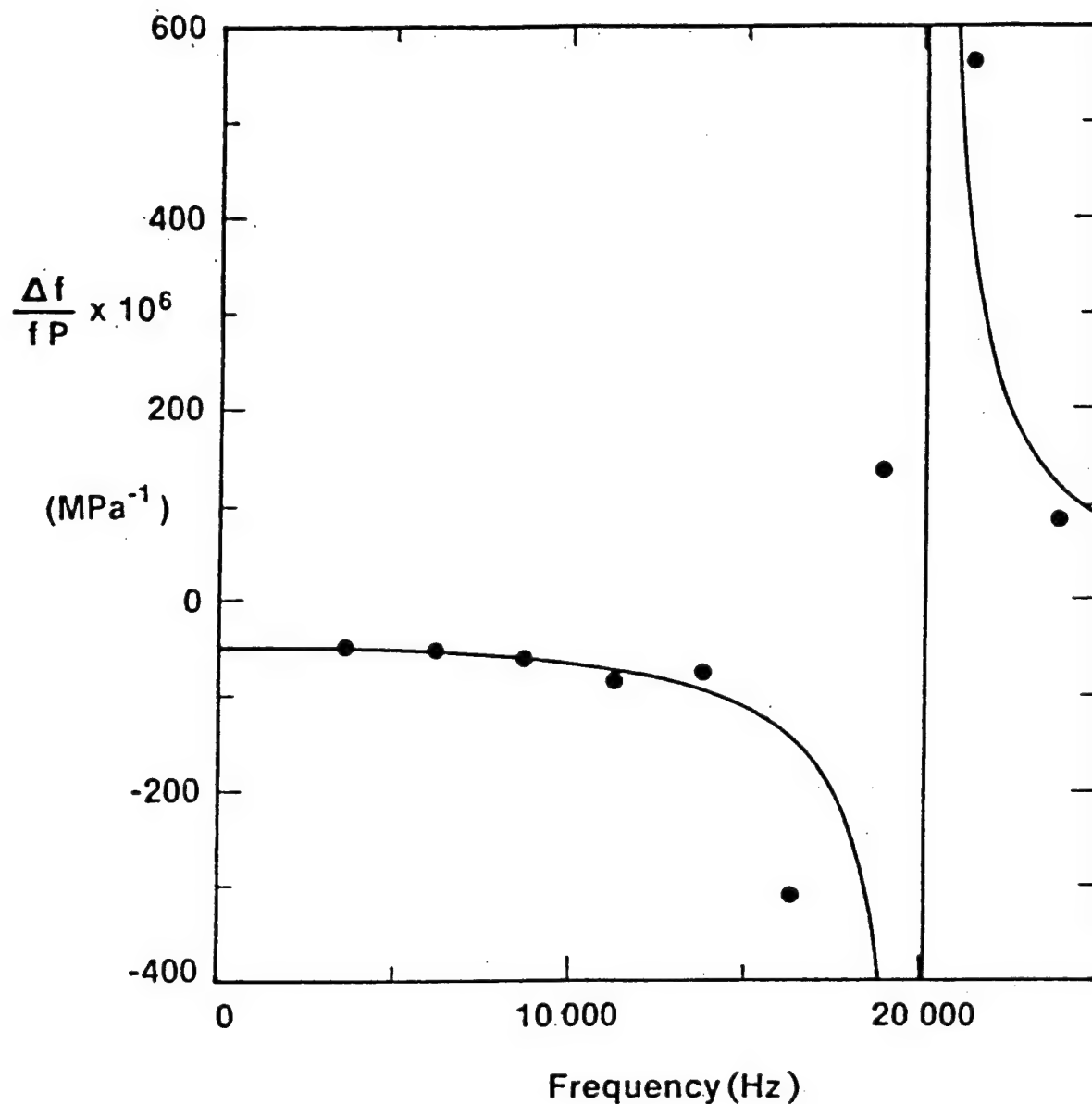


FIG. 11. Response of the shell to radially symmetric excitation as a function of frequency. The points are the average slopes of the curves in Fig. 9. The curve is calculated for an isotropic seamless shell using the theory of elasticity and the elastic constants tabulated for aluminum (see Table V). The idealized shell has a breathing resonance near 20.2 kHz.

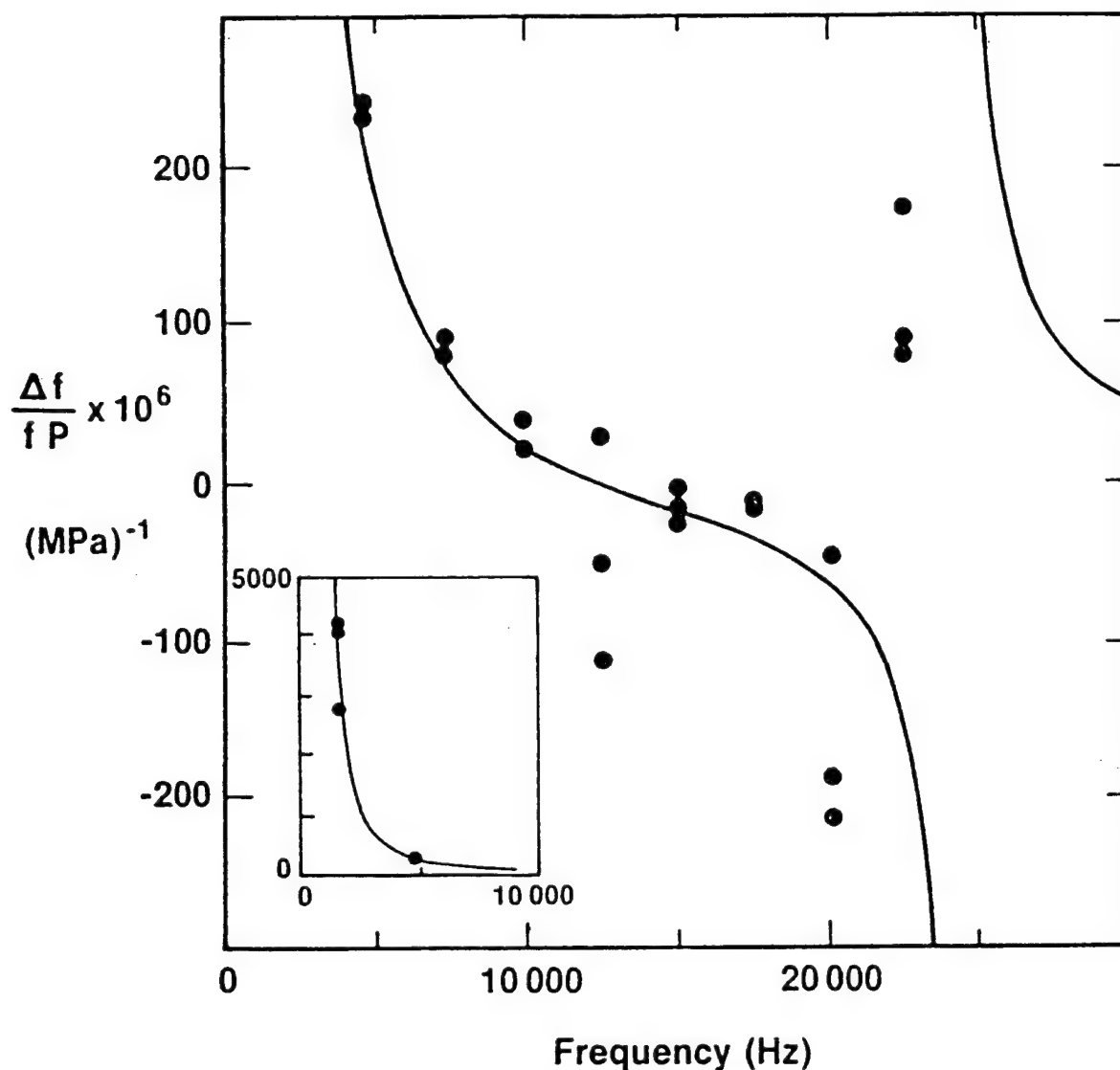


FIG. 12. Elastic response of the shell to excitation with symmetry of  $Y_{1m}(\theta, \phi)$  as a function of frequency. The points are the average slopes of curves such as those shown in Fig. 10. The curve is calculated for an isotropic, seamless shell using the theory of elasticity and the elastic constants tabulated for aluminum. The idealized shell has  $Y_{1m}$  resonances at 0 and 24.5 kHz.



**Sampled Continuous Wave  
measurement on single crystals of  
YBCO and BSCCO.**

**Debashis Dasgupta  
Jeff Feller**

**Carsten Hucho  
Moises Levy  
Bimal K Sarma**

**Resonance Meeting, 1999  
Univ. of Mississippi**

**Graduate School, UW-Milwaukee  
Physics Department, UW-Milwaukee  
Office of Naval Research**

**Small size of HTSC single crystals.  
(typically ~ 1000 X 800 X 200 microns)**

**Objective:**

**Develop an ultrasonic technique for  
investigating properties of these small  
HTSC crystals.**



**Sampled CW Technique**



# **OUTLINE**

## **Type-II Superconductors**

**Essentials (Magnetic phase diagram)**

**Flux lines and Pinning sites**

**Description of Sampled CW Setup**

**Experimental Results On YBCO**

**Untwinned Crystal**

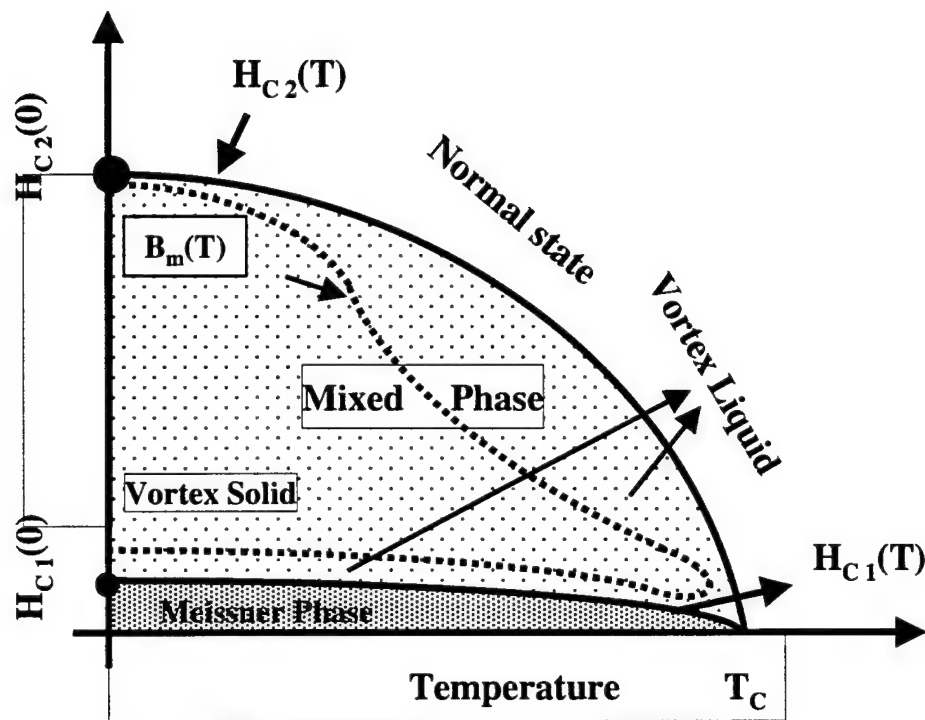
**Twinned Crystal**

**➡ Susceptibility Results (Twinned)**

**Experimental Results on BSCCO**

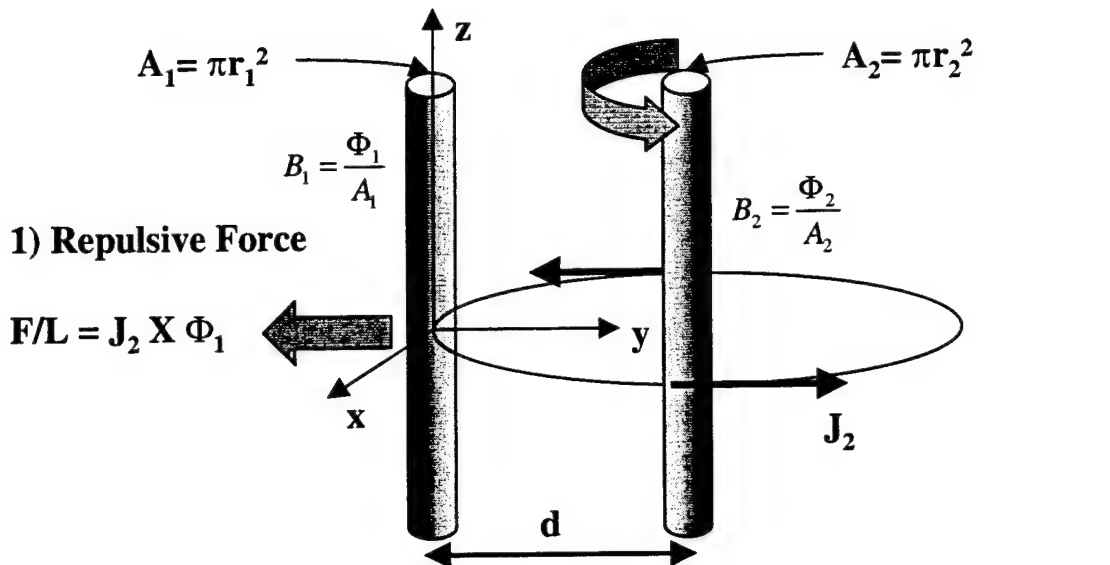
**Conclusions**

**Future**



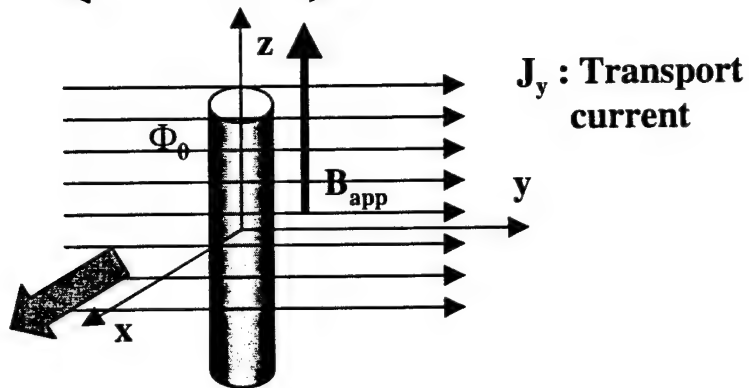
A schematic of complex magnetic phase diagram of a Type-II superconductor. Some of the regimes shown are magnified for clarity.

## Forces acting on flux lines:



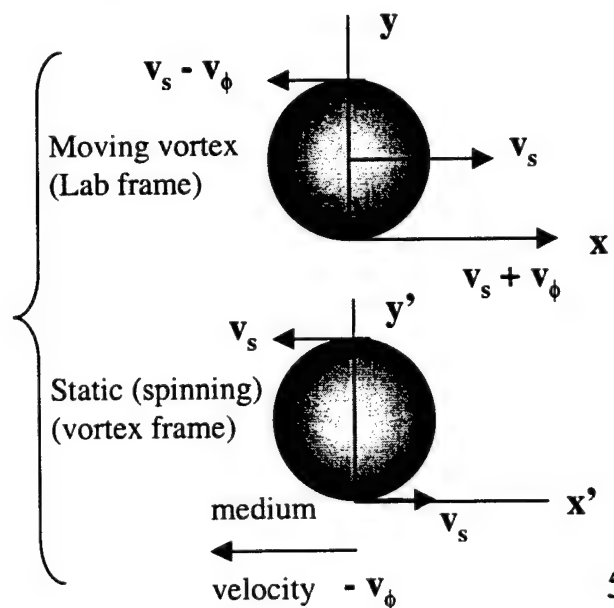
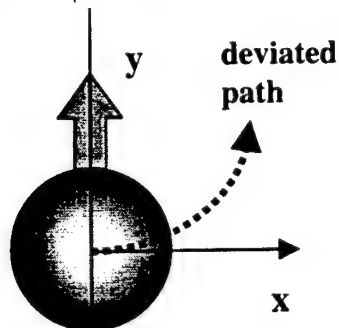
## 2) Lorentz Force

$$F = J \times B$$



## 3) Magnus Force

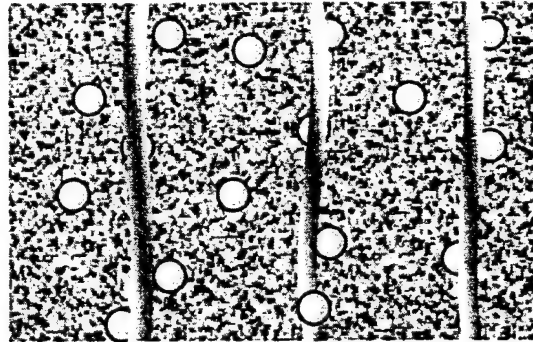
$$F_M \sim -v_\phi \times \Phi_0$$



## Pinning sources for flux lines

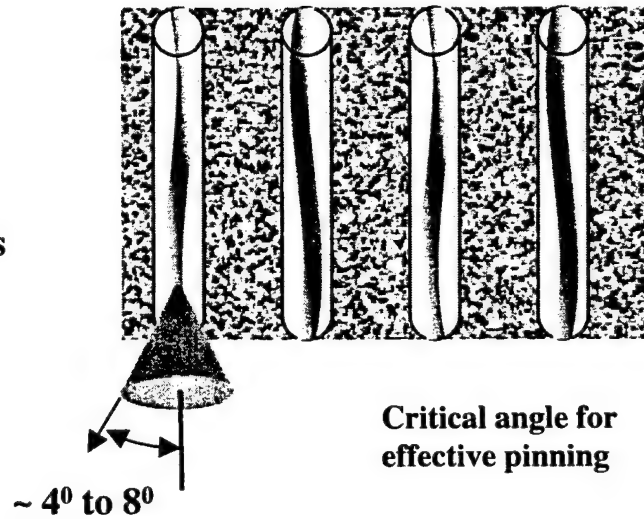
### Point defects

- Oxygen Vacancies
- Electron Irradiation induced point defects



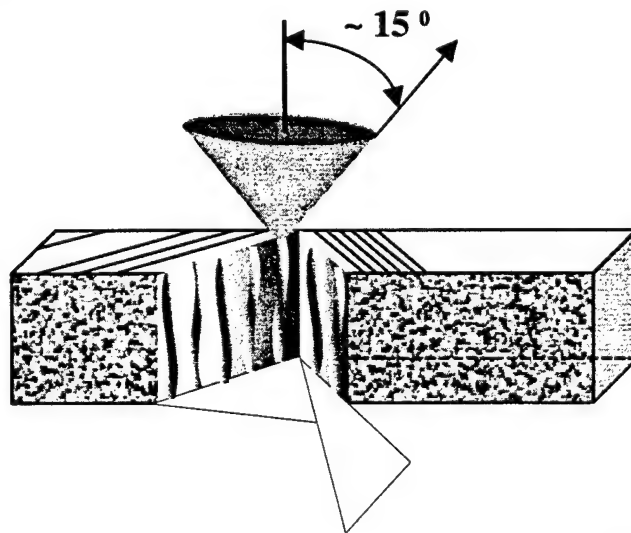
### Line Defects

- Heavy ion irradiation induced columnar tracks

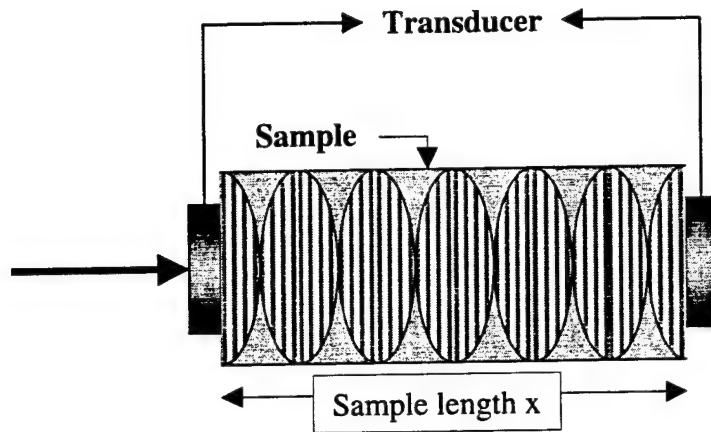


### Planar Defects

- Twin Boundaries
- Layered Structure

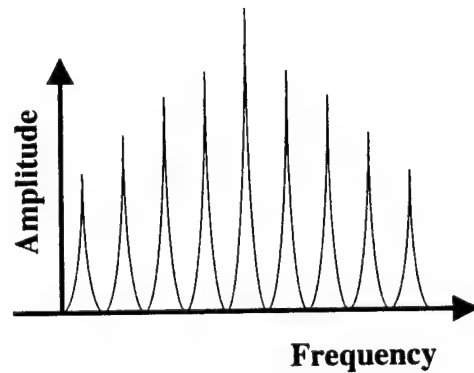


## Continuous Wave Technique

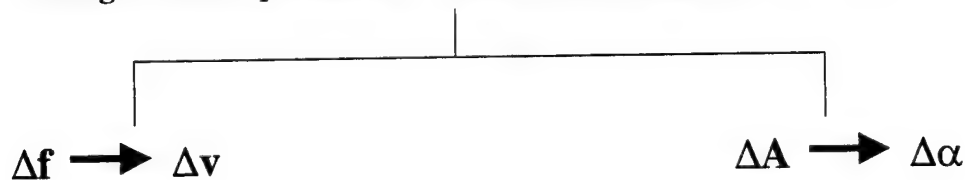


- Setup standing wave resonances

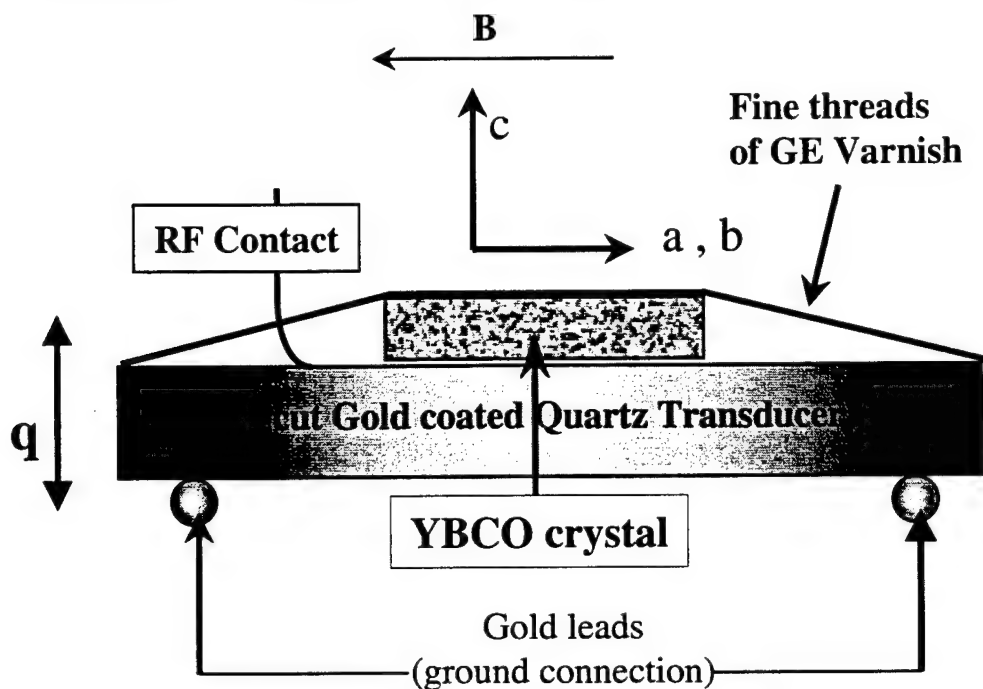
$$Q_m = \frac{\omega_m}{\Delta\omega}$$



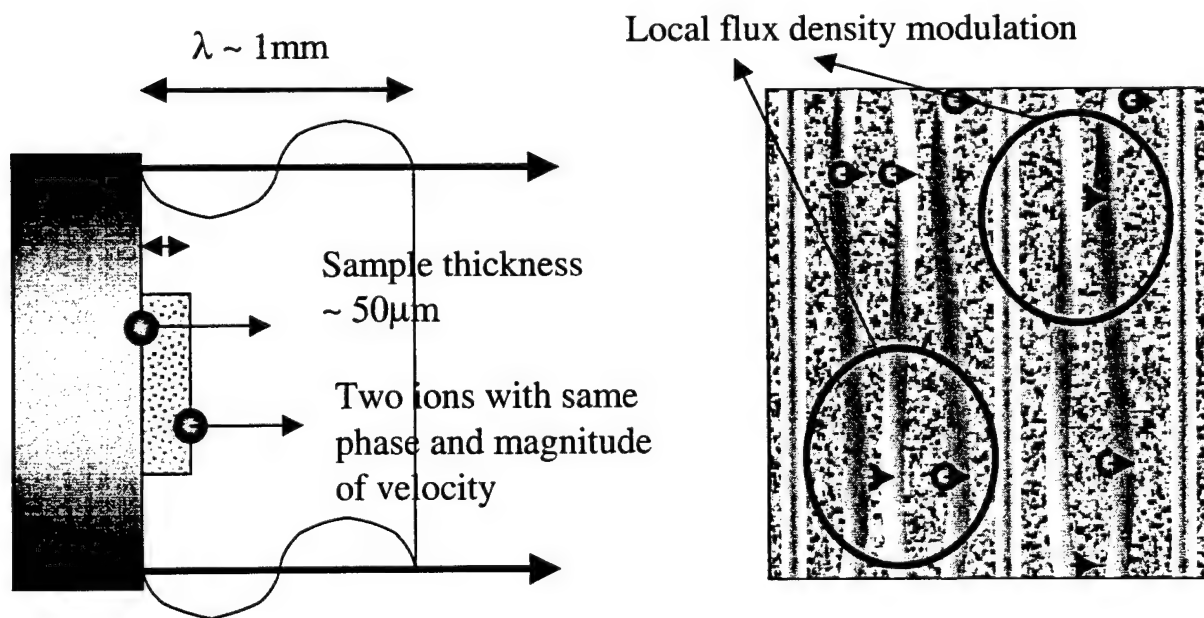
Lock in to mechanical resonance  
Change external parameters (Temperature, Magnetic Field ... )

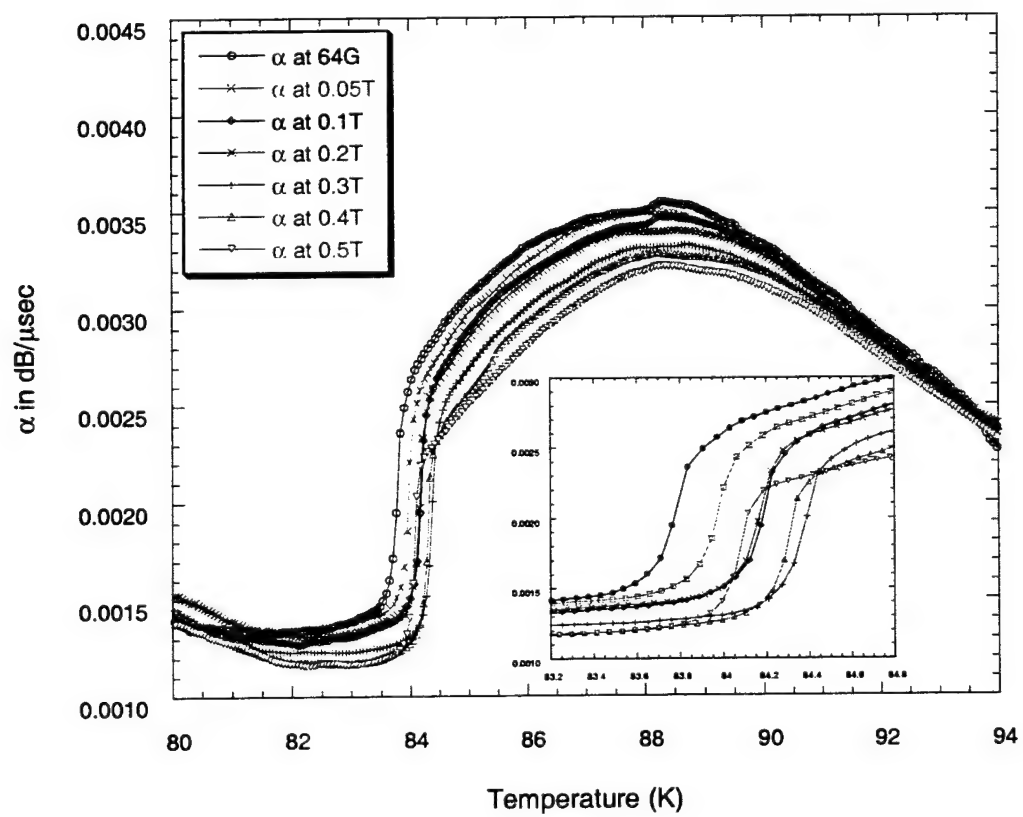


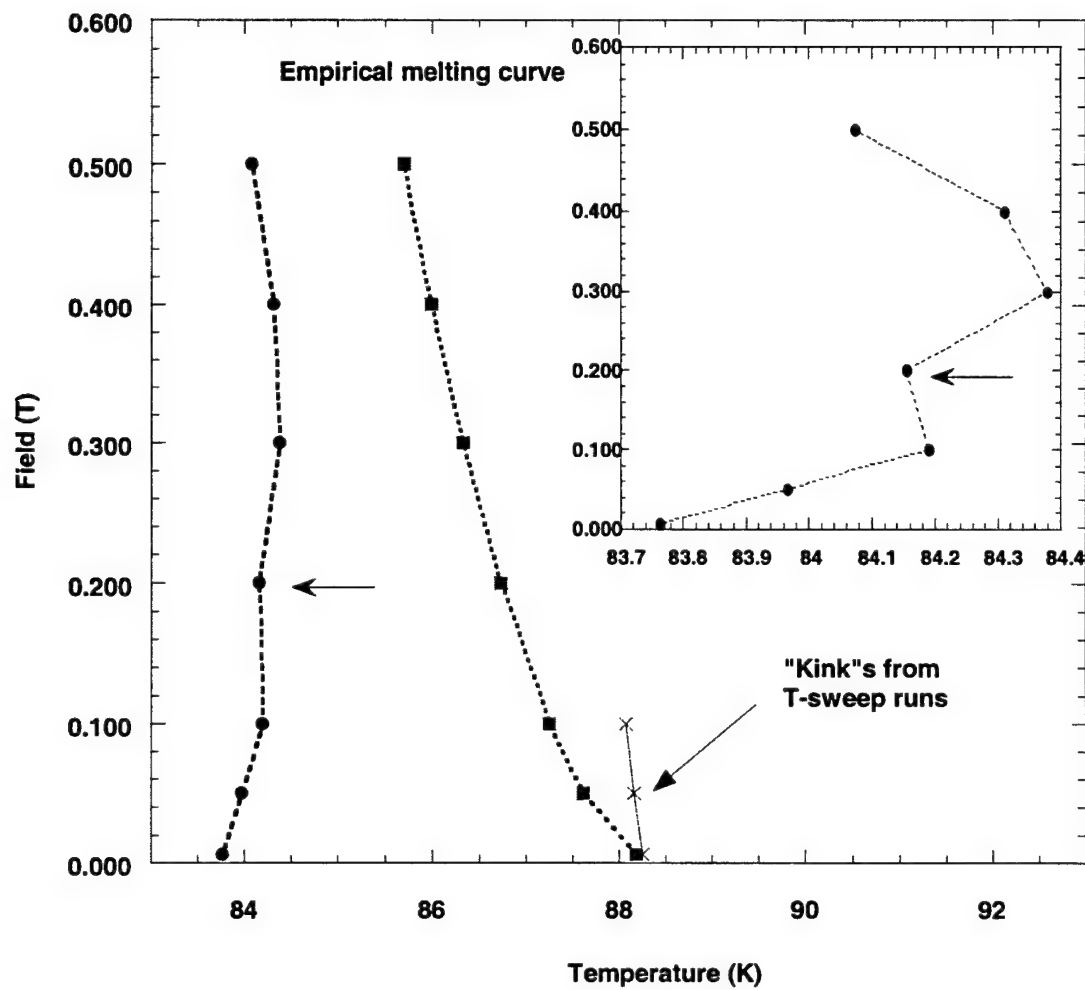
# Sampled CW Experimental setup: transducer + crystal



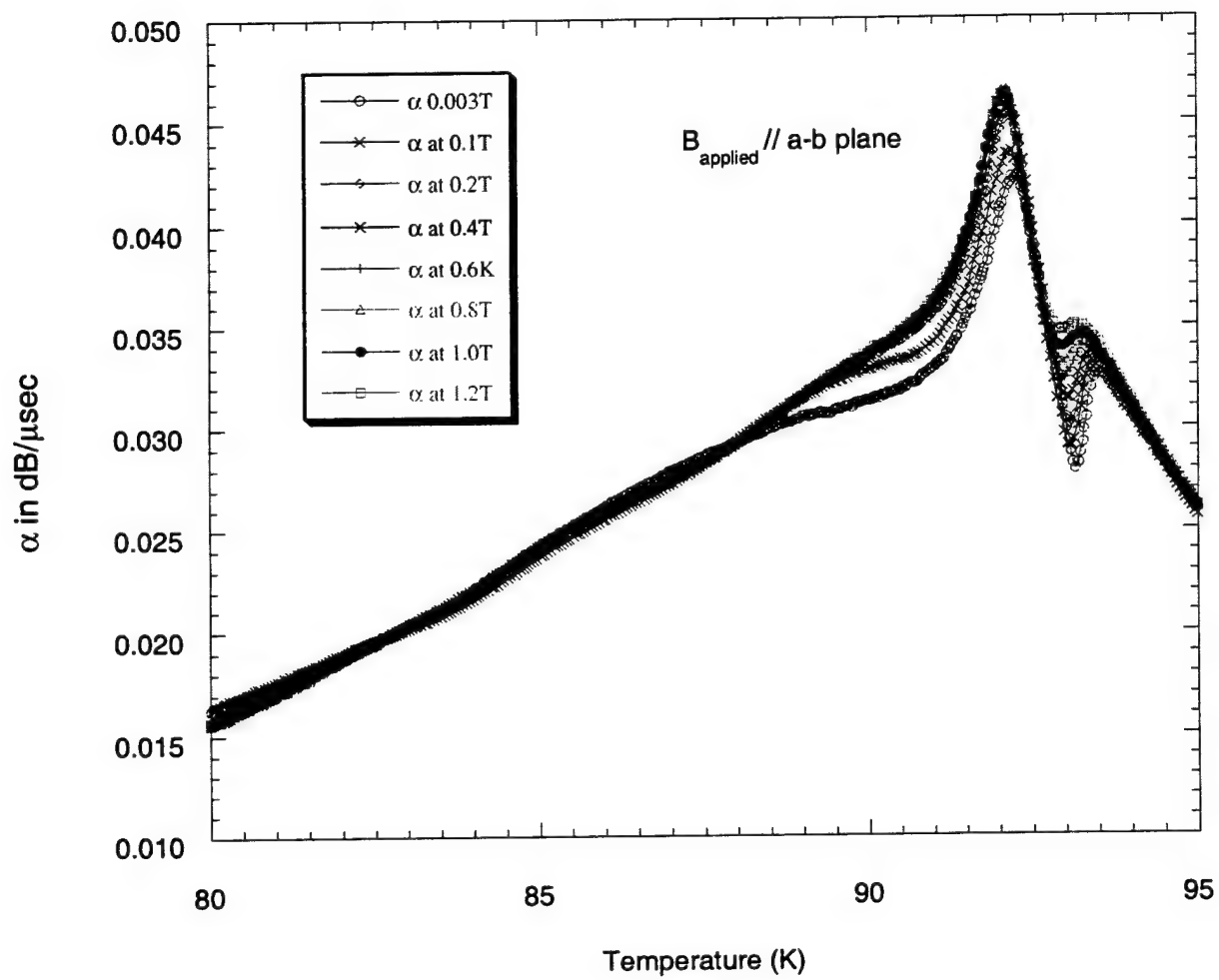
$\lambda \gg$  thickness of the sample assures no displacement gradient for the ions  
 sample thickness:  $\sim 50\mu\text{m}$  obtained through polishing (optical microscope for thickness)  
 sound velocity  $c: \sim 5000 \text{ m/s}$ , frequency:  $5\text{MHz} \Rightarrow \lambda \approx 1 \text{ mm} \gg$  sample thickness

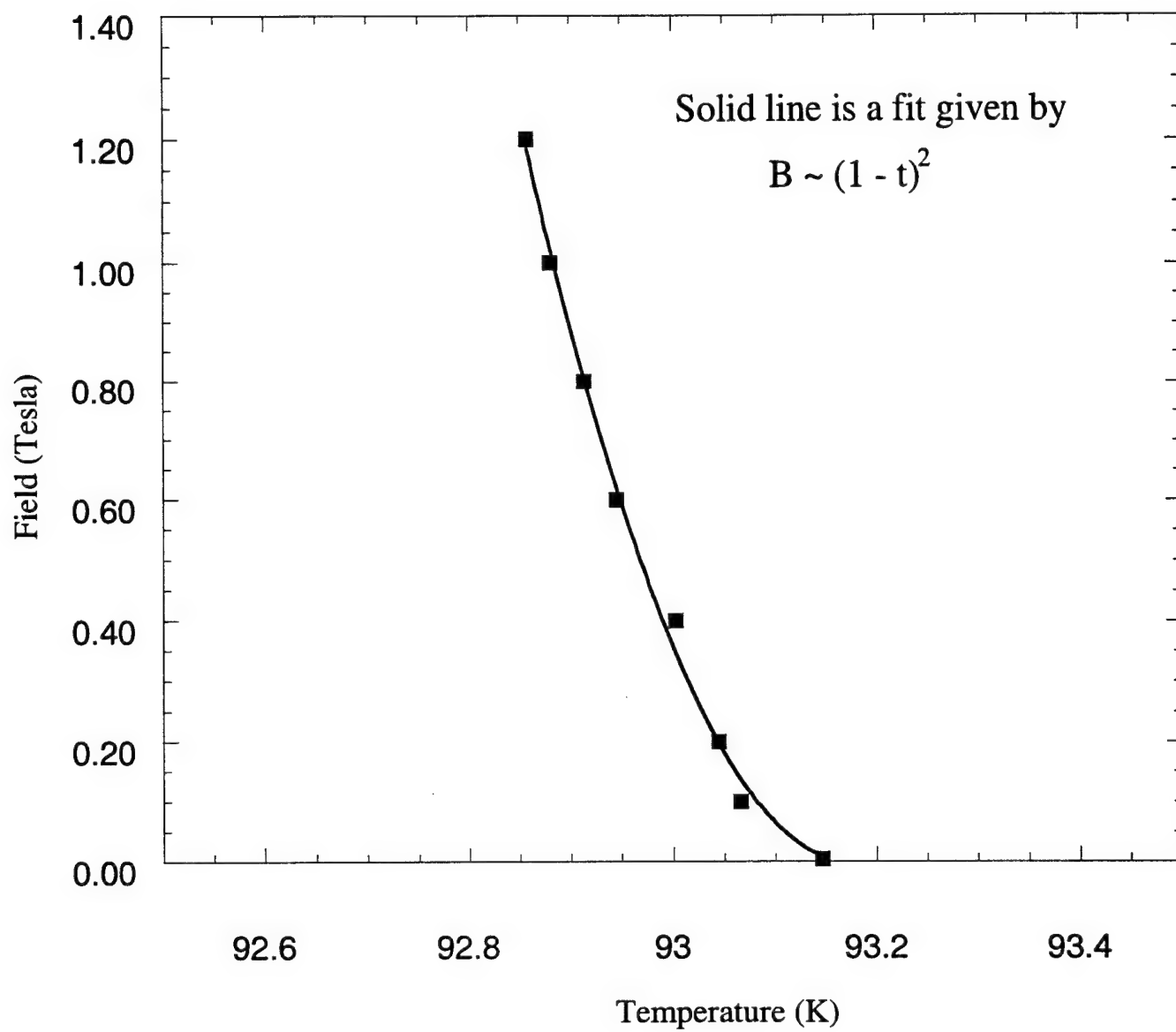




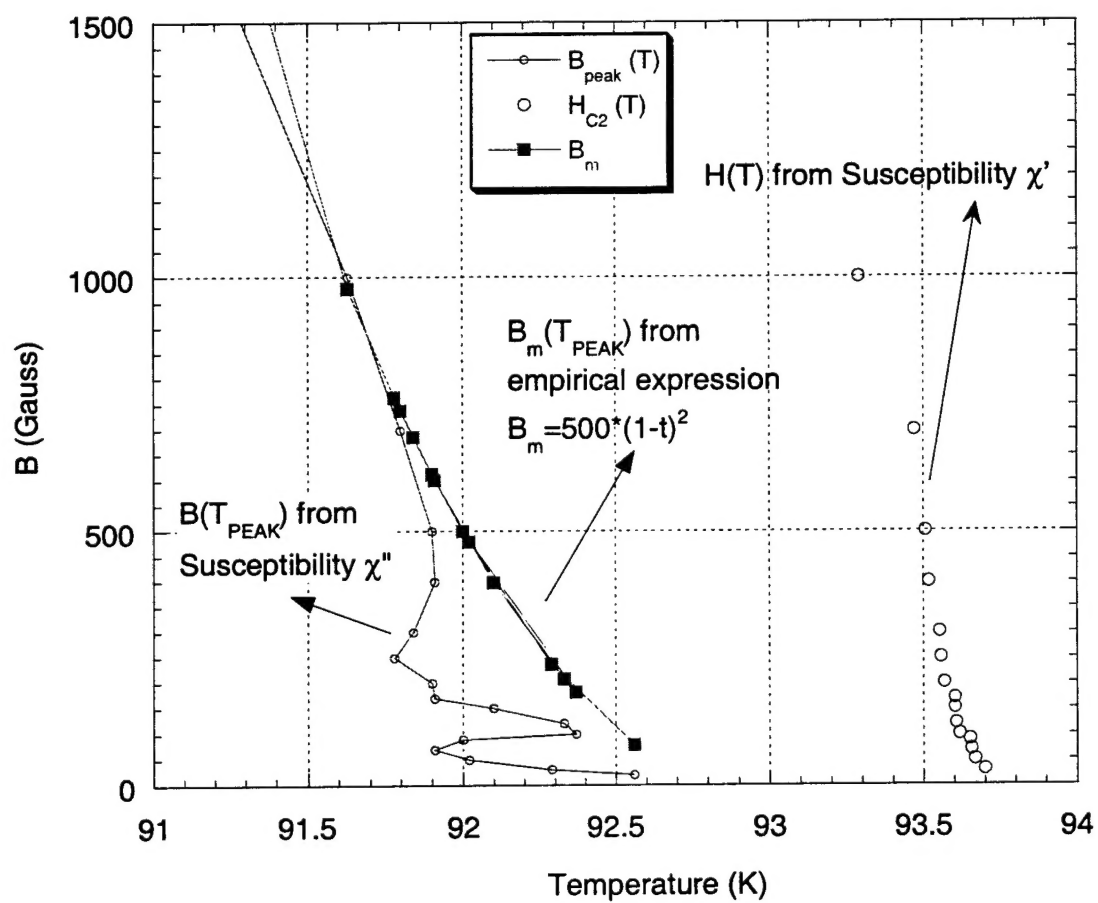


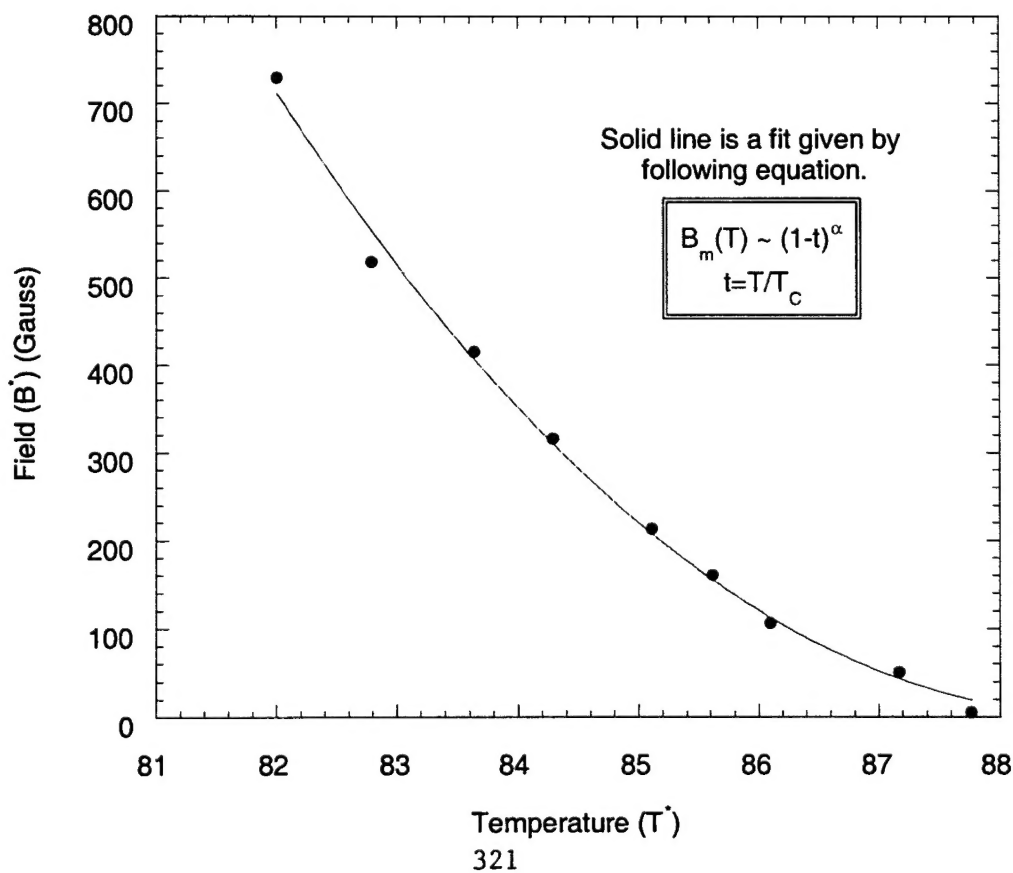
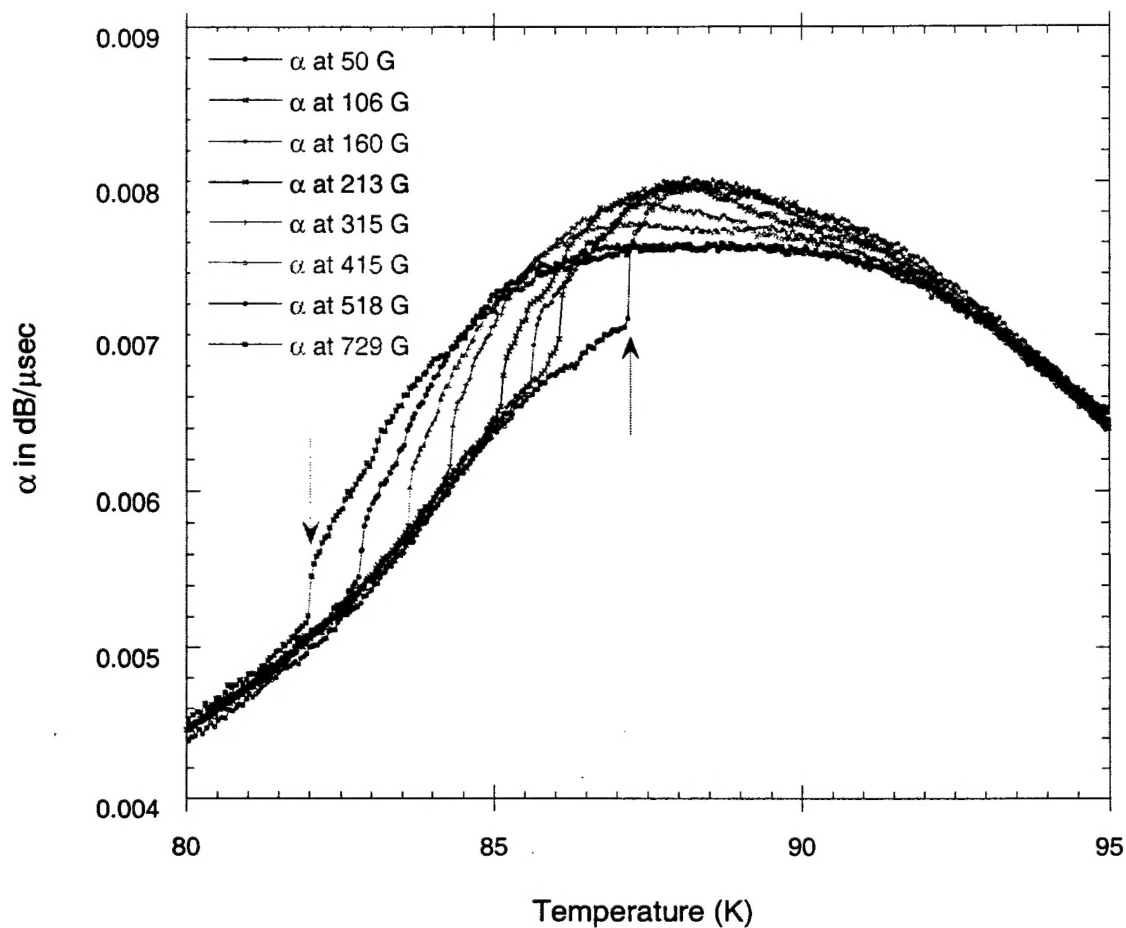






$H_{C2}$  and melting curves for a twinned YBCO crystal.  
(  $H \parallel c$ -axis)





## **Conclusions:**

### **Sampled CW:**

- “The ultrasonic technique”
- Minimum external perturbations → thermodynamic measurements
- Bulk elastic properties of flux lines
  - melting of flux lines in YBCO and BSCCO
  - soft lattice at low fields
  - rigid lattice at higher fields
  - possibly soft-rigid contour follows the melting curve with 0.3T as a turnaround field
- Sensitive to the “depinning” transition
  - change in activation energy around 0.3T for YBCO
- Basic electron-phonon interaction  
SC transition at  $H_{C2}$

### **Susceptibility:**

- Peak effect for YBCO
- Re-entrant nature of the peak effect has been observed

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